Combinatorics, nCr, nPr & Pascal's Triangle

Number of arrangements	Explanation	Example
$n!$	Number of distinct ways of arranging letters A,B,C are ABC, ACB, BAC, BCA, CAB, CBA i.e. **6** = 3 x 2 x 1 = **3!** Number of distinct ways of arranging *n* letters A,B,C ..... are ***n*!** These are the number of ***permutations*** of a set of *n* distinct objects.	How many permutations are there of ABCDEFG? 7! = **5040**
$\frac{n!}{r!}$	If out of *n* objects, *r* are **repeated**, we must divide the total number of permutations by *r*! Why? Write out *n*! permutations of *n* objects, with *r* marked, into a *r!* x *n! /r!* grid. Each *row* corresponds to a different permutation of the marked objects, and each *column* corresponds to a different permutation of the unmarked objects. If *r* marked objects are the same, each row is identical. Hence the total number of distinct permutations is *n!/r!*	Permutations of VOODOO are 6!/4! = 6 x 5 = **30**
$\frac{n!}{r!k!....}$	Consider *n* objects with *r* repeats of one object, *k* repeats of another etc. Following the same 'gridding' ideas as above, we must successively divide the number of permutations by the factorial of the repeats	Permutations of **MISSISSIPPI** (M x 1, I x 4, S x 4, P x 2, total 11 letters) are 11! / (4! x 4! x 2!) = **34,650** Permutations of **RADAR** are 5!/(2!2!) = **30** Permutations of **BAZOOKA** are 7!/(2!2!) = **1260**
$\frac{n!}{(n-r)!r!}$	Consider *n* objects comprising of *r* repetitions of one object and *n-r* repetitions of another object. i.e. only two types of object. **NOTE THIS IS ALSO ELEMENT *(n,r)* OF PASCAL'S TRIANGLE**	How many ways can a tennis player win three matches out of ten fixtures? Assume no draws are allowed. (So 3 wins and 7 losses). 10!/(7! x 3!) = 120
${}^nP_r = \frac{n!}{(n-r)!}$	Consider the number of permutations of a ***subset*** of *r* letters from an alphabet of *n*. The number of ways of marking *r* letters for inclusion in the subset is $n!/(n-r)!r!$ The number of permutations of *r* letters is *r!* so the total number of permutations is $\frac{n!}{(n-r)!r!} \times r! = \frac{n!}{(n-r)!}$	Ginger Twos has 16 flavours of ice cream. How many *permutations* of three *distinct* flavours can we have? 16! / (13! x 3!) = 16 x 15 x 14 = **3360**
${}^nC_r = \frac{n!}{(n-r)!r!}$	The number of ***combinations*** of *r* objects from the list of *n* is when the order doesn't matter. i.e. A,B,C is the same combination as B,A,C as it contains the letters A, B and C. In this case we have *r*! permutations for each set of *r* objects and therefore the total number of combinations is ${}^nP_r / r! = n!/(n-r)!r!$	How many *combinations* of two *distinct* flavours are there to be tried at Ginger Twos? 16!/ / (14! x 2!) = 16 x 15 / 2 = **120** The number of combinations of four letter 'words' from an alphabet of 26 letters is 26!/(22! x 4!) = 26 x 25 x 24 x 23 / (4 x 3 x 2) = **14,950**