19.2 Transforming Quadratic Functions

Essential Question: How can you obtain the graph of $g(x) = a(x - h)^2 + k$ from the graph of $f(x) = x^2$?

⊘ Explore   Understanding Quadratic Functions of the Form $g(x) = a(x - h)^2 + k$

Every quadratic function can be represented by an equation of the form $g(x) = a(x - h)^2 + k$. The values of the parameters *a, h,* and *k* determine how the graph of the function compares to the graph of the parent function, $y = x^2$. Use the method shown to graph $g(x) = 2(x - 3)^2 + 1$ by transforming the graph of $f(x) = x^2$.
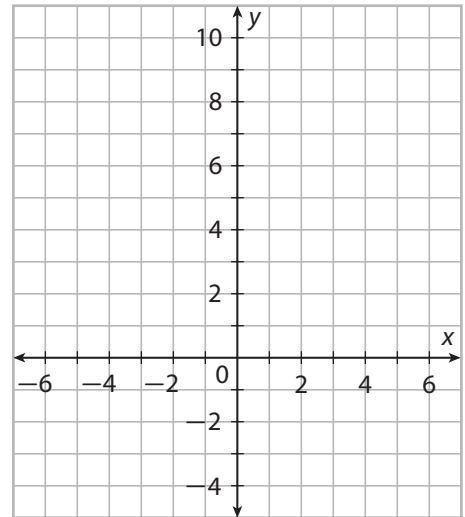
(A) Graph $f(x) = x^2$.

(B) Stretch the graph vertically by a factor of ____ to obtain the graph of $y = 2x^2$. Graph $y = 2x^2$.

Notice that point $(2, 4)$ moves to point ☐.

(C) Translate the graph of $y = 2x^2$ right 3 units and up 1 unit to obtain the graph of $g(x) = 2(x - 3)^2 + 1$. Graph $g(x) = 2(x - 3)^2 + 1$.

Notice that point $(2, 8)$ moves to point ☐.

(D) The vertex of the graph of $f(x) = x^2$ is _____ while the vertex of the graph of $g(x) = 2(x - 3)^2 + 1$ is _____.

1. **Discussion** Compare the minimum values of $f(x) = x^2$ and $g(x) = 2(x - 3)^2 + 1$. How is the minimum value related to the vertex?

_____

_____

2. **Discussion** What is the axis of symmetry of the function $g(x) = 2(x - 3)^2 + 1$? How is the axis of symmetry related to the vertex?

_____

_____

_____

⚙ Explain 1   Understanding Vertical Translations

A **vertical translation** of a parabola is a shift of the parabola up or down, with no change in the shape of the parabola.

| Vertical Translations of a Parabola |
| :--- |
| The graph of the function $f(x) = x^2 + k$ is the graph of $f(x) = x^2$ translated vertically. |
| If $k > 0$, the graph $f(x) = x^2$ is translated k units up. |
| If $k < 0$, the graph $f(x) = x^2$ is translated $\lvert k \rvert$ units down. |

Example 1   **Graph each quadratic function. Give the minimum or maximum value and the axis of symmetry.**

Ⓐ $g(x) = x^2 + 2$

Make a table of values for the parent function $f(x) = x^2$ and for $g(x) = x^2 + 2$. Graph the functions together.

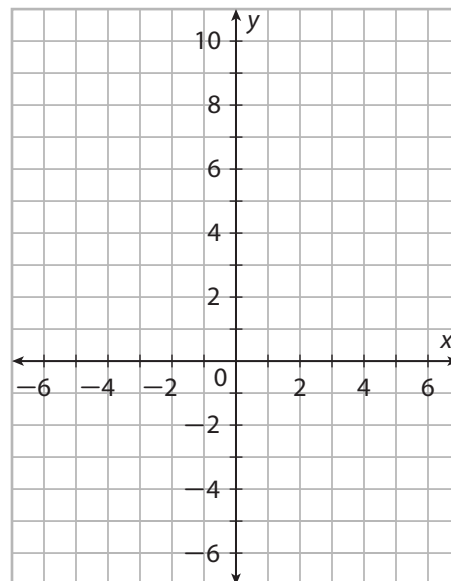| x	$f(x) = x^2$	$g(x) = x^2 + 2$
−3	9	11
−2	4	6
−1	1	3
0	0	2
1	1	3
2	4	6
3	9	11
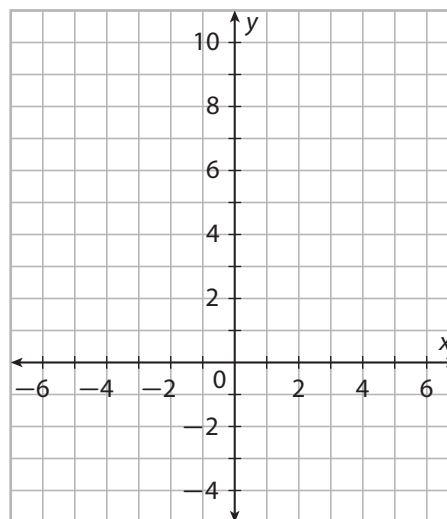
The function $g(x) = x^2 + 2$ has a minimum value of 2.

The axis of symmetry of $g(x) = x^2 + 2$ is $x = 0$.

(B) $g(x) = x^2 - 5$

Make a table of values for the parent function $f(x) = x^2$
and for $g(x) = x^2 - 5$. Graph the functions together.

x	$f(x) = x^2$	$g(x) = x^2 - 5$
−3	☐	☐
−2	☐	☐
−1	☐	☐
0	☐	☐
1	☐	☐
2	☐	☐
3	☐	☐

The function $g(x) = x^2 - 5$ has a minimum value of ____.

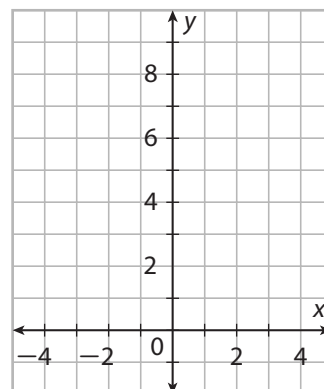The axis of symmetry of $g(x) = x^2 - 5$ is _____.

Reflect

3. How do the values in the table for $g(x) = x^2 + 2$ compare with the values in the table for the parent
function $f(x) = x^2$?

_____

4. How do the values in the table for $g(x) = x^2 - 5$ compare with the values in the table for the parent
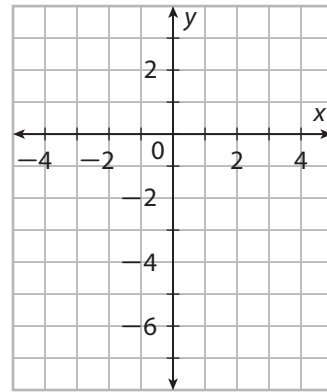function $f(x) = x^2$?

_____

Your Turn

Graph each quadratic function. Give the minimum or maximum value and the axis of symmetry.

5. $g(x) = x^2 + 4$

6. $g(x) = x^2 - 7$

Explain 2 Understanding Horizontal Translations

A **horizontal translation** of a parabola is a shift of the parabola left or right, with no change in the shape of the parabola.

Horizontal Translations of a Parabola

The graph of the function $f(x) = (x - h)^2$ is the graph of $f(x) = x^2$ translated horizontally.

If $h > 0$, the graph $f(x) = x^2$ is translated h units right.
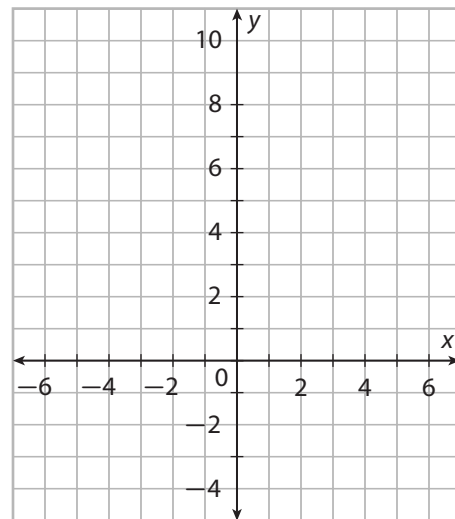
If $h < 0$, the graph $f(x) = x^2$ is translated $|h|$ units left.

Example 2 Graph each quadratic function. Give the minimum or maximum value and the axis of symmetry.

(A) $g(x) = (x - 1)^2$

Make a table of values for the parent function $f(x) = x^2$ and for $g(x) = (x - 1)^2$. Graph the functions together.

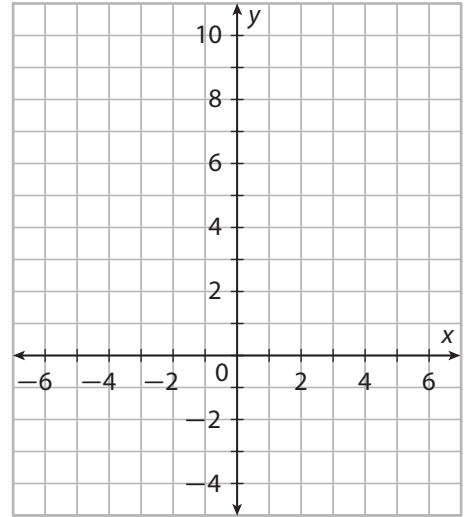| x	$f(x) = x^2$	$g(x) = (x-1)^2$
−3	9	16
−2	4	9
−1	1	4
0	0	1
1	1	0
2	4	1
3	9	4

The function $g(x) = (x - 1)^2$ has a minimum value of 0.

The axis of symmetry of $g(x) = (x - 1)^2$ is $x = 1$.

Ⓑ $g(x) = (x + 1)^2$

Make a table of values and graph the functions together.

x	$f(x) = x^2$	$g(x) = (x + 1)^2$
−3	9	
−2	4	
−1	1	
0	0	
1	1	
2	4	
3	9	

The function $g(x) = (x + 1)^2$ has a minimum value of ____.

The axis of symmetry of $g(x) = (x + 1)^2$ is _____.

Reflect

7. How do the values in the table for $g(x) = (x - 1)^2$ compare with the values in the table for the parent function $f(x) = x^2$?

_____

8. How do the values in the table for $g(x) = (x + 1)^2$ compare with the values in the table for the parent function $f(x) = x^2$?

_____

Your Turn

Graph each quadratic function. Give the minimum or maximum value and the axis of symmetry.

9. $g(x) = (x - 2)^2$

10. $g(x) = (x + 3)^2$
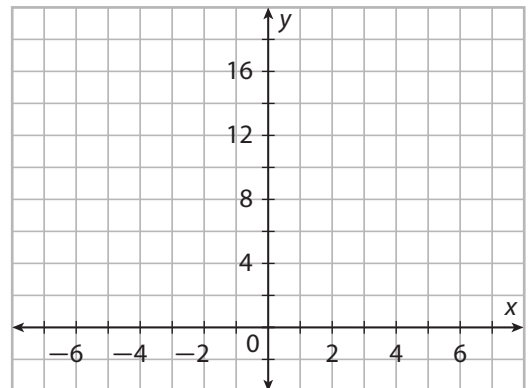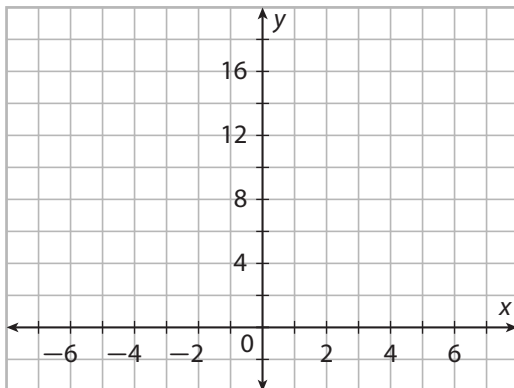
⚙ Explain 3 Graphing $g(x) = a(x - h)^2 + k$

The **vertex form of a quadratic function** is $g(x) = a(x - h)^2 + k$, where the point (h, k) is the vertex. The *axis of symmetry* of a quadratic function in this form is the vertical line $x = h$.

To graph a quadratic function in the form $g(x) = a(x - h)^2 + k$, first identify the vertex (h, k). Next, consider the sign of a to determine whether the graph opens upward or downward. If a is positive, the graph opens upward. If a is negative, the graph opens downward. Then generate two points on each side of the vertex. Using those points, sketch the graph of the function.

Example 3 Graph each quadratic function.

Ⓐ $g(x) = -3(x + 1)^2 - 2$

Identify the vertex.

The vertex is at $(-1, -2)$.

Make a table for the function. Find two points on each side of the vertex.

x	−3	−2	−1	0	1
g(x)	−14	−5	−2	−5	−14

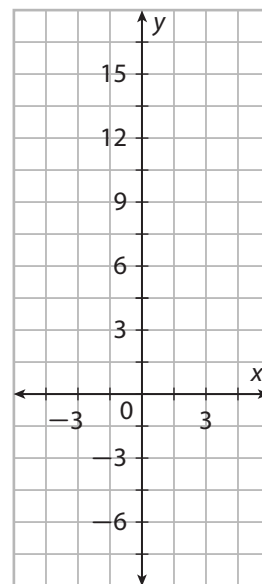Plot the points and draw a parabola through them.
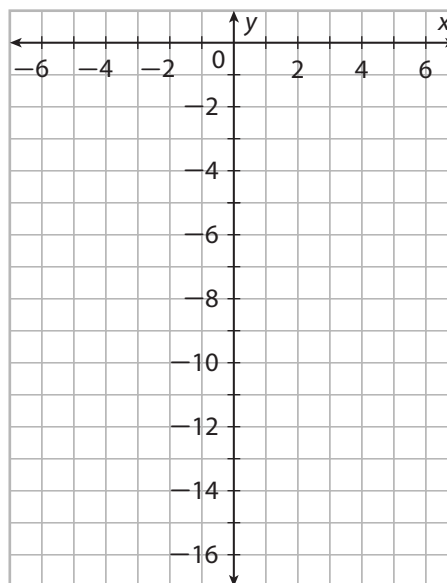
Ⓑ $g(x) = 2(x - 1)^2 - 7$

Identify the vertex.

The vertex is at _____.

Make a table for the function. Find two points on each side of the vertex.

x	−2	0	1	2	4
g(x)					

Plot the points and draw a parabola through them.
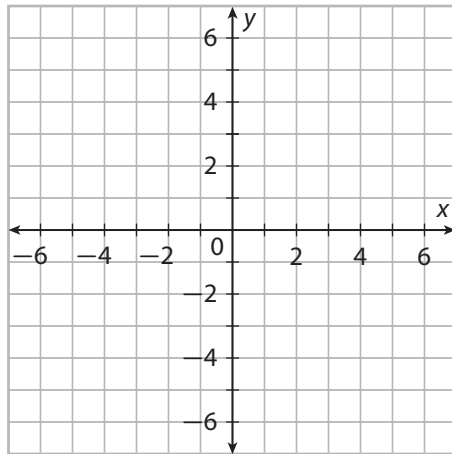

Reflect

11. How do you tell from the equation whether the vertex is a maximum value or a minimum value?
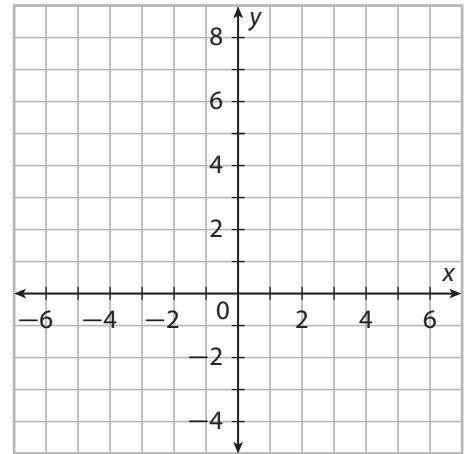
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_____

Graph each quadratic function.

12. $g(x) = -(x-2)^2 + 4$

x					
g(x)					

13. $g(x) = 2(x+3)^2 - 1$

x					
g(x)					

Elaborate

14. How does the value of k in $g(x) = x^2 + k$ affect the translation of $f(x) = x^2$?

_____

_____

15. How does the value of h in $g(x) = (x-h)^2$ affect the translation of $f(x) = x^2$?

_____

_____

16. In $g(x) = a(x-h)^2 + k$, what are the coordinates of the vertex?

_____

_____

17. **Essential Question Check-In** How can you use the values of a, h, and k, to obtain the graph of $g(x) = a(x-h)^2 + k$ from the graph $f(x) = x^2$?
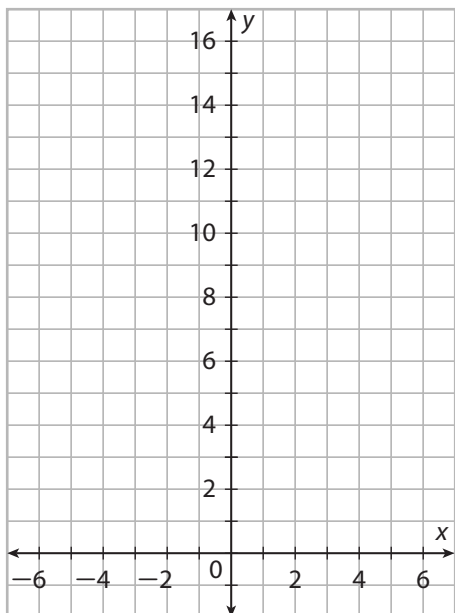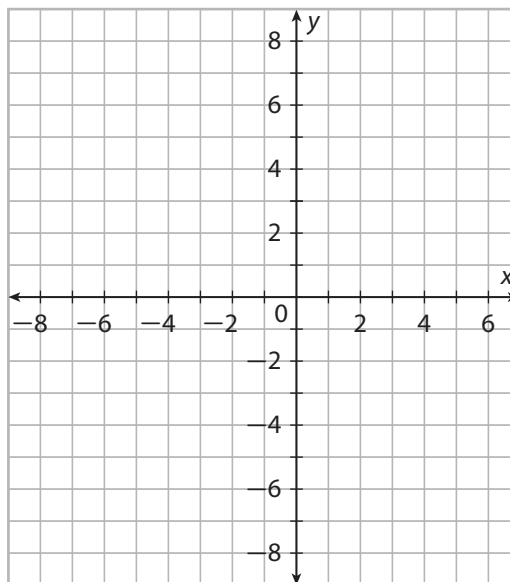
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• Online Homework
• Hints and Help
• Extra Practice

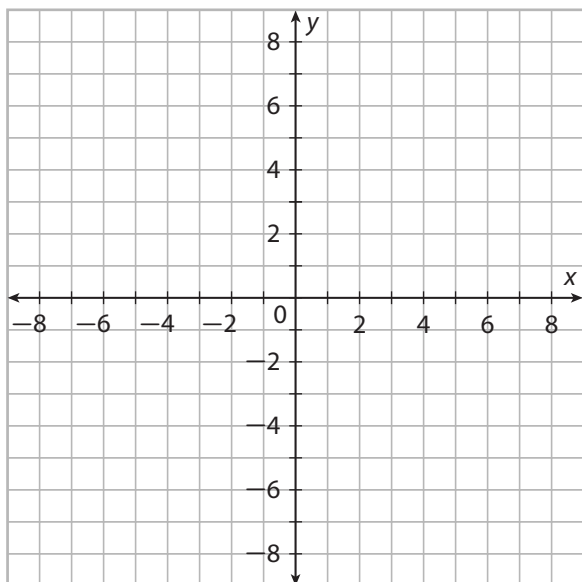**Graph each quadratic function by transforming the graph of $f(x) = x^2$. Describe the transformations.**
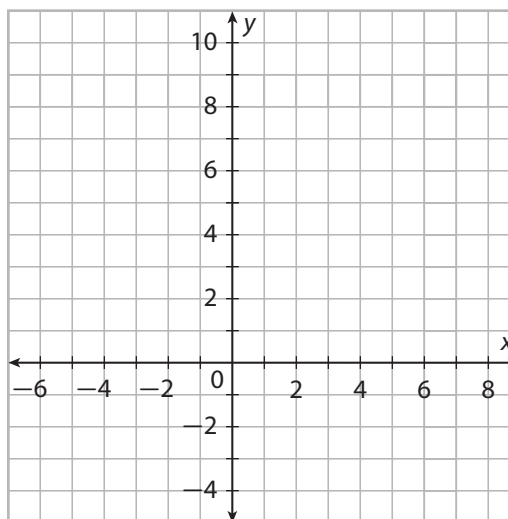
1. $g(x) = 2(x - 2)^2 + 5$

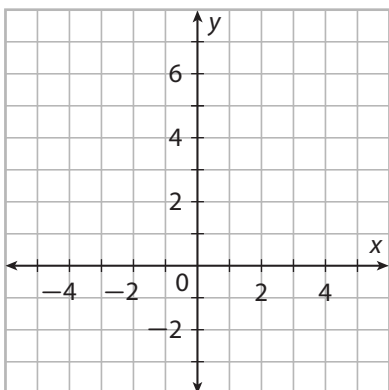**2.** $g(x) = 2(x + 3)^2 - 6$

3. $g(x) = \frac{1}{2}(x - 3)^2 - 4$
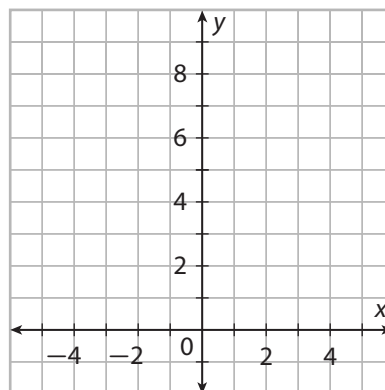
4. $g(x) = 3(x - 4)^2 - 2$

Graph each quadratic function.

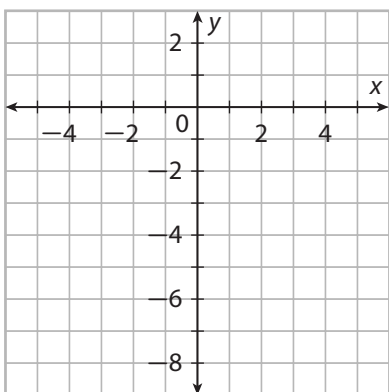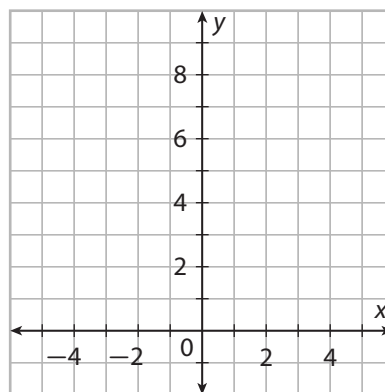**5.** $g(x) = x^2 - 2$
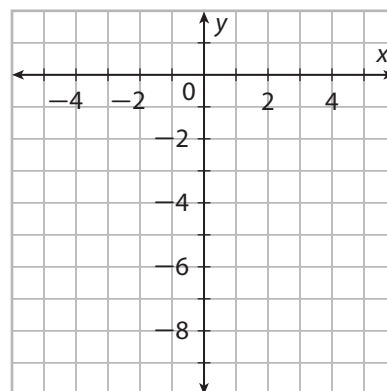

6. $g(x) = x^2 + 5$

7. $g(x) = x^2 - 6$
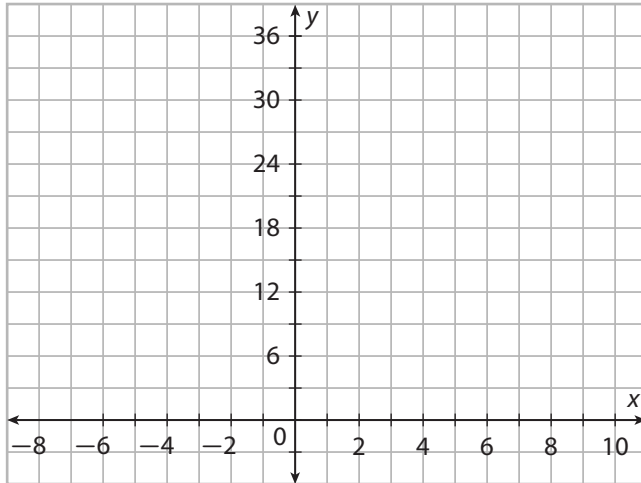

8. $g(x) = x^2 + 3$

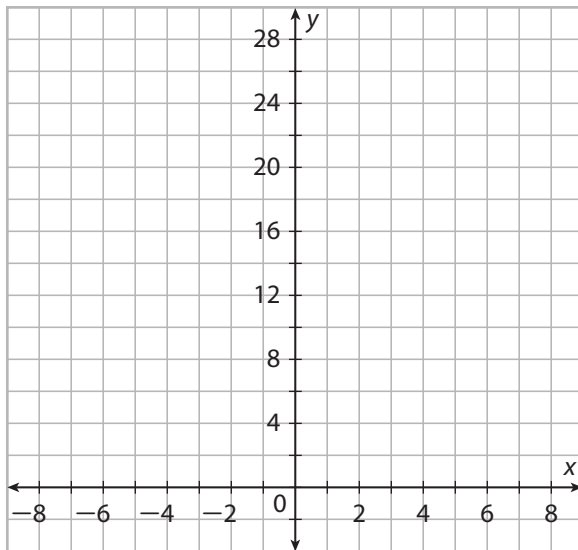**9.** Graph $g(x) = x^2 - 9$. Give the minimum or maximum value and the axis of symmetry.

10. How is the graph of $g(x) = x^2 + 12$ related to the graph of $f(x) = x^2$?

Graph each quadratic function. Give the minimum or maximum value and the axis of symmetry.
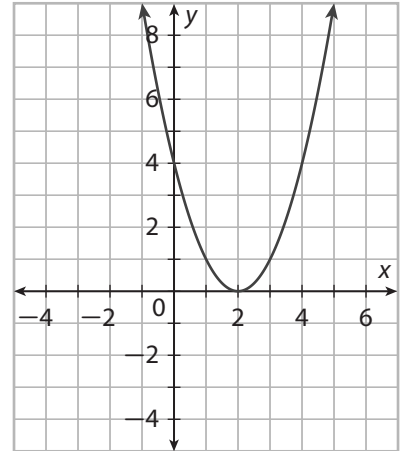
11. $g(x) = (x - 3)^2$

12. $g(x) = (x + 2)^2$

13. How is the graph of $g(x) = (x + 12)^2$ related to the graph of $f(x) = x^2$?

14. How is the graph of $g(x) = (x - 10)^2$ related to the graph of $f(x) = x^2$?

15. Compare the given graph to the graph of the parent function $f(x) = x^2$. Describe how the parent function must be translated to get the graph shown here.

16. For the function $g(x) = (x - 9)^2$ give the minimum or maximum value and the axis of symmetry.

Graph each quadratic function. Give the minimum or maximum value and the axis of symmetry.

17. $g(x) = (x - 1)^2 - 5$

18. $g(x) = -(x + 2)^2 + 5$

19. $g(x) = \frac{1}{4}(x + 1)^2 - 7$

20. $g(x) = -\frac{1}{3}(x + 3)^2 + 8$

21. Compare the given graph to the graph of the parent function $f(x) = x^2$. Describe how the parent function must be translated to get the graph shown here.
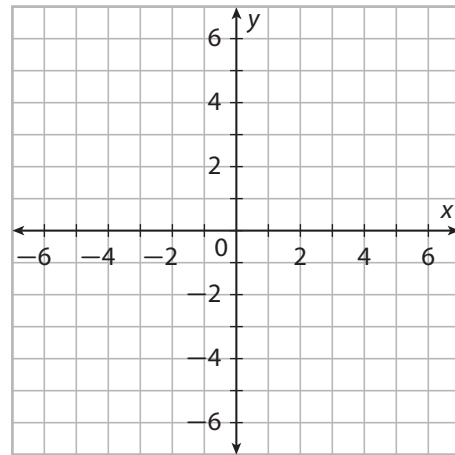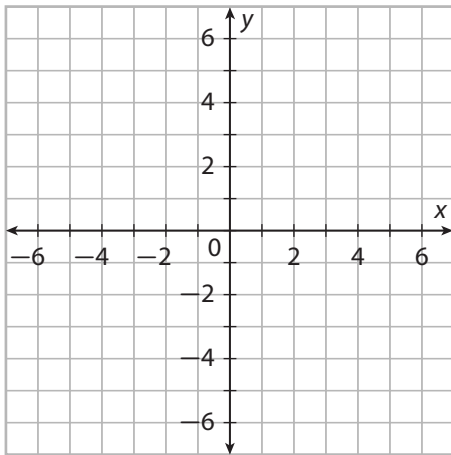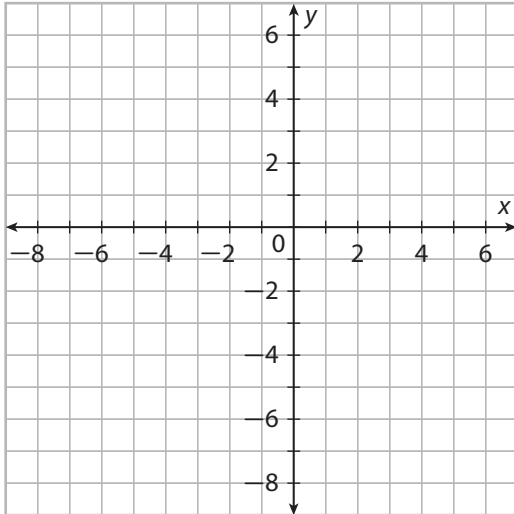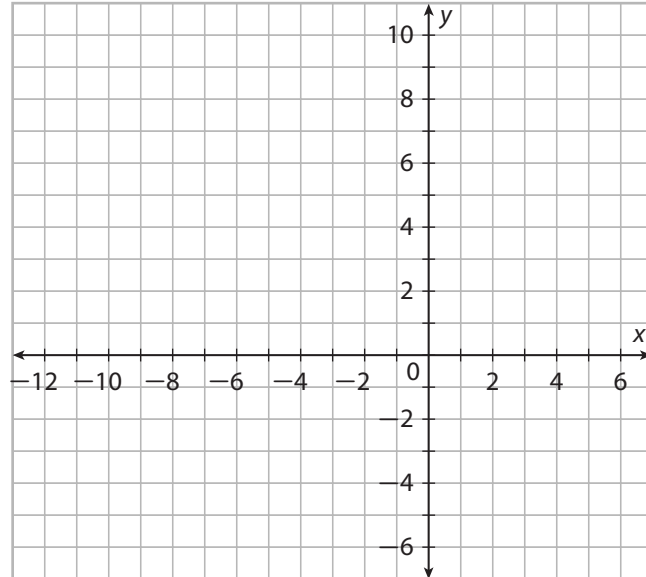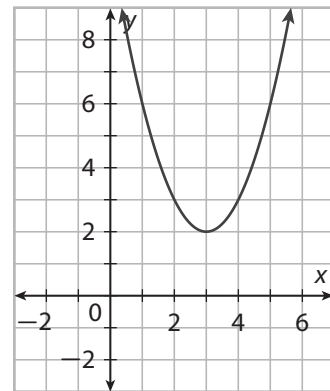
22. Multiple Representations Select the equation for the function represented by the graph of a parabola that is a translation of $f(x) = x^2$. The graph has been translated 11 units to the left and 5 units down.

   a. $g(x) = (x - 11)^2 - 5$

   b. $g(x) = (x + 11)^2 - 5$

   c. $g(x) = (x + 11)^2 + 5$

   d. $g(x) = (x - 11)^2 + 5$

   e. $g(x) = (x - 5)^2 - 11$

   f. $g(x) = (x - 5)^2 + 11$

   g. $g(x) = (x + 5)^2 - 11$

   h. $g(x) = (x + 5)^2 + 11$

Critical Thinking Use a graphing calculator to compare the graphs of $y = (2x)^2$, $y = (3x)^2$, and $y = (4x)^2$ with the graph of the parent function $y = x^2$. Then compare the graphs of $y = \left(\frac{1}{2}x\right)^2$, $y = \left(\frac{1}{3}x\right)^2$, and $y = \left(\frac{1}{4}x\right)^2$ with the graph of the parent function $y = x^2$.

23. Explain how the parameter b horizontally stretches or compresses the graph of $y = (bx)^2$ when $b > 1$.

24. Explain how the parameter b horizontally stretches or compresses the graph of $y = (bx)^2$ when $0 < b < 1$.

**25. Explain the Error** Nina is trying to write an equation for the function represented by the graph of a parabola that is a translation of $f(x) = x^2$. The graph has been translated 4 units to the right and 2 units up. She writes the function as $g(x) = (x + 4)^2 + 2$. Explain the error.

26. Multiple Representations A group of engineers drop an experimental tennis ball from a catwalk and let it fall to the ground. The tennis ball's height above the ground (in feet) is given by a function of the form $f(t) = a(t - h)^2 + k$ where t is the time (in seconds) after the tennis ball was dropped. Use the graph to find the equation for $f(t)$.
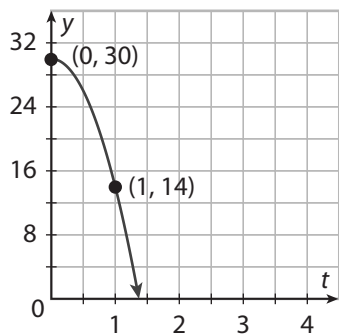
27. Make a Prediction For what values of a and c will the graph of $f(x) = ax^2 + c$ have one x-intercept?

Lesson Performance Task

The path a baseball takes after it has been hit is modeled by the graph. The baseball's height above the ground is given by a function of the form $f(t) = a(t - h)^2 + k$, where t is the time in seconds since the baseball was hit.

a. What is the baseball's maximum height? At what time was the baseball at its maximum height?

b. When does the baseball hit the ground?

c. Find an equation for $f(t)$.

d. A player hits a second baseball. The second baseball's path is modeled by the function $g(t) = -16(t - 4)^2 + 256$. Which baseball has a greater maximum height? Which baseball is in the air for the longest?

Baseball's Height