Name _____ Class _____ Date _____

4.1 Identifying and Graphing Sequences

Essential Question: **What is a sequence and how are sequences and functions related?**

Resource Locker

Explore Understanding Sequences

A go-kart racing track charges $5 for a go-kart license and $2 for each lap. If you list the charges for 1 lap, 2 laps, 3 laps, and so on, in order, the list forms a sequence of numbers:

$$7, 9, 11, 13, \ldots$$

A **sequence** is a list of numbers in a specific order. Each element in a sequence is called a **term**. In a sequence, each term has a position number. In the sequence 7, 9, 11, 13, . . . , the second term is 9, so its position number is 2.

(A) The total cost (term) of riding a go-kart for different numbers of laps (position) is shown below. Complete the table.

Position number, *n*	1	2	3		5		*Domain*
Term of the sequence, *f*(*n*)	7	9		13		17	*Range*

(B) You can use the term and position number of a sequence to write a function. Using function notation, $f(2) = 9$ indicates that the second term is 9. Use the table to complete the following statements.

$f(1) =$ ☐ $f(3) =$ ☐ $f(6) =$ ☐ $f(\square) = 13$ $f(\square) = 15$

(C) Identify the domain of the function $f(n)$. _____

(D) Identify the range of the function $f(n)$. _____

Reflect

1. **Discussion** What does $f(4) = 13$ mean in the context of the go-kart problem?

_____

2. **Discussion** Explain how to find the missing values in the table.

_____

3. **Communicate Mathematical Ideas** Explain why the relationship between the position numbers and the corresponding terms of a sequence can be considered a function.

_____

Explain 1 Generating Sequences Using an Explicit Rule

An **explicit rule** for a sequence defines the nth term as a function of n for any whole number n greater than 0. Explicit rules can be used to find any specific term in a sequence without finding any of the previous terms.

Example 1 **Write the first 4 terms of the sequence defined by the explicit rule.**

(A) $f(n) = n^2 + 2$

Make a table and substitute values for $n = 1, 2, 3, 4$ to find the first 4 terms.

The first 4 terms of the sequence defined by the explicit rule $f(n) = n^2 + 2$ are 3, 6, 11, and 18.

n	$f(n) = n^2 + 2$	$f(n)$
1	$f(1) = 1^2 + 2 = 3$	3
2	$f(2) = 2^2 + 2 = 6$	6
3	$f(3) = 3^2 + 2 = 11$	11
4	$f(4) = 4^2 + 2 = 18$	18

(B) $f(n) = 3n^2 + 1$

Make a table and substitute values for $n =$ _____.

The first 4 terms are _____.

n	$f(n) = 3n^2 + 1$	$f(n)$
1	$f(\square) = 3(\square)^2 + 1 = \square$	
2	$f(\square) = 3(\square)^2 + 1 = \square$	
3	$f(\square) = 3(\square)^2 + 1 = \square$	
4	$f(\square) = 3(\square)^2 + 1 = \square$	

Reflect

4. **Communicate Mathematical Ideas** Explain how to find the 20th term of the sequence defined by the explicit rule $f(n) = n^2 + 2$.

5. **Justify Reasoning** The number 125 is a term of the sequence defined by the explicit rule $f(n) = 3n + 2$. Which term in the sequence is 125? Justify your answer.

Your Turn

6. Write the first 4 terms of the sequence defined by the explicit rule. $f(n) = n^2 - 5$

7. Find the 15^{th} term of the sequence defined by the explicit rule. $f(n) = 4n - 3$.

Explain 2 Generating Sequences Using a Recursive Rule

A **recursive rule** for a sequence defines the nth term by relating it to one or more previous terms.

The following is an example of a recursive rule:

$f(1) = 4, f(n) = f(n - 1) + 10$ for each whole number n greater than 1

This rule means that after the first term of the sequence, every term $f(n)$ is the sum of the pervious term $f(n - 1)$ and 10.

Example 2 **Write the first 4 terms of the sequence defined by the recursive rule.**

(A) $f(1) = 2, f(n) = f(n - 1) + 3$ for each whole number n greater than 1

For the first 4 terms, the domain of the function is 1, 2, 3, and 4.

The first term of the sequence is 2.

n	$f(n) = f(n-1) + 3$	$f(n)$
1	$f(1) = 2$	2
2	$f(2) = f(1) + 3 = 2 + 3 = 5$	5
3	$f(3) = f(2) + 3 = 5 + 3 = 8$	8
4	$f(4) = f(3) + 3 = 8 + 3 = 11$	11

The first 4 terms are 2, 5, 8, and 11.

(B) $f(1) = 4, f(n) = f(n - 1) + 5$ for each whole number n greater than 1

For the first 4 terms, the domain of the function is _____

The first term of the sequence is ☐ .

n	$f(n) = f(n-1) + 5$	$f(n)$
1	$f(1) =$ ☐	☐
2	$f(2) = f($☐$) + 5 =$ ☐ $+ 5 =$ ☐	☐
3	$f(3) = f($☐$) + 5 =$ ☐ $+ 5 =$ ☐	☐
4	$f(4) = f($☐$) + 5 =$ ☐ $+ 5 =$ ☐	☐

The first 4 terms are _____ .

Reflect

8. Describe how to find the 12^{th} term of the sequence in Example 2A.

9. Suppose you want to find the 40^{th} term of a sequence. Would you rather use a recursive rule or an explicit rule? Explain your reasoning.

Your Turn

Write the first 5 terms of the sequence.

10. $f(1) = 35$ and $f(n) = f(n - 1) - 2$ for each whole number n greater than 1.

11. $f(1) = 45$ and $f(n) = f(n - 1) - 4$ for each whole number n greater than 1.

Explain 3 Constructing and Graphing Sequences

You can graph a sequence on a coordinate plane by plotting the points $(n, f(n))$ indicated in a table that you use to generate the terms.

Example 3 **Construct and graph the sequence described.**

(A) The go-kart racing charges are \$5 for a go-kart license and \$2 for each lap. Use the explicit rule $f(n) = 2n + 5$.

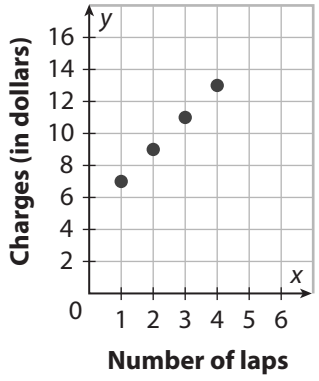
Complete the table to represent the cost for the first 4 laps.

n	$f(n) = 2n + 5$	$f(n)$
1	$f(1) = 2(1) + 5 = 2 + 5 = 7$	7
2	$f(2) = 2(2) + 5 = 4 + 5 = 9$	9
3	$f(3) = 2(3) + 5 = 6 + 5 = 11$	11
4	$f(4) = 2(4) + 5 = 8 + 5 = 13$	13

The ordered pairs are (1, 7), (2, 9), (3, 11), (4, 13).

Graph the sequence using the ordered pairs.

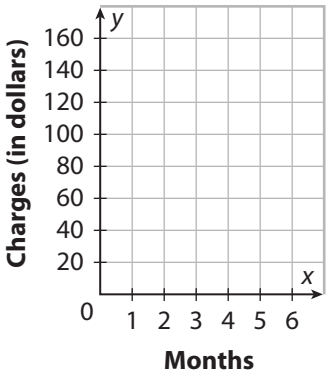
Notice that the graph is a set of points that are not connected.



(B) A movie rental club charges $20 a month plus a $5 membership fee. Use the explicit rule $f(n) = 20n + 5$.

Complete the table to represent the charges paid for 6 months.

n	$f(n) = \square\, n + \square$	$f(n)$
1	$f(\square) = \square(\square) + \square = \square$	
2	$f(\square) = \square(\square) + \square = \square$	
3	$f(\square) = \square(\square) + \square = \square$	
4	$f(\square) = \square(\square) + \square = \square$	
5	$f(\square) = \square(\square) + \square = \square$	
6	$f(\square) = \square(\square) + \square = \square$	



The ordered pairs are _____.

Graph the sequence using the ordered pairs.

Notice that the graph is a set of points that are not connected.

Reflect

12. Explain why the points in the graphs in Example 3 are not connected.

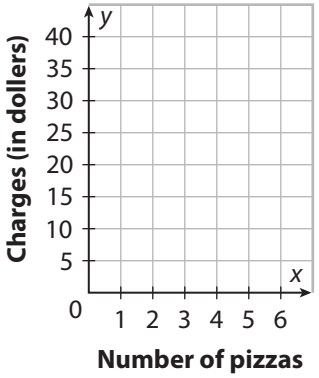
_____

_____

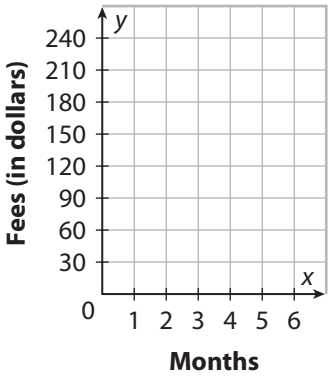
Your Turn

Construct and graph the sequence described.

13. A pizza place is having a special. If you order a large pizza for a regular price $17, you can order any number of additional pizzas for $8.50 each. Use the recursive rule $f(1) = 17$ and $f(n) = f(n - 1) + 8.5$ for each whole number n greater than 1.



14. A gym charges \$100 as the membership fee and \$20 monthly fee. Use the explicit rule $f(n) = 20n + 100$ to construct and graph the sequence.



Elaborate

15. What is the difference between an explicit rule and a recursive rule?

16. Describe how to use an explicit rule to find the position number of a given term in a sequence.

17. Explain why the graph of a sequence is a set of points that are not connected.

18. **Essential Question Check-In** Why can the rule for a sequence be considered a function?

Evaluate: Homework and Practice

- Online Homework
- Hints and Help
- Extra Practice

Complete the table, and state the domain and range for the sequence it represents. Assume that the sequence continues without end.

1.

n	1	2	3		5	
$f(n)$	15	30		60		90

2.

n	1		3	4		6
$f(n)$	6	8	10		14	

Write the first 4 terms of the sequence defined by the given rule.

3. $f(1) = 65{,}536,\ f(n) = \sqrt{f(n-1)}$

4. $f(n) = n^3 - 1$

5. $f(1) = 7,\ f(n) = -4 \cdot f(n-1) + 15$

6. $f(n) = 2n^2 + 4$

7. $f(1) = 3,\ f(n) = [f(n-1)]^2$

8. $f(n) = (2n-1)^2$

Find the 10th term of the sequence defined by the given rule.

9. $f(1) = 2, f(n) = f(n - 1) + 7$

10. $f(n) = \sqrt{n + 2}$

11. $f(1) = 30, f(n) = 2 \cdot f(n - 1) - 50$

12. $f(n) = \frac{1}{2}(n - 1) + 3$

The explicit rule for a sequence and one of the specific terms is given. Find the position of the given term.

13. $f(n) = 1.25n + 6.25; 25$

14. $f(n) = -3(n - 1); -51$

15. $f(n) = (2n - 2) + 2; 52$

The recursive rule for a sequence and one of the specific terms is given. Find the position of the given term.

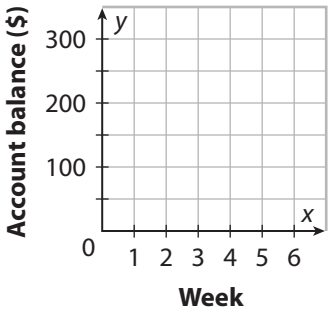
16. $f(1) = 8\frac{1}{2}; f(n) = f(n - 1) - \frac{1}{2}; 5\frac{1}{2}$

17. $f(1) = 99, f(n) = f(n - 1) + 4; 119$

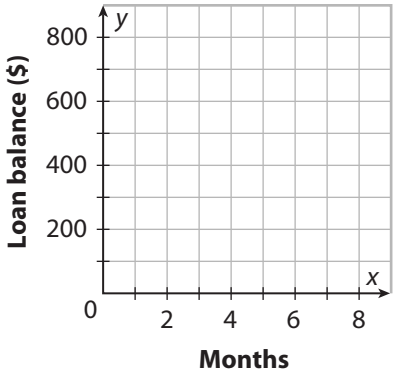
18. $f(1) = 33.3, f(n) = f(n - 1) + 0.2; 34.9$

Graph the sequence that represents the situation on a coordinate plane.

19. Jessica had $150 in her savings account after her first week of work. She then started adding $35 each week to her account for the next 5 weeks. The savings account balance can be represented by a sequence.



20. Carrie borrowed $840 from a friend to pay for a car repair. Carrie promises to repay her friend in 8 equal monthly payments. The remaining amount Carrie has to repay can be represented by a sequence.



H.O.T. Focus on Higher Order Thinking

21. A park charges $12 for one round of miniature golf and a reduced fee for each additional round played. If Tom paid $47 for 6 rounds of miniature golf, what is the reduced fee for each additional round played?

22. **Analyze Relationships** Construct a recursive rule to describe the sequence: 2, 4, 6, 8,…

23. **Explain the Error** To find the 5th term of a sequence where $f(1) = 4$ and $f(n) = 2 \cdot f(n-1) + 1$ for each whole number greater than 1, Shane calculates $(4 \cdot 2 \cdot 2 \cdot 2 \cdot 2) + 1 = 65$. Is this correct? Justify your answer.

24. **Critical Thinking** Write a recursive rule for a sequence where every term is the same.

Lesson Performance Task

A museum charges $10 per person for admission and $2 for each of 8 special exhibits.

a. Use function notation to write an equation to represent the cost for attending n events.

b. Make a table to represent the total cost of admission plus 1, 2, and 3 special exhibits.

c. What would $f(0) = 10$ represent?

d. What would the total cost be for going to all 8 special exhibits?

e. Determine an explicit rule for the total cost if the first special exhibit were free.