Name _____ Class _____ Date _____

2.2 Creating and Solving Equations

Essential Question: How do you use an equation to model and solve a real-world problem?

Resource Locker

Explore Creating Equations from Verbal Descriptions

You can use what you know about writing algebraic expressions to write an equation that represents a real-world situation.

Suppose Cory and his friend Walter go to a movie. Each of their tickets costs the same amount, and they share a frozen yogurt that costs $5.50. The total amount they spend is $19.90. How can you write an equation that describes the situation?

A **Identify the important information.**

The word _____ tells you that the relationship describes an equation.

The word *total* tells you that the operation involved in the relationship is _____.

What numerical information do you have? _____

_____

What is the unknown quantity? _____

B **Write a verbal description.**

Choose a name for the variable. In this case, use c for _____.

The verbal description is: Twice the cost of _____ plus the cost

of _____ equals _____.

C To write an equation, write a numerical or _____ expression for each quantity

and insert an equal sign in the appropriate place. An equation is: _____.

Reflect

1. How can you use a verbal model to write an equation for the situation described?

2. Could you write a different equation to describe the situation? Explain your reasoning.

Explain 1 Creating and Solving Equations Involving the Distributive Property

When you create an equation to model a real-world problem, your equation may involve the Distributive Property. When you solve a real-world problem, you should always check that your answer makes sense.

Example 1 **Write and solve an equation to solve each problem.**

(A) Aaron and Alice are bowling. Alice's score is twice the difference of Aaron's score and 5. The sum of their scores is 320. Find each student's bowling score.

Write a verbal description of the basic situation.

The sum of Aaron's score and Alice's score is 320.

Choose a variable for the unknown quantity and write an equation to model the detailed situation.

Let a represent Aaron's score. Then $2(a - 5)$ represents Alice's score.

$a + 2(a - 5) = 320$

Solve the equation for a.

$a + 2(a - 5) = 320$

$a + 2a - 10 = 320$ Distributive Property

$3a - 10 = 320$

$3a - 10 + 10 = 320 + 10$ Addition Property of Equality

$3a = 330$

$\frac{3a}{3} = \frac{330}{3}$ Division Property of Equality

$a = 110$

So, Aaron's score is 110 and Alice's score is $2(a - 5) = 2(110 - 5) = 2(105) = 210$.

Check that the answer makes sense.

$110 + 210 = 320$, so the answer makes sense.

(B) Mari, Carlos, and Amanda collect stamps. Carlos has five more stamps than Mari, and Amanda has three times as many stamps as Carlos. Altogether, they have 100 stamps. Find the number of stamps each person has.

Write a verbal description of the basic situation.

Choose a variable for the unknown quantity and write an equation to model the detailed situation.

Let s represent the number of stamps Mari has. Then Carlos has _____ stamps, and Amanda has _____ stamps.

$s + \square + 3\left(\square\right) = \square$

Solve the equation for s.

$s + \square + 3\left(\square\right) = \square$

$s + s + 5 + 3s + \square = \square$ Distributive Property

$\square + \square = \square$ Combine like terms

$\square s = \square$ Subtraction Property of Equality

$s = \square$ Division Property of Equality

So, Mari has _____ stamps, Carlos has _____ stamps, and Amanda has _____ stamps.

Check that the answer makes sense.

$\square + \square + \square = \square$

$\square = \square$

; the answer makes sense.

Reflect

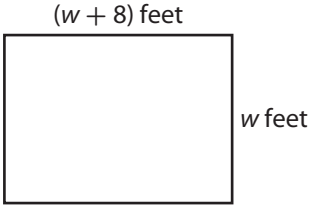
3. Would a fractional answer make sense in this situation?

4. **Discussion** What might it mean if a check revealed that the answer to a real-world problem did not make sense?

Your Turn

Write and solve an equation to solve the problem.

5. A rectangular garden is fenced on all sides with 256 feet of fencing. The garden is 8 feet longer than it is wide. Find the length and width of the garden.



Explain 2 Creating and Solving Equations with Variables on Both Sides

In some equations, variables appear on both sides. You can use the properties of equality to collect the variable terms so that they all appear on one side of the equation.

Example 2 **Write and solve an equation to solve each problem.**

(A) Janine has job offers at two companies. One company offers a starting salary of \$28,000 with a raise of \$3000 each year. The other company offers a starting salary of \$36,000 with a raise of \$2000 each year. In how many years would Janine's salary be the same with both companies? What will the salary be?

Write a verbal description of the basic situation.

Let n represent the number of years it takes for the salaries to be equal.

Base Salary A plus \$3000 per year raise = Base Salary B + \$2000 per year raise

$$28{,}000 + 3000n = 36{,}000 + 2{,}000n$$

$$28{,}000 + 3000n - 2000n = 36{,}000 + 2{,}000n - 2000n \quad \text{Subtraction Property of Equality}$$

$$28{,}000 + 1000n = 36{,}000 \quad \text{Combine like terms.}$$

$$28{,}000 + 1000n - 28{,}000 = 36{,}000 - 28{,}000 \quad \text{Subtraction Property of Equality}$$

$$1000n = 8000$$

$$\frac{1000n}{1000} = \frac{8000}{1000} \quad \text{Division Property of Equality}$$

$$n = 8$$

$$28{,}000 + 3{,}000(8) = 36{,}000 + 2{,}000(8)$$

$$52{,}000 = 52{,}000$$

In 8 years, the salaries offered by both companies will be \$52,000.

Ⓑ One moving company charges $800 plus $16 per hour. Another moving company charges $720 plus $21 per hour. At what number of hours will the charge by both companies be the same? What is the charge?

Write a verbal description of the basic situation. Let t represent the number of hours that the move takes.

Moving Charge A plus $16 per hour = Moving Charge B plus $21 per hour

$$800 + \square\, t = 720 + \square\, t$$

$$800 + \square\, t - \square\, t = 720 + \square\, t - \square\, t \qquad \text{Subtraction Property of Equality}$$

$$800 = 720 + \square\, t$$

$$800 - 720 = 720 + \square\, t - 720 \qquad \text{Subtraction Property of Equality}$$

$$\square = \square\, t$$

$$\frac{\square}{\square} = \frac{\square\, t}{\square} \qquad \text{Division Property of Equality}$$

$$t = \square$$

The charges are the same for a job that takes _____ hours.

Substitute the value 16 in the original equation.

$$800 + 16t = 720 + 21t$$

$$800 + 16\left(\square\right) = 720 + 21\left(\square\right)$$

$$800 + \square = 720 + \square$$

$$\square = \square$$

After _____ hours, the moving charge for both companies will be _____.

Reflect

6. Suppose you collected the variable terms on the other side of the equal sign to solve the equation. Would that affect the solution?

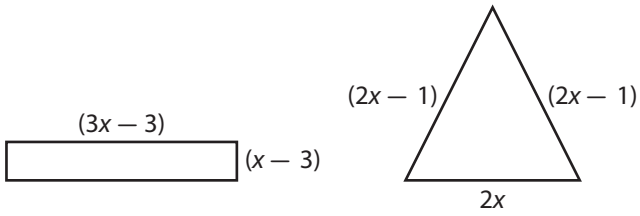
_____

_____

Your Turn

Write and solve an equation to solve each problem.

7. Claire bought just enough fencing to enclose either a rectangular garden or a triangular garden, as shown. The two gardens have the same perimeter. How many feet of fencing did she buy?



8. A veterinarian is changing the diets of two animals, Simba and Cuddles. Simba currently consumes 1200 Calories per day. That number will increase by 100 Calories each day. Cuddles currently consumes 3230 Calories a day. That number will decrease by 190 Calories each day. The patterns will continue until both animals are consuming the same number of Calories each day. In how many days will that be? How many Calories will each animal be consuming each day then?

Explain 3 Constructing Equations from an Organized Table

You can use a table to organize information and see relationships.

Example 3 **Construct and solve an equation to solve the problem.**

Kim works 4 hours more each day than Jill does, and Jack works 2 hours less each day than Jill does. Over 2 days, the number of hours Kim works is equal to the difference of 4 times the number of hours Jack works and the number of hours Jill works. How many hours does each person work each day?

Analyze Information

Identify the important information.

- Kim works ☐ hours more per day than Jill does.
- Jack works ☐ hours less per day than Jill does.

Formulate a Plan

Make a table using the information given. Let x be the number of hours Jill works in one day.

	Hours Worked Per Day	Hours Worked Over 2 Days
Kim		
Jill		
Jack		

Over 2 days, the number of hours Kim works is equal to the difference of 4 times the number of hours Jack works and the number of hours Jill works.

Solve

$2(x + 4) = 4 \cdot 2(x - 2) - 2x$

$2(x + 4) = \square(x - 2) - 2x$ Simplify.

$2x + \square = 8x - \square - 2x$ Distributive Property

$2x + 8 = \square x - 16$

$2x + 8 + \square = 6x - 16 + \square$ Addition Property of Equality

$2x + \square = 6x$

$2x + 24 - \square x = 6x - \square x$ Subtraction Property of Equality

$24 = \square x$

$\frac{24}{\square} = \frac{\square x}{\square}$ Division Property of Equality

$\square = x$

Jill works $\square$ hours per day, Kim works $\square$ hours per day, and Jack works $\square$ hours per day.

Justify and Evaluate

Substitute $x = 6$ into the original equation.

$2(6 + 4) = 4 \cdot 2(6 - 2) - 2x$

$2\left(\square\right) = 8\left(\square\right) - 2\left(\square\right)$

$\square = \square$

Your Turn

Write and solve an equation to solve the problem.

9. Lisa is 10 centimeters taller than her friend Ian. Ian is 14 centimeters taller than Jim. Every month, their heights increase by 2 centimeters. In 7 months, the sum of Ian's and Jim's heights will be 170 centimeters more than Lisa's height. How tall is Ian now?

Elaborate

10. How can you use properties to solve equations with variables on both sides?

11. How is a table helpful when constructing equations?

12. When solving a real-world problem to find a person's age, would a negative solution make sense? Explain.

13. **Essential Question Check-In** How do you write an equation to represent a real-world situation?

Evaluate: Homework and Practice

- Online Homework
- Hints and Help
- Extra Practice

Write an equation for each description.

1. The sum of 14 and a number is equal to 17.

2. A number increased by 10 is 114.

3. The difference between a number and 12 is 20.

4. Ten times the sum of half a number and 6 is 8.

5. Two-thirds a number plus 4 is 7.

6. Tanmayi wants to raise $175 for a school fundraiser. She has raised $120 so far. How much more does she need to reach her goal?

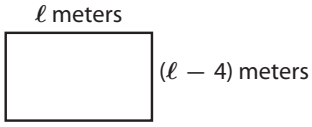
7. Hector is visiting a cousin who lives 350 miles away. He has driven 90 miles. How many more miles does he need to drive to reach his cousin's home?

8. The length of a rectangle is twice its width. The perimeter of the rectangle is 126 feet.

Write and solve an equation for each situation.

9. In one baseball season, Peter hit twice the difference of the number of home runs Alice hit and 6. Altogether, they hit 18 home runs. How many home runs did each player hit that season?

10. The perimeter of a parallelogram is 72 meters. The width of the parallelogram is 4 meters less than its length. Find the length and the width of the parallelogram.



11. One month, Ruby worked 6 hours more than Isaac, and Svetlana worked 4 times as many hours as Ruby. Together they worked 126 hours. Find the number of hours each person worked.

12. In one day, Annie traveled 5 times the sum of the number of hours Brian traveled and 2. Together they traveled 20 hours. Find the number of hours each person traveled.

13. Xian and his cousin Kai both collect stamps. Xian has 56 stamps, and Kai has 80 stamps. The boys recently joined different stamp-collecting clubs. Xian's club will send him 12 new stamps per month. Kai's club will send him 8 new stamps per month. After how many months will Xian and Kai have the same number of stamps? How many stamps will each have?

14. Kenya plans to make a down payment plus monthly payments in order to buy a motorcycle. At one dealer she would pay $2,500 down and $150 each month. At another dealer, she would pay $3,000 down and $125 each month. After how many months would the total amount paid be the same for both dealers? What would that amount be?

15. Community Gym charges a $50 membership fee and a $55 monthly fee. Workout Gym charges a $200 membership fee and a $45 monthly fee. After how many months will the total amount of money paid to both gyms be the same? What will the amount be?

16. Tina is saving to buy a notebook computer. She has two options. The first option is to put $200 away initially and save $10 every month. The second option is to put $100 away initially and save $30 every month. After how many months would Tina save the same amount using either option? How much would she save with either option?

Use the table to answer each question.

	Starting Salary	Yearly Salary Increase
Company A	$24,000	$3000
Company B	$30,000	$2400
Company C	$36,000	$2000

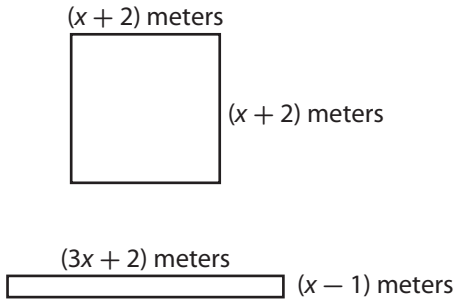
17. After how many years are the salaries offered by Company A and Company B the same?

18. After how many years are the salaries offered by Company B and Company C the same?

19. Paul started work at Company B ten years ago at the salary shown in the table. At the same time, Sharla started at Company C at the salary shown in the table. Who earned more during the last year? How much more?

20. George's page contains twice as many typed words as Bill's page and Bill's page contains 50 fewer words than Charlie's page. If each person can type 60 words per minute, after one minute, the difference between twice the number of words on Bill's page and the number of words on Charlie's page is 210. How many words did Bill's page contain initially? Use a table to organize the information.

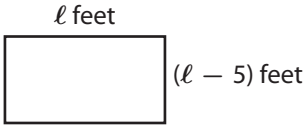
21. **Geometry** Sammie bought just enough fencing to border either a rectangular plot or a square plot, as shown. The perimeters of the plots are the same. How many meters of fencing did she buy?



H.O.T. Focus on Higher Order Thinking

22. **Justify Reasoning** Suppose you want to solve the equation $2a + b = 2a$, where a and b are nonzero real numbers. Describe the solution to this equation. Justify your description.

23. **Multi-Step** A patio in the shape of a rectangle, is fenced on all sides with 134 feet of fencing. The patio is 5 feet less wide than it is long.



a. What information can be used to solve the problem? How can you find the information?

b. Describe how to find the area of the patio. What is the area of the patio?

24. **Explain the Error** Kevin and Brittany write an equation to represent the following relationship, and both students solve their equation. Who found the correct equation and solution? Why is the other person incorrect?

5 times the difference of a number and 20 is the same as half the sum of 4 more than 4 times a number.

Kevin:

$$5(x - 20) = \frac{1}{2}(4x + 4)$$
$$5x - 100 = 2x + 2$$
$$3x - 100 = 2$$
$$3x = 102$$
$$x = 34$$

Brittany:

$$5(20 - x) = \frac{1}{2}(4x + 4)$$
$$100 - 5x = 2x + 2$$
$$100 - 7x = 2$$
$$-7x = -98$$
$$x = 14$$

25. **What If?** Alexa and Zack are solving the following problem.

The number of miles on Car A is 50 miles more than the number of miles on Car B, and the number of miles on Car B is 30 miles more than the number of miles on
Car C. All the cars travel 50 miles in 1 hour. After 1 hour, twice the number of miles on Car A is 70 miles less than 3 times the number of miles on Car C. How many miles were there on Car B initially?

Alexa assumes there are m miles on Car B. Zack assumes there are m miles on Car C. Will Zack's answer be the same as Alexa's answer? Explain.

Lesson Performance Task

Stacy, Oliver, and Jivesh each plan to put a certain amount of money into their savings accounts that earn simple interest of 6% per year. Stacy puts $550 more than Jivesh, and Oliver puts in 2 times as much as Jivesh. After a year, the amount in Stacy's account is 2 times the sum of $212 and the amount in Oliver's account. How much does each person initially put into his or her account? Who had the most money in his or her account after a year? Who had the least? Explain.