Trig Identities Packet

$\sin\theta = \frac{1}{\csc\theta}$	$\csc\theta = \frac{1}{\sin\theta}$	$\cos\theta = \frac{1}{\sec\theta}$	$\sec\theta = \frac{1}{\cos\theta}$
$\tan\theta = \frac{\sin\theta}{\cos\theta}$	$\tan\theta = \frac{1}{\cot\theta}$	$\cot\theta = \frac{\cos\theta}{\sin\theta}$	$\cot\theta = \frac{1}{\tan\theta}$
$\cos^2\theta + \sin^2\theta = 1$	$\sin^2\theta = 1 - \cos^2\theta$	$\cos^2\theta = 1 - \sin^2\theta$	
$1 + \tan^2\theta = \sec^2\theta$	$\tan^2\theta = \sec^2\theta - 1$	$-\tan^2\theta = 1 - \sec^2\theta$	
$1 + \cot^2\theta = \csc^2\theta$	$\cot^2\theta = \csc^2\theta - 1$	$-\cot^2\theta = 1 - \csc^2\theta$	

Advanced Math – March 2018

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
4	**5** **Quiz 6.6**	**6** **Trig Identities [D1]** HW: Worksheet	**7** **Trig Identities [D2]** HW: Worksheet	**8** **Trig Identities [D3]** HW: Worksheet	**9** **Review Trig Identities: Odds**	**10**
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$\tan\theta =$ $\cot\theta =$

$\csc\theta =$ $\sec\theta =$

Pythagorean Identity

Solve the Pythagorean Identity for $\cos^2\theta$

Solve the Pythagorean Identity for $\sin^2\theta$

Take the Pythagorean Identity and divide every single term by $\cos^2\theta$

$$\cos^2\theta + \sin^2\theta = 1$$

Solve the above equation for $\tan^2\theta$

Take the Pythagorean Identity and divide every single term by $\sin^2\theta$

$$\cos^2\theta + \sin^2\theta = 1$$

Solve the above equation for $\cot^2\theta$

Some other identities:

$\sin\theta =$ $\cos\theta =$ $\tan\theta =$

$\sin\theta = \frac{1}{\csc\theta}$	$\csc\theta = \frac{1}{\sin\theta}$	$\cos\theta = \frac{1}{\sec\theta}$	$\sec\theta = \frac{1}{\cos\theta}$
$\tan\theta = \frac{\sin\theta}{\cos\theta}$	$\tan\theta = \frac{1}{\cot\theta}$	$\cot\theta = \frac{\cos\theta}{\sin\theta}$	$\cot\theta = \frac{1}{\tan\theta}$
$\cos^2\theta + \sin^2\theta = 1$	$\sin^2\theta = 1 - \cos^2\theta$	$\cos^2\theta = 1 - \sin^2\theta$	
$1 + \tan^2\theta = \sec^2\theta$	$\tan^2\theta = \sec^2\theta - 1$	$-\tan^2\theta = 1 - \sec^2\theta$	
$1 + \cot^2\theta = \csc^2\theta$	$\cot^2\theta = \csc^2\theta - 1$	$-\cot^2\theta = 1 - \csc^2\theta$	

Example 1: Use Trigonometric Identities to write each expression in terms of a single trigonometric identity or a constant.

a. $\tan\theta\cos\theta$

b. $\frac{1-\cos^2\theta}{\cos^2\theta}$

c. $\cos\theta\csc\theta$

d. $\frac{\sin\theta\sec\theta}{\tan\theta}$

Example 2: Simplify the complex fraction.

a. $\frac{\frac{2}{3}}{\frac{4}{15}}$

b. $\frac{\frac{4}{5}}{\frac{4}{35}}$

c. $\frac{\frac{2}{5}}{\frac{3}{5}}$

d. $\frac{\frac{1}{2}}{2}$

$\sin\theta = \frac{1}{\csc\theta}$	$\csc\theta = \frac{1}{\sin\theta}$	$\cos\theta = \frac{1}{\sec\theta}$	$\sec\theta = \frac{1}{\cos\theta}$
$\tan\theta = \frac{\sin\theta}{\cos\theta}$	$\tan\theta = \frac{1}{\cot\theta}$	$\cot\theta = \frac{\cos\theta}{\sin\theta}$	$\cot\theta = \frac{1}{\tan\theta}$
$\cos^2\theta + \sin^2\theta = 1$	$\sin^2\theta = 1 - \cos^2\theta$	$\cos^2\theta = 1 - \sin^2\theta$	
$1 + \tan^2\theta = \sec^2\theta$	$\tan^2\theta = \sec^2\theta - 1$	$-\tan^2\theta = 1 - \sec^2\theta$	
$1 + \cot^2\theta = \csc^2\theta$	$\cot^2\theta = \csc^2\theta - 1$	$-\cot^2\theta = 1 - \csc^2\theta$	

Example 3: Simplify the complex fraction.

a. $\frac{\csc\theta}{\cot\theta}$

b. $\frac{1-\cos^2\theta}{\tan^2\theta}$

c. $\frac{\cos\theta\sec\theta}{\tan\theta}$

d. $\frac{\sin\theta}{\csc\theta}$

Use Trigonometric Identities to write each expression in terms of a single trigonometric identity or a constant.

1. $\cot\theta \sin\theta$

1.)_____

2. $\frac{1-\sin^2\theta}{\sin^2\theta}$

2.)_____

3. $\sin\theta \sec\theta$

3.)_____

4. $\frac{\cos\theta \csc\theta}{\cot\theta}$

4.)_____

Simplify the complex fraction.

5. $\frac{\sec\theta}{\tan\theta}$

5.)_____

6. $\frac{1-\sin^2\theta}{\cot^2\theta}$

6.)_____

7. $\frac{\sin\theta \csc\theta}{\cot\theta}$

7.)_____

8. $\frac{\cos\theta}{\sec\theta}$

8.)_____

Example 1: **Simplify**

a. $\frac{\tan\theta + \cot\theta}{\tan\theta}$

b. $\frac{\cos^2\theta}{1-\sin\theta}$

c. $\frac{\sec^2\theta - 1}{\sec^2\theta}$

d. $\tan\theta \csc\theta \cos\theta$

To **VERIFY AN IDENTITY**: Work on each side separately and make sure you don't move things from one side to the other! You can work on both sides at the same time – but you just can't move things from one side to the other.

Verify the identity.

Example 1: $\sin\theta \cot\theta \sec\theta = 1$

Example 2: $1 - 2\sin^2\theta = 2\cos^2\theta - 1$

Example 3: **Factor**

a. $a^2 - a^2b$

b. $x^2 - 2x + 1$

Example 4: ***Verify the identity.*** $\csc^2\theta - \cos^2\theta\csc^2\theta = 1$

Example 5: **Simplify**

a. $(\sin\theta - \cos\theta)(\sin\theta + \cos\theta)$

There are two different ways you can leave this answer! In the notes, leave it in terms of $sin^2\theta$. *In the homework, you will be "verifying" and leaving it in terms of* $cos^2\theta$

b. $(\tan\theta + 1)^2$

c. $\sin^2\theta - 2\sin\theta + 1$

Simplify the complex fraction.

1. $\dfrac{\csc\theta - \sin\theta}{\csc\theta}$

2. $\dfrac{\sin^2\theta}{1+\cos\theta}$

1.)_____

2.)_____

3. $\dfrac{\csc^2\theta - 1}{\csc^2\theta}$

4. $\tan\theta \sec\theta \sin\theta$

3.)_____

4.)_____

Verify the identity. Both sides should end up being equal, so you will not find these on the answer key.

5. $\tan\theta \csc\theta \cos\theta = 1$

6. $(\sin\theta - \cos\theta)(\sin\theta + \cos\theta) = 1 - 2\cos^2\theta$

7. $\dfrac{\sin\theta}{1+\cos\theta} \cdot \dfrac{1-\cos\theta}{1-\cos\theta} = \dfrac{1-\cos\theta}{\sin\theta}$

8. $\sin^2\theta(1 + \cot^2\theta) = 1$

Verify the identity. Both sides should end up being equal, so you will not find these on the answer key.

9. $\frac{\sec\theta - \cos\theta}{\sec\theta} = \sin^2\theta$

10. $\frac{\cot\theta \sec\theta}{\csc\theta} = 1$

11. $\frac{1+\tan^2\theta}{\sec\theta} = \sec\theta$

12. $(1-\cos\theta)(1+\cos\theta) = \frac{1}{\csc^2\vartheta}$

Example 1: Simplify

a. $\frac{2}{3} + \frac{1}{4}$

b. $\frac{1}{\cos\theta} + \frac{1}{\sin\theta}$

c. $\frac{1}{1-\cos\theta} + \frac{1}{1+\cos\theta}$

d. $\tan\theta - \frac{\sec^2\theta}{\tan\theta}$

e. $\frac{\tan\theta}{\cot\theta} + 1$

f. $\frac{1}{\cos\theta} + \frac{1}{\sin\theta}$

Simplify.

1. $\frac{\sin\theta}{\csc\theta} + \frac{\cos\theta}{\sec\theta}$

2. $\frac{\csc^2\theta - 1}{\cot\theta}$

Verify the identity. Both sides should end up being equal, so you will not find these on the answer key.

3. $\frac{1+\sec^2\theta}{\sec^2\theta} = 1 + \cos^2\theta$

4. $\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = \frac{1}{\cos\theta\sin\theta}$

5. $\sec^2\theta - \sin^2\theta\sec^2\theta = 1$

6. $\frac{\sin^2\theta - 2\sin\theta + 1}{\sin\theta - 1} = \sin\theta - 1$

7. $\frac{1}{1-\sin\theta} + \frac{1}{1+\sin\theta}$

8. $\cot\theta - \frac{\csc^2\theta}{\cot\theta}$

SOLUTIONS

D1

1.	$\cos\theta$	**2**.	$\cot^2\theta$	**3**.	$\tan\theta$	**4**.	1
5.	$\csc\theta$	**6**.	$\sin^2\theta$	**7**.	$\tan\theta$	**8**.	$\cos^2\theta$

D2

1.	$\cos^2\theta$	**2**.	$1-\cos\theta$	**3**.	$\cos^2\theta$	**4**.	$\tan^2\theta$

D3

1.	1	**2**.	$\cot\theta$