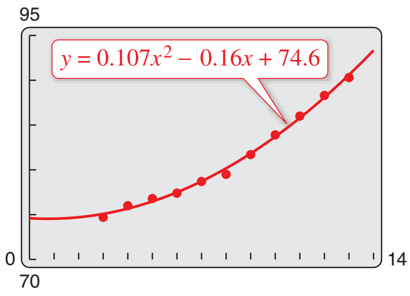
2 Polynomial and Rational Functions



Section 2.8, Example 4
People Not in the
U.S. Labor Force

2.1 Quadratic Functions

The Graph of a Quadratic Function

In this and the next section, you will study the graphs of polynomial functions.

Definition of Polynomial Function

Let n be a nonnegative integer and let $a_n, a_{n-1}, \ldots a_2, a_1, a_0$ be real numbers with $a_n \neq 0$. The function given by

$$f(x) = a_nx^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0$$

is called a **polynomial function of x with degree n.**

Polynomial functions are classified by degree. For instance, the polynomial function

$$f(x) = a, \quad a \neq 0 \qquad \text{Constant function}$$

has degree 0 and is called a **constant function.** In Chapter 1, you learned that the graph of this type of function is a horizontal line. The polynomial function

$$f(x) = mx + b, \quad m \neq 0 \qquad \text{Linear function}$$

has degree 1 and is called a **linear function.** You learned in Chapter 1 that the graph of $f(x) = mx + b$ is a line whose slope is m and whose y-intercept is $(0, b)$. In this section, you will study second-degree polynomial functions, which are called **quadratic functions.**

Definition of Quadratic Function

Let a, b, and c be real numbers with $a \neq 0$. The function given by

$$f(x) = ax^2 + bx + c \qquad \text{Quadratic function}$$

is called a **quadratic function.**

Often real-life data can be modeled by quadratic functions. For instance, the table at the right shows the height h (in feet) of a projectile fired from a height of 6 feet with an initial velocity of 256 feet per second at any time t (in seconds). A quadratic model for the data in the table is

$$h(t) = -16t^2 + 256t + 6 \quad \text{for} \quad 0 \leq t \leq 16.$$

Time, (in seconds) t	Height, (in feet) h
0	6
2	454
4	774
6	966
8	1030
10	966
12	774
14	454
16	6

The graph of a quadratic function is a special type of U-shaped curve called a **parabola.** Parabolas occur in many real-life applications, especially those involving reflective properties, such as satellite dishes or flashlight reflectors. You will study these properties in a later chapter.

All parabolas are symmetric with respect to a line called the **axis of symmetry,** or simply the **axis** of the parabola. The point where the axis intersects the parabola is called the **vertex** of the parabola. These and other basic characteristics of quadratic functions are summarized on the next page.

What you should learn

- Analyze graphs of quadratic functions.
- Write quadratic functions in standard form and use the results to sketch graphs of functions.
- Find minimum and maximum values of quadratic functions in real-life applications.

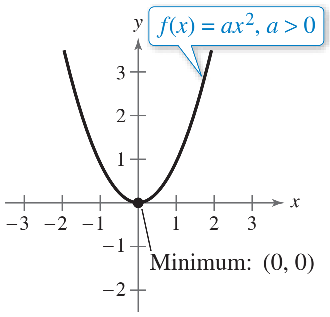
Why you should learn it

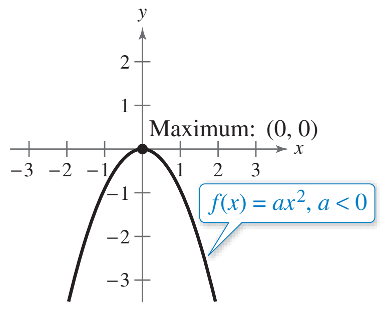
Quadratic functions can be used to model the design of a room. For instance, Exercise 63 on page 97 shows how the size of an indoor fitness room with a running track can be modeled.

Basic Characteristics of Quadratic Functions

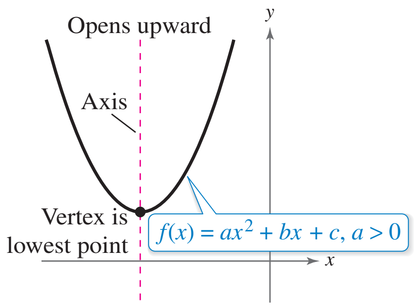
Graph of $f(x) = ax^2, a > 0$
Domain: $(-\infty, \infty)$
Range: $[0, \infty)$
Intercept: $(0, 0)$
Decreasing on $(-\infty, 0)$
Increasing on $(0, \infty)$
Even function
Axis of symmetry: $x = 0$
Relative minimum or vertex: $(0, 0)$

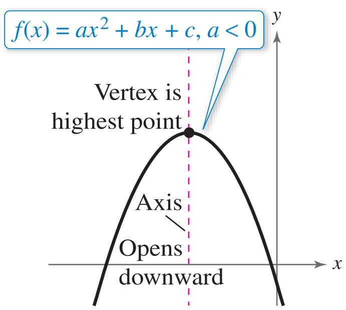
Graph of $f(x) = ax^2, a < 0$
Domain: $(-\infty, \infty)$
Range: $(-\infty, 0]$
Intercept: $(0, 0)$
Increasing on $(-\infty, 0)$
Decreasing on $(0, \infty)$
Even function
Axis of symmetry: $x = 0$
Relative maximum or vertex: $(0, 0)$



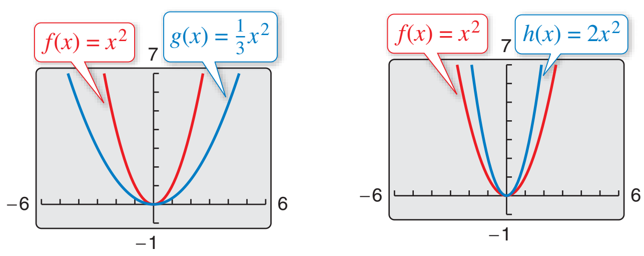


For the general quadratic form $f(x) = ax^2 + bx + c$, when the leading coefficient a is positive, the parabola opens upward; and when the leading coefficient a is negative, the parabola opens downward. Later in this section you will learn ways to find the coordinates of the vertex of a parabola.





When sketching the graph of $f(x) = ax^2$, it is helpful to use the graph of $y = x^2$ as a reference, as discussed in Section 1.4. There you saw that when $a > 1$, the graph of $y = af(x)$ is a vertical stretch of the graph of $y = f(x)$. When $0 < a < 1$, the graph of $y = af(x)$ is a vertical shrink of the graph of $y = f(x)$. Notice in Figure 2.1 that the coefficient a determines how widely the parabola given by $f(x) = ax^2$ opens. When $|a|$ is small, the parabola opens more widely than when $|a|$ is large.



Vertical shrink ***Vertical stretch***

Figure 2.1

Library of Parent Functions: Quadratic Functions

The *parent quadratic function* is $f(x) = x^2$, also known as the *squaring function*. The basic characteristics of the parent quadratic function are summarized below and on the inside cover of this text.

Graph of $f(x) = x^2$

Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

Intercept: $(0, 0)$

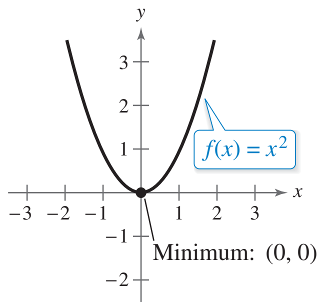
Decreasing on $(-\infty, 0)$

Increasing on $(0, \infty)$

Even function

Axis of symmetry: $x = 0$

Relative minimum or vertex: $(0, 0)$



Recall from Section 1.4 that the graphs of $y = f(x \pm c)$, $y = f(x) \pm c$, $y = -f(x)$, and $y = f(-x)$ are rigid transformations of the graph of $y = f(x)$.

$y = f(x \pm c)$	Horizontal shift	$y = -f(x)$	Reflection in x-axis
$y = f(x) \pm c$	Vertical shift	$y = f(-x)$	Reflection in y-axis

Emphasize that the technique illustrated in Example 1 [comparing functions with the parent function $f(x) = x^2$] is very useful when analyzing functions and preparing to graph them. This kind of exercise (see Exercises 9–16 in the exercise set) also helps build conceptual understanding. You may want to cover extra examples of this technique during class for practice.

EXAMPLE 1 Library of Parent Functions: $f(x) = x^2$

See LarsonPrecalculus.com for an interactive version of this type of example.

Sketch the graph of each function by hand and compare it with the graph of $f(x) = x^2$.

a. $g(x) = -x^2 + 1$ **b.** $h(x) = (x + 2)^2 - 3$

Solution

a. With respect to the graph of $f(x) = x^2$, the graph of g is obtained by a *reflection* in the x-axis and a vertical shift one unit *upward*, as shown in Figure 2.2. Confirm this with a graphing utility.

b. With respect to the graph of $f(x) = x^2$, the graph of h is obtained by a horizontal shift two units *to the left* and a vertical shift three units *downward*, as shown in Figure 2.3. Confirm this with a graphing utility.

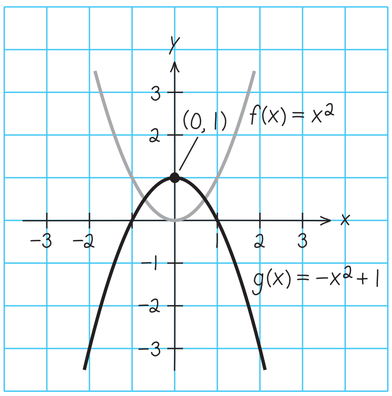


Figure 2.2

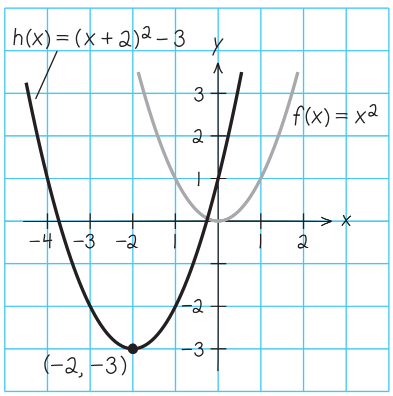


Figure 2.3

✔ Checkpoint *Audio-video solution in English & Spanish at LarsonPrecalculus.com.*

Sketch the graph of each function by hand and compare it with the graph of $f(x) = x^2$.

a. $g(x) = x^2 - 4$ **b.** $h(x) = (x - 3)^2 + 1$

The Standard Form of a Quadratic Function

The equation in Example 1(b) is written in the **standard form**

$$f(x) = a(x - h)^2 + k.$$

This form is especially convenient for sketching a parabola because it identifies the vertex of the parabola as (h, k).

Standard Form of a Quadratic Function

The quadratic function given by

$$f(x) = a(x - h)^2 + k, \quad a \neq 0$$

is in **standard form.** The graph of f is a parabola whose axis is the vertical line $x = h$ and whose vertex is the point (h, k). When $a > 0$, the parabola opens upward, and when $a < 0$, the parabola opens downward.

EXAMPLE 2 Identifying the Vertex of a Quadratic Function

Describe the graph of

$$f(x) = 2x^2 + 8x + 7$$

and identify the vertex.

Solution

Write the quadratic function in standard form by completing the square. Recall that the first step is to factor out any coefficient of x^2 that is not 1.

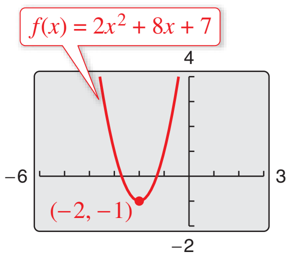
$f(x) = 2x^2 + 8x + 7$	Write original function.
$= (2x^2 + 8x) + 7$	Group x-terms.
$= 2(x^2 + 4x) + 7$	Factor 2 out of x-terms.
$= 2(x^2 + 4x + \underbrace{4}_{(4/2)^2} - 4) + 7$	Add and subtract $(4/2)^2 = 4$ within parentheses to complete the square.
$= 2(x^2 + 4x + 4) - 2(4) + 7$	Regroup terms.
$= 2(x + 2)^2 - 1$	Write in standard form.

From the standard form, you can see that the graph of f is a parabola that opens upward with vertex

$$(-2, -1)$$

as shown in the figure. This corresponds to a left shift of two units and a downward shift of one unit relative to the graph of

$$y = 2x^2.$$



Checkpoint Audio-video solution in English & Spanish at LarsonPrecalculus.com.

Describe the graph of $f(x) = 3x^2 - 6x + 4$ and identify the vertex.

To find the x-intercepts of the graph of $f(x) = ax^2 + bx + c$, solve the equation $ax^2 + bx + c = 0$. When $ax^2 + bx + c$ does not factor, you can use the Quadratic Formula to find the x-intercepts, or a graphing utility to approximate the x-intercepts. Remember, however, that a parabola may not have x-intercepts.

Explore the Concept

Use a graphing utility to graph $y = ax^2$ with $a = -2, -1, -0.5, 0.5, 1$, and 2. How does changing the value of a affect the graph?

Use a graphing utility to graph $y = (x - h)^2$ with $h = -4, -2, 2$, and 4. How does changing the value of h affect the graph?

Use a graphing utility to graph $y = x^2 + k$ with $k = -4, -2, 2$, and 4. How does changing the value of k affect the graph?

EXAMPLE 3 Identifying x-Intercepts of a Quadratic Function

Describe the graph of $f(x) = -x^2 + 6x - 8$ and identify any x-intercepts.

Solution

$$f(x) = -x^2 + 6x - 8 \qquad \text{Write original function.}$$

$$= -(x^2 - 6x) - 8 \qquad \text{Factor } -1 \text{ out of } x\text{-terms.}$$

$$= -(x^2 - 6x + \underbrace{9 - 9}_{(-6/2)^2}) - 8 \qquad \text{Add and subtract } (-6/2)^2 = 9 \text{ within parentheses.}$$

$$= -(x^2 - 6x + 9) - (-9) - 8 \qquad \text{Regroup terms.}$$

$$= -(x - 3)^2 + 1 \qquad \text{Write in standard form.}$$

The graph of f is a parabola that opens downward with vertex $(3, 1)$, as shown in Figure 2.4. The x-intercepts are determined as follows.

$$-(x^2 - 6x + 8) = 0 \qquad \text{Factor out } -1.$$

$$-(x - 2)(x - 4) = 0 \qquad \text{Factor.}$$

$$x - 2 = 0 \Rightarrow x = 2 \qquad \text{Set 1st factor equal to 0.}$$

$$x - 4 = 0 \Rightarrow x = 4 \qquad \text{Set 2nd factor equal to 0.}$$

So, the x-intercepts are $(2, 0)$ and $(4, 0)$, as shown in Figure 2.4.

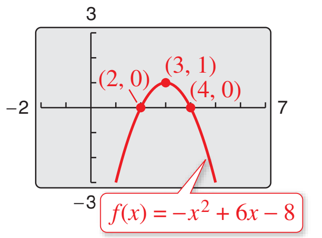


Figure 2.4

Checkpoint Audio-video solution in English & Spanish at LarsonPrecalculus.com.

Describe the graph of $f(x) = x^2 - 4x + 3$ and identify any x-intercepts.

EXAMPLE 4 Writing the Equation of a Parabola

Write the standard form of the equation of the parabola whose vertex is $(1, 2)$ and that passes through the point $(3, -6)$.

Solution

Because the vertex of the parabola is $(h, k) = (1, 2)$, the equation has the form

$$f(x) = a(x - 1)^2 + 2. \qquad \text{Substitute for } h \text{ and } k \text{ in standard form.}$$

Because the parabola passes through the point $(3, -6)$, it follows that $f(3) = -6$. So, you obtain

$$f(x) = a(x - 1)^2 + 2 \qquad \text{Write in standard form.}$$

$$-6 = a(3 - 1)^2 + 2 \qquad \text{Substitute } -6 \text{ for } f(x) \text{ and 3 for } x.$$

$$-6 = 4a + 2 \qquad \text{Simplify.}$$

$$-8 = 4a \qquad \text{Subtract 2 from each side.}$$

$$-2 = a. \qquad \text{Divide each side by 4.}$$

The equation in standard form is $f(x) = -2(x - 1)^2 + 2$. You can confirm this answer by graphing f with a graphing utility, as shown in Figure 2.5. Use the *zoom* and *trace* features or the *maximum* and *value* features to confirm that its vertex is $(1, 2)$ and that it passes through the point $(3, -6)$.

Remark

In Example 4, there are infinitely many different parabolas that have a vertex at $(1, 2)$. Of these, however, the only one that passes through the point $(3, -6)$ is the one given by

$$f(x) = -2(x - 1)^2 + 2.$$

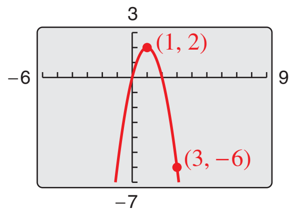


Figure 2.5

Checkpoint Audio-video solution in English & Spanish at LarsonPrecalculus.com.

Write the standard form of the equation of the parabola whose vertex is $(-4, 11)$ and that passes through the point $(-6, 15)$.

Finding Minimum and Maximum Values

Many applications involve finding the maximum or minimum value of a quadratic function. By completing the square of the quadratic function $f(x) = ax^2 + bx + c$, you can rewrite the function in standard form.

$$f(x) = a\left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right) \qquad \text{Standard form}$$

So, the vertex of the graph of f is $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$, which implies the following.

Minimum and Maximum Values of Quadratic Functions

Consider the function $f(x) = ax^2 + bx + c$ with vertex $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$.

1. If $a > 0$, then f has a *minimum* at $x = -\frac{b}{2a}$.

The minimum value is $f\left(-\frac{b}{2a}\right)$.

2. If $a < 0$, then f has a *maximum* at $x = -\frac{b}{2a}$.

The maximum value is $f\left(-\frac{b}{2a}\right)$.

EXAMPLE 5 The Maximum Height of a Projectile

The path of a baseball is given by the function $f(x) = -0.0032x^2 + x + 3$, where $f(x)$ is the height of the baseball (in feet) and x is the horizontal distance from home plate (in feet). What is the maximum height reached by the baseball?

Algebraic Solution

For this quadratic function, you have

$$f(x) = ax^2 + bx + c = -0.0032x^2 + x + 3$$

which implies that $a = -0.0032$ and $b = 1$. Because the function has a maximum when $x = -b/(2a)$, you can conclude that the baseball reaches its maximum height when it is x feet from home plate, where x is

$$x = -\frac{b}{2a}$$

$$= -\frac{1}{2(-0.0032)}$$

$$= 156.25 \text{ feet.}$$

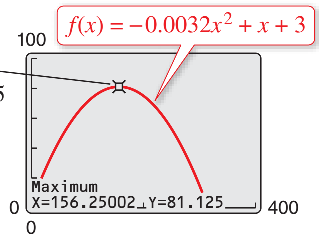
At this distance, the maximum height is

$$f(156.25) = -0.0032(156.25)^2 + 156.25 + 3$$

$$= 81.125 \text{ feet.}$$

Graphical Solution

The maximum height is $y \approx 81.125$ feet at $x \approx 156.25$ feet.



Checkpoint Audio-video solution in English & Spanish at LarsonPrecalculus.com.

Rework Example 5 when the path of the baseball is given by the function

$$f(x) = -0.007x^2 + x + 4.$$

2.1 Exercises

See *CalcChat.com* for tutorial help and worked-out solutions to odd-numbered exercises.
For instructions on how to use a graphing utility, see Appendix A.

Vocabulary and Concept Check

In Exercises 1 and 2, fill in the blanks.

1. A polynomial function with degree n and leading coefficient a_n is a function of the form $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdot \cdot \cdot + a_2x^2 + a_1x + a_0,\ a_n \neq 0$, where n is a ________ and $a_n, a_{n-1}, \ldots, a_2, a_1, a_0$ are ________ numbers.

2. A ________ function is a second-degree polynomial function, and its graph is called a ________ .

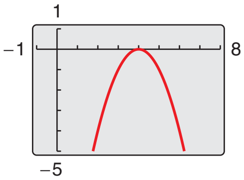
3. Is the quadratic function $f(x) = (x - 2)^2 + 3$ written in standard form? Identify the vertex of the graph of f.

4. Does the graph of the quadratic function $f(x) = -3x^2 + 5x + 2$ have a relative minimum value at its vertex?

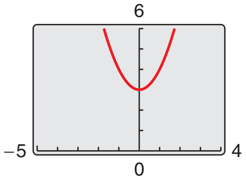
Procedures and Problem Solving

Graphs of Quadratic Functions **In Exercises 5–8, match the quadratic function with its graph. [The graphs are labeled (a), (b), (c), and (d).]**

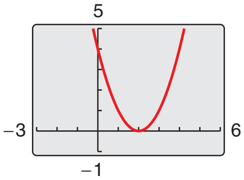
(a)



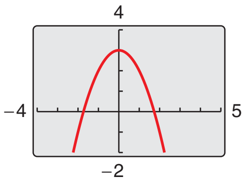
(b)



(c)



(d)



5. $f(x) = (x - 2)^2$

6. $f(x) = 3 - x^2$

7. $f(x) = x^2 + 3$

8. $f(x) = -(x - 4)^2$

Library of Parent Functions **In Exercises 9–16, sketch the graph of the function and compare it with the graph of $y = x^2$.**

9. $y = -x^2$

10. $y = x^2 - 1$

11. $y = (x + 3)^2$

12. $y = -(x + 3)^2 - 1$

13. $y = (x + 1)^2$

14. $y = -x^2 + 2$

15. $y = (x - 3)^2$

16. $y = -(x - 3)^2 + 1$

Identifying the Vertex of a Quadratic Function **In Exercises 17–30, describe the graph of the function and identify the vertex. Use a graphing utility to verify your results.**

17. $f(x) = 20 - x^2$

18. $f(x) = x^2 + 8$

19. $f(x) = \frac{1}{2}x^2 - 5$

20. $f(x) = -6 - \frac{1}{4}x^2$

21. $f(x) = (x + 3)^2 - 4$

22. $f(x) = (x - 7)^2 + 2$

23. $h(x) = x^2 - 2x + 1$

24. $g(x) = x^2 + 16x + 64$

25. $f(x) = x^2 - x + \frac{5}{4}$

26. $f(x) = x^2 + 3x + \frac{1}{4}$

27. $f(x) = -x^2 + 2x + 5$

28. $f(x) = -x^2 - 4x + 1$

29. $h(x) = 4x^2 - 4x + 21$

30. $f(x) = 2x^2 - x + 1$

Identifying *x*-Intercepts of a Quadratic Function **In Exercises 31–36, describe the graph of the quadratic function. Identify the vertex and *x*-intercept(s). Use a graphing utility to verify your results.**

31. $g(x) = x^2 + 8x + 11$

32. $f(x) = x^2 + 10x + 14$

33. $f(x) = -(x^2 - 2x - 15)$

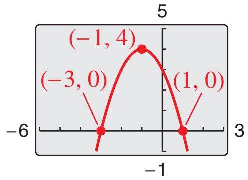
34. $f(x) = -(x^2 + 3x - 4)$

35. $f(x) = -2x^2 + 16x - 31$

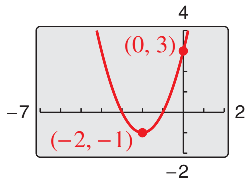
36. $f(x) = -4x^2 + 24x - 41$

Writing the Equation of a Parabola in Standard Form **In Exercises 37 and 38, write an equation of the parabola in standard form. Use a graphing utility to graph the equation and verify your result.**

37.



38.

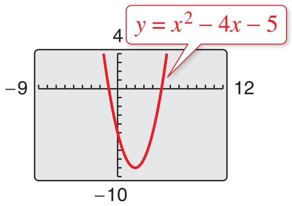


Writing the Equation of a Parabola in Standard Form **In Exercises 39–44, write the standard form of the quadratic function that has the indicated vertex and whose graph passes through the given point. Use a graphing utility to verify your result.**

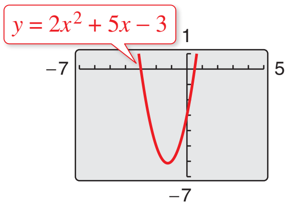
39. Vertex: $(-2, 5)$;	Point: $(0, 9)$
40. Vertex: $(4, 1)$;	Point: $(6, -7)$
41. Vertex: $(1, -2)$;	Point: $(-1, 14)$
42. Vertex: $(-4, -1)$;	Point: $(-2, 4)$
43. Vertex: $\left(\frac{1}{2}, 1\right)$;	Point: $\left(-2, -\frac{21}{5}\right)$
44. Vertex: $\left(-\frac{1}{4}, -1\right)$;	Point: $\left(0, -\frac{17}{16}\right)$

Using a Graph to Identify x-Intercepts **In Exercises 45–48, determine the x-intercept(s) of the graph visually. Then find the x-intercept(s) algebraically to verify your answer.**

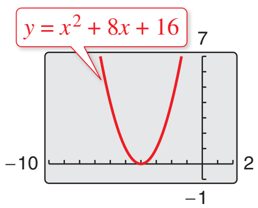
45.



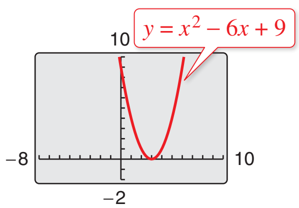
46.



47.



48.



Graphing to Identify x-Intercepts **In Exercises 49–54, use a graphing utility to graph the quadratic function and find the x-intercepts of the graph. Then find the x-intercepts algebraically to verify your answer.**

49. $y = x^2 - 4x$	**50.** $y = -2x^2 + 10x$
51. $y = 2x^2 - 7x - 30$	**52.** $y = 4x^2 + 25x - 21$
53. $y = -\frac{1}{2}(x^2 - 6x - 7)$	**54.** $y = \frac{7}{10}(x^2 + 12x - 45)$

Using the x-Intercepts to Write Equations **In Exercises 55–58, find two quadratic functions, one that opens upward and one that opens downward, whose graphs have the given x-intercepts. (There are many correct answers.)**

55. $(-1, 0), (3, 0)$	**56.** $(0, 0), (10, 0)$
57. $(-3, 0), \left(-\frac{1}{2}, 0\right)$	**58.** $\left(-\frac{5}{2}, 0\right), (2, 0)$

Maximizing a Product of Two Numbers **In Exercises 59–62, find the two positive real numbers with the given sum whose product is a maximum.**

59. The sum is 110.	**60.** The sum is 66.

61. The sum of the first and twice the second is 24.

62. The sum of the first and three times the second is 42.

63. ***Why you should learn it*** *(p. 90)* An indoor physical fitness room consists of a rectangular region with a semicircle on each end. The perimeter of the room is to be a 200-meter single-lane running track.

(a) Draw a diagram that illustrates the problem. Let x and y represent the length and width of the rectangular region, respectively.

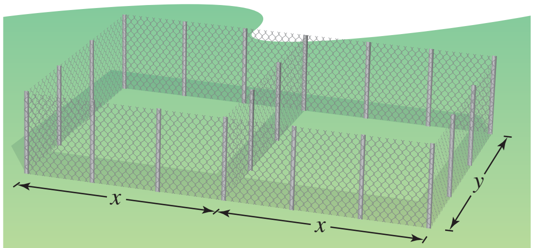
(b) Determine the radius of the semicircular ends of the track. Determine the distance, in terms of y, around the inside edge of the two semicircular parts of the track.

(c) Use the result of part (b) to write an equation, in terms of x and y, for the distance traveled in one lap around the track. Solve for y.

(d) Use the result of part (c) to write the area A of the rectangular region as a function of x.

(e) Use a graphing utility to graph the area function from part (d). Use the graph to approximate the dimensions that will produce a rectangle of maximum area.

64. **Algebraic-Graphical-Numerical** A child-care center has 200 feet of fencing to enclose two adjacent rectangular safe play areas (see figure). Use the following methods to determine the dimensions that will produce a maximum enclosed area.



(a) Write the total area A of the play areas as a function of x.

(b) Use the *table* feature of a graphing utility to create a table showing possible values of x and the corresponding total area A of the play areas. Use the table to estimate the dimensions that will produce the maximum enclosed area.

(c) Use the graphing utility to graph the area function. Use the graph to approximate the dimensions that will produce the maximum enclosed area.

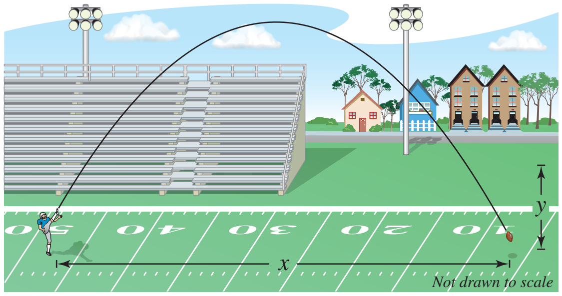
(d) Write the area function in standard form to find algebraically the dimensions that will produce the maximum enclosed area.

(e) Compare your results from parts (b), (c), and (d).

65. Height of a Projectile The height y (in feet) of a punted football is approximated by

$$y = -\frac{16}{2025}x^2 + \frac{9}{5}x + \frac{3}{2}$$

where x is the horizontal distance (in feet) from where the football is punted. (See figure.)



(a) Use a graphing utility to graph the path of the football.

(b) How high is the football when it is punted? (*Hint:* Find y when $x = 0$.)

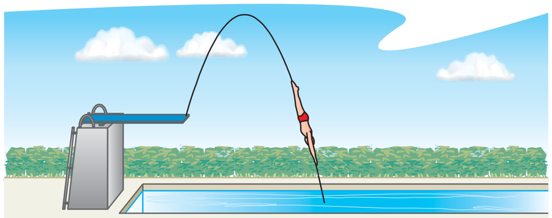
(c) What is the maximum height of the football?

(d) How far from the punter does the football strike the ground?

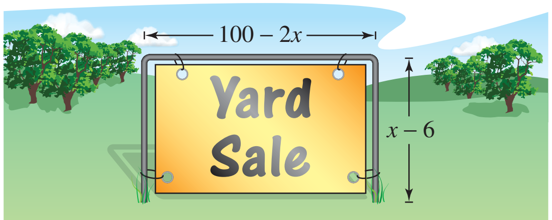
66. Physics The path of a diver is approximated by

$$y = -\frac{4}{9}x^2 + \frac{24}{9}x + 12$$

where y is the height (in feet) and x is the horizontal distance (in feet) from the end of the diving board (see figure). What is the maximum height of the diver?



67. Geometry You have a steel wire that is 100 inches long. To make a sign holder, you bend the wire x inches from each end to form two right angles. To use the sign holder, you insert each end 6 inches into the ground. (See figure.)



(a) Write a function for the rectangular area A enclosed by the sign holder in terms of x.

(b) Use the *table* feature of a graphing utility to determine the value of x that maximizes the rectangular area enclosed by the sign holder.

68. Economics The monthly revenue R (in thousands of dollars) from the sales of a digital picture frame is approximated by $R(p) = -10p^2 + 1580p$, where p is the price per unit (in dollars).

(a) Find the monthly revenues for unit prices of \$50, \$70, and \$90.

(b) Find the unit price that will yield a maximum monthly revenue.

(c) What is the maximum monthly revenue?

69. Public Health For selected years from 1955 through 2010, the annual per capita consumption C of cigarettes by Americans (ages 18 and older) can be modeled by

$$C(t) = -1.39t^2 + 36.5t + 3871, \quad 5 \le t \le 60$$

where t is the year, with $t = 5$ corresponding to 1955. (Sources: Centers for Disease Control and Prevention and U.S. Census Bureau)

(a) Use a graphing utility to graph the model.

(b) Use the graph of the model to approximate the year when the maximum annual consumption of cigarettes occurred. Approximate the maximum average annual consumption.

(c) Beginning in 1966, all cigarette packages were required by law to carry a health warning. Do you think the warning had any effect? Explain.

(d) In 2010, the U.S. population (ages 18 and older) was 234,564,000. Of those, about 45,271,000 were smokers. What was the average annual cigarette consumption *per smoker* in 2010? What was the average daily cigarette consumption *per smoker*?

70. Demography The population P of Germany (in thousands) from 2000 through 2013 can be modeled by

$$P(t) = -14.82t^2 + 95.9t + 82{,}276, \quad 0 \le t \le 13$$

where t is the year, with $t = 0$ corresponding to 2000. (Source: U.S. Census Bureau)

(a) According to the model, in what year did Germany have its greatest population? What was the population?

(b) According to the model, what will Germany's population be in the year 2075? Is this result reasonable? Explain.

Conclusions

True or False? **In Exercises 71–74, determine whether the statement is true or false. Justify your answer.**

71. The function $f(x) = -12x^2 - 1$ has no x-intercepts.

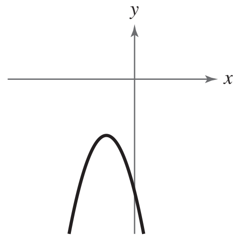
72. The function $f(x) = a(x - 5)^2$ has exactly one x-intercept for any nonzero value of a.

73. The functions $f(x) = 3x^2 + 6x + 7$ and $g(x) = 3x^2 + 6x - 1$ have the same vertex.

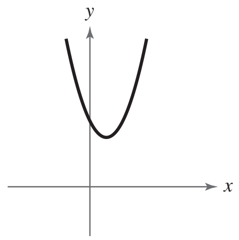
74. The graphs of $f(x) = -4x^2 - 10x + 7$ and $g(x) = 12x^2 + 30x + 1$ have the same axis of symmetry.

Library of Parent Functions **In Exercises 75 and 76, determine which equation(s) may be represented by the graph shown. (There may be more than one correct answer.)**

75. (a) $f(x) = -(x-4)^2 + 2$
(b) $f(x) = -(x+2)^2 + 4$
(c) $f(x) = -(x+2)^2 - 4$
(d) $f(x) = -x^2 - 4x - 8$
(e) $f(x) = -(x-2)^2 - 4$
(f) $f(x) = -x^2 + 4x - 8$



76. (a) $f(x) = (x-1)^2 + 3$
(b) $f(x) = (x+1)^2 + 3$
(c) $f(x) = (x-3)^2 + 1$
(d) $f(x) = x^2 + 2x + 4$
(e) $f(x) = (x+3)^2 + 1$
(f) $f(x) = x^2 + 6x + 10$



Describing Parabolas **In Exercises 77–80, let z represent a positive real number. Describe how the family of parabolas represented by the given function compares with the graph of $g(x) = x^2$.**

77. $f(x) = (x - z)^2$
78. $f(x) = x^2 - z$
79. $f(x) = z(x-3)^2$
80. $f(x) = zx^2 + 4$

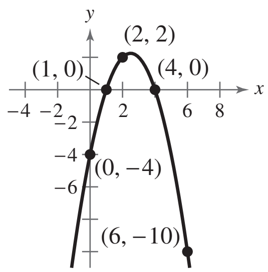
Think About It **In Exercises 81–84, find the value of b such that the function has the given maximum or minimum value.**

81. $f(x) = -x^2 + bx - 75$; Maximum value: 25
82. $f(x) = -x^2 + bx - 16$; Maximum value: 48
83. $f(x) = x^2 + bx + 26$; Minimum value: 10
84. $f(x) = x^2 + bx - 25$; Minimum value: -50

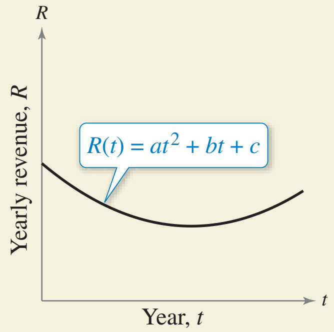
85. Proof Let x and y be two positive real numbers whose sum is S. Show that the maximum product of x and y occurs when x and y are both equal to $S/2$.

86. Proof Assume that the function $f(x) = ax^2 + bx + c$, $a \neq 0$, has two real zeros. Show that the x-coordinate of the vertex of the graph is the average of the zeros of f. (*Hint:* Use the Quadratic Formula.)

87. Writing The parabola in the figure has an equation of the form $y = ax^2 + bx - 4$. Find the equation of this parabola two different ways, by hand and with technology. Write a paragraph describing the methods you used and comparing the results.



88. HOW DO YOU SEE IT? The graph shows a quadratic function of the form $R(t) = at^2 + bt + c$, which represents the yearly revenues for a company, where $R(t)$ is the revenue in year t.



(a) Is the value of a positive, negative, or zero?
(b) Write an expression in terms of a and b that represents the year t when the company had the least revenue.
(c) The company made the same yearly revenues in 2004 and 2014. Estimate the year in which the company had the least revenue.
(d) Assume that the model is still valid today. Are the yearly revenues currently increasing, decreasing, or constant? Explain.

89. Think About It The annual profit P (in dollars) of a company is modeled by a function of the form $P = at^2 + bt + c$, where t represents the year. Discuss which of the following models the company might prefer.
(a) a is positive and $t \geq -b/(2a)$.
(b) a is positive and $t \leq -b/(2a)$.
(c) a is negative and $t \geq -b/(2a)$.
(d) a is negative and $t \leq -b/(2a)$.

Cumulative Mixed Review

Finding Points of Intersection **In Exercises 90–93, determine algebraically any point(s) of intersection of the graphs of the equations. Verify your results using the *intersect* feature of a graphing utility.**

90. $x + y = 8$
$-\frac{2}{3}x + y = 6$

91. $y = 3x - 10$
$y = \frac{1}{4}x + 1$

92. $y = 9 - x^2$
$y = x + 3$

93. $y = x^3 + 2x - 1$
$y = -2x + 15$

94. *Make a Decision* To work an extended application analyzing the heights of a softball after it has been dropped, visit this textbook's website at *LarsonPrecalculus.com*.