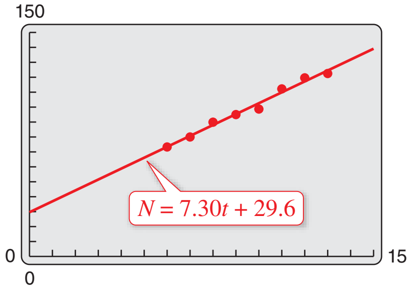
1 Functions and Their Graphs



Section 1.7, Example 4
e-Filed Tax Returns

Andres Rodriguez | Dreamstime

Introduction to Library of Parent Functions

In Chapter 1, you will be introduced to the concept of a *function*. As you proceed through the text, you will see that functions play a primary role in modeling real-life situations.

There are three basic types of functions that have proven to be the most important in modeling real-life situations. These functions are algebraic functions, exponential and logarithmic functions, and trigonometric and inverse trigonometric functions. These three types of functions are referred to as the *elementary functions*, though they are often placed in the two categories of *algebraic functions* and *transcendental functions*. Each time a new type of function is studied in detail in this text, it will be highlighted in a box similar to those shown below. The graphs of these functions are shown on the inside covers of this text.

Algebraic Functions

These functions are formed by applying algebraic operations to the linear function $f(x) = x$.

Name	*Function*	*Location*
Linear	$f(x) = x$	Section 1.1
Quadratic	$f(x) = x^2$	Section 2.1
Cubic	$f(x) = x^3$	Section 2.2
Rational	$f(x) = \dfrac{1}{x}$	Section 2.7
Square root	$f(x) = \sqrt{x}$	Section 1.2

Transcendental Functions

These functions cannot be formed from the linear function by using algebraic operations.

Name	*Function*	*Location*
Exponential	$f(x) = a^x, a > 0, a \neq 1$	Section 3.1
Logarithmic	$f(x) = \log_a x, x > 0, a > 0, a \neq 1$	Section 3.2
Trigonometric	$f(x) = \sin x$	Section 4.5
	$f(x) = \cos x$	Section 4.5
	$f(x) = \tan x$	Section 4.6
	$f(x) = \csc x$	Section 4.6
	$f(x) = \sec x$	Section 4.6
	$f(x) = \cot x$	Section 4.6
Inverse trigonometric	$f(x) = \arcsin x$	Section 4.7
	$f(x) = \arccos x$	Section 4.7
	$f(x) = \arctan x$	Section 4.7

Nonelementary Functions

Some useful nonelementary functions include the following.

Name	*Function*	*Location*
Absolute value	$f(x) = \lvert x \rvert$	Section 1.2
Greatest integer	$f(x) = [\![x]\!]$	Section 1.3

1.1 Lines in the Plane

What you should learn

- Find the slopes of lines.
- Write linear equations given points on lines and their slopes.
- Use slope-intercept forms of linear equations to sketch lines.
- Use slope to identify parallel and perpendicular lines.

Why you should learn it

The slope of a line can be used to solve real-life problems. For instance, in Exercise 97 on page 14, you will use a linear equation to model student enrollment at Penn State University.

The Slope of a Line

In this section, you will study lines and their equations. The **slope** of a nonvertical line represents the number of units the line rises or falls vertically for each unit of horizontal change from left to right. For instance, consider the two points

(x_1, y_1) and (x_2, y_2)

on the line shown in Figure 1.1.

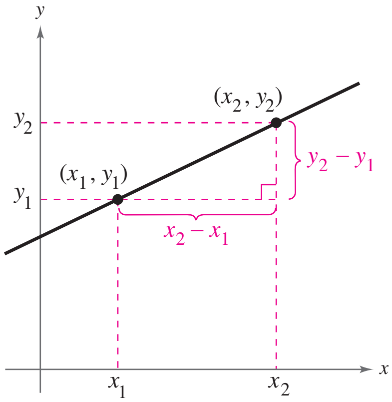


Figure 1.1

As you move from left to right along this line, a change of $(y_2 - y_1)$ units in the vertical direction corresponds to a change of $(x_2 - x_1)$ units in the horizontal direction. That is,

$y_2 - y_1 =$ the change in y

and

$x_2 - x_1 =$ the change in x.

The slope of the line is given by the ratio of these two changes.

Definition of the Slope of a Line

The **slope** m of the nonvertical line through (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{change in } y}{\text{change in } x}$$

where $x_1 \neq x_2$.

When this formula for slope is used, the *order of subtraction* is important. Given two points on a line, you are free to label either one of them as (x_1, y_1) and the other as (x_2, y_2). Once you have done this, however, you must form the numerator and denominator using the same order of subtraction.

$$\underbrace{m = \frac{y_2 - y_1}{x_2 - x_1}}_{\text{Correct}} \qquad \underbrace{m = \frac{y_1 - y_2}{x_1 - x_2}}_{\text{Correct}} \qquad \underbrace{\cancel{m = \frac{y_2 - y_1}{x_1 - x_2}}}_{\text{Incorrect}}$$

Throughout this text, the term *line* always means a *straight* line.

EXAMPLE 1 Finding the Slope of a Line

Find the slope of the line passing through each pair of points.

a. $(-2, 0)$ and $(3, 1)$ **b.** $(-1, 2)$ and $(2, 2)$ **c.** $(0, 4)$ and $(1, -1)$

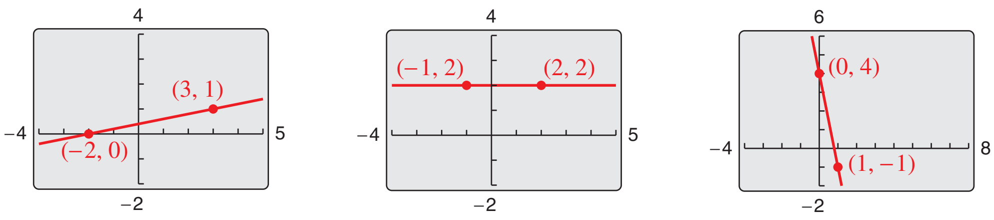
Solution

$$\textbf{a.}\quad m = \frac{\overbrace{y_2 - y_1}^{\text{Difference in } y\text{-values}}}{\underbrace{x_2 - x_1}_{\text{Difference in } x\text{-values}}} = \frac{1 - 0}{3 - (-2)} = \frac{1}{3 + 2} = \frac{1}{5}$$

$$\textbf{b.}\quad m = \frac{2 - 2}{2 - (-1)} = \frac{0}{3} = 0$$

$$\textbf{c.}\quad m = \frac{-1 - 4}{1 - 0} = \frac{-5}{1} = -5$$

The graphs of the three lines are shown in Figure 1.2. Note that the *square setting* gives the correct "steepness" of the lines.



(a) (b) (c)

Figure 1.2

✔ ***Checkpoint*** *Audio-video solution in English & Spanish at LarsonPrecalculus.com.*

Find the slope of the line passing through each pair of points.

a. $(-5, -6)$ and $(2, 8)$ **b.** $(4, 2)$ and $(2, 5)$ **c.** $(0, -1)$ and $(3, -1)$

The definition of slope does not apply to vertical lines. For instance, consider the points $(3, 4)$ and $(3, 1)$ on the vertical line shown in Figure 1.3. Applying the formula for slope, you obtain

$$m = \frac{4 - 1}{3 - 3} = \cancel{\frac{3}{0}}. \qquad \text{Undefined}$$

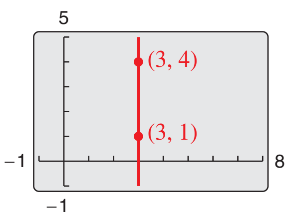


Figure 1.3

Because division by zero is undefined, the slope of a vertical line is undefined.

From the lines shown in Figures 1.2 and 1.3, you can make the following generalizations about the slope of a line.

The Slope of a Line

1. A line with positive slope $(m > 0)$ *rises* from left to right.
2. A line with negative slope $(m < 0)$ *falls* from left to right.
3. A line with zero slope $(m = 0)$ is *horizontal*.
4. A line with undefined slope is *vertical*.

Explore the Concept

Use a graphing utility to compare the slopes of the lines $y = 0.5x$, $y = x$, $y = 2x$, and $y = 4x$. What do you observe about these lines? Compare the slopes of the lines $y = -0.5x$, $y = -x$, $y = -2x$, and $y = -4x$. What do you observe about these lines? (*Hint:* Use a *square setting* to obtain a true geometric perspective.)

Common Error

A common error when finding the slope of a line is combining x- and y-coordinates in either the numerator or denominator, or both, as in

$$\cancel{m = \frac{y_2 - x_1}{x_2 - y_1}}.$$

Point out to your students that the vertical line shown in Figure 1.3 must be drawn on a graphing utility with a special command because there is no way to express the line's equation in the "$y =$" format.

The Point-Slope Form of the Equation of a Line

When you know the slope of a line *and* you also know the coordinates of one point on the line, you can find an equation of the line. For instance, in Figure 1.4, let (x_1, y_1) be a point on the line whose slope is m. When (x, y) is any *other* point on the line, it follows that

$$\frac{y - y_1}{x - x_1} = m.$$

This equation in the variables x and y can be rewritten in the **point-slope form** of the equation of a line.

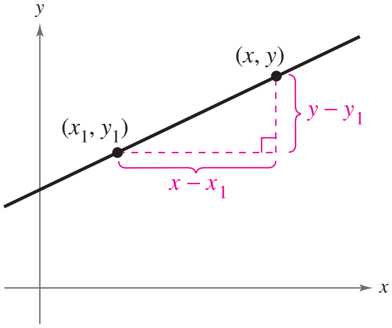


Figure 1.4

Point-Slope Form of the Equation of a Line

The **point-slope form** of the equation of the line that passes through the point (x_1, y_1) and has a slope of m is

$$y - y_1 = m(x - x_1).$$

EXAMPLE 2 The Point-Slope Form of the Equation of a Line

Find an equation of the line that passes through the point

$(1, -2)$

and has a slope of 3.

Solution

$y - y_1 = m(x - x_1)$	Point-slope form
$y - (-2) = 3(x - 1)$	Substitute for y_1, m, and x_1.
$y + 2 = 3x - 3$	Simplify.
$y = 3x - 5$	Solve for y.

The line is shown in Figure 1.5.

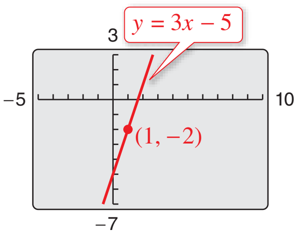


Figure 1.5

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Find an equation of the line that passes through the point $(3, -7)$ and has a slope of 2.

The point-slope form can be used to find an equation of a nonvertical line passing through two points

(x_1, y_1) and (x_2, y_2).

First, find the slope of the line.

$$m = \frac{y_2 - y_1}{x_2 - x_1}, \quad x_1 \neq x_2$$

Then use the point-slope form to obtain the equation

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1).$$

This is sometimes called the **two-point form** of the equation of a line.

Remark

When you find an equation of the line that passes through two given points, you need to substitute the coordinates of only one of the points into the point-slope form. It does not matter which point you choose because both points will yield the same result.

EXAMPLE 3 A Linear Model for Profits Prediction

In 2011, Tyson Foods had sales of \$32.266 billion, and in 2012, sales were \$33.278 billion. Write a linear equation giving the sales y in terms of the year x. Then use the equation to predict the sales for 2013. (Source: Tyson Foods, Inc.)

Solution

Let $x = 0$ represent 2000. In Figure 1.6, let $(11, 32.266)$ and $(12, 33.278)$ be two points on the line representing the sales. The slope of this line is

$$m = \frac{33.278 - 32.266}{12 - 11} = 1.012.$$

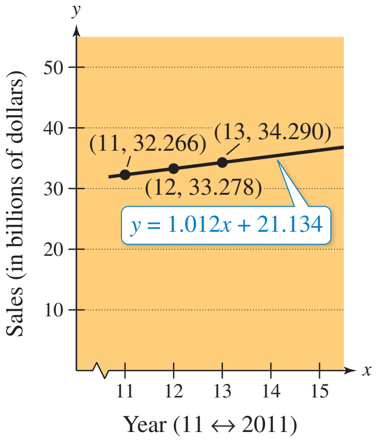

Figure 1.6

Next, use the point-slope form to find the equation of the line.

$$y - 32.266 = 1.012(x - 11)$$

$$y = 1.012x + 21.134$$

Now, using this equation, you can predict the 2013 sales ($x = 13$) to be

$$y = 1.012(13) + 21.134 = 13.156 + 21.134 = \$34.290 \text{ billion.}$$

(In this case, the prediction is quite good—the actual sales in 2013 were \$34.374 billion.)

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In 2012, Apple had sales of \$156.508 billion, and in 2013, sales were \$170.910 billion. Write a linear equation giving the sales y in terms of the year x. Then use the equation to predict the sales for 2014. (Source: Apple, Inc.)

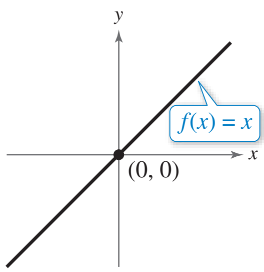
Library of Parent Functions: Linear Function

In the next section, you will be introduced to the precise meaning of the term *function*. The simplest type of function is the *parent linear function*

$$f(x) = x.$$

As its name implies, the graph of the parent linear function is a line. The basic characteristics of the parent linear function are summarized below and on the inside cover of this text. (Note that some of the terms below will be defined later in the text.)

Graph of $f(x) = x$
Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$
Intercept: $(0, 0)$
Increasing



The function $f(x) = x$ is also referred to as the *identity function*. Later in this text, you will learn that the graph of the linear function $f(x) = mx + b$ is a line with slope m and y-intercept $(0, b)$. When $m = 0$, $f(x) = b$ is called a *constant function* and its graph is a horizontal line.

Sketching Graphs of Lines

Many problems in coordinate geometry can be classified in two categories.

1. Given a graph (or parts of it), find its equation.
2. Given an equation, sketch its graph.

For lines, the first problem can be solved by using the point-slope form. This formula, however, is not particularly useful for solving the second type of problem. The form that is better suited to graphing linear equations is the **slope-intercept form** of the equation of a line, $y = mx + b$.

Slope-Intercept Form of the Equation of a Line

The graph of the equation

$$y = mx + b$$

is a line whose slope is m and whose y-intercept is $(0, b)$.

EXAMPLE 4 Using the Slope-Intercept Form

See LarsonPrecalculus.com for an interactive version of this type of example.

Determine the slope and y-intercept of each linear equation. Then describe its graph.

a. $x + y = 2$ **b.** $y = 2$

Algebraic Solution

a. Begin by writing the equation in slope-intercept form.

$$x + y = 2 \quad \text{Write original equation.}$$
$$y = 2 - x \quad \text{Subtract } x \text{ from each side.}$$
$$y = -x + 2 \quad \text{Write in slope-intercept form.}$$

From the slope-intercept form of the equation, the slope is -1 and the y-intercept is

$(0, 2)$.

Because the slope is negative, you know that the graph of the equation is a line that falls one unit for every unit it moves to the right.

b. By writing the equation $y = 2$ in slope-intercept form

$$y = (0)x + 2$$

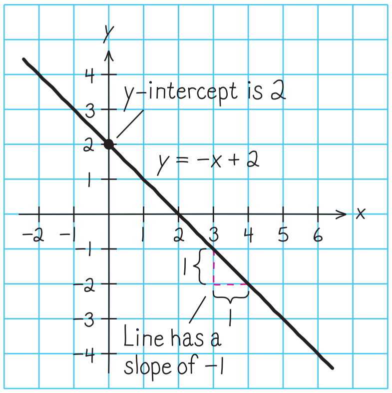
you can see that the slope is 0 and the y-intercept is

$(0, 2)$.

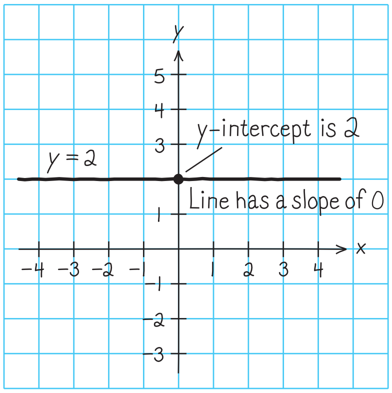
A zero slope implies that the line is horizontal.

Graphical Solution

a.



b.



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Determine the slope and y-intercept of $x - 2y = 4$. Then describe its graph.

From the slope-intercept form of the equation of a line, you can see that a horizontal line ($m = 0$) has an equation of the form $y = b$. This is consistent with the fact that each point on a horizontal line through $(0, b)$ has a y-coordinate of b. Similarly, each point on a vertical line through $(a, 0)$ has an x-coordinate of a. So, a vertical line has an equation of the form $x = a$. This equation cannot be written in slope-intercept form because the slope of a vertical line is undefined. However, every line has an equation that can be written in the **general form**

$$Ax + By + C = 0 \qquad \text{General form of the equation of a line}$$

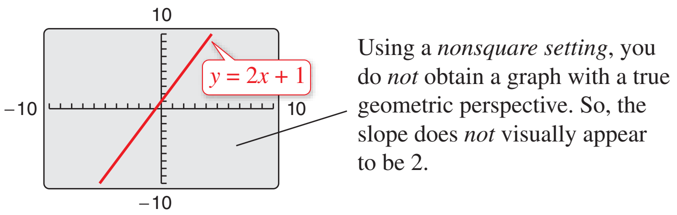
where A and B are not *both* zero.

Summary of Equations of Lines

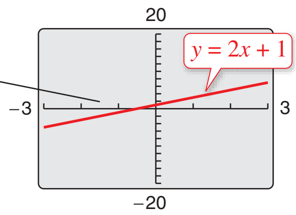
1. General form: $Ax + By + C = 0$
2. Vertical line: $x = a$
3. Horizontal line: $y = b$
4. Slope-intercept form: $y = mx + b$
5. Point-slope form: $y - y_1 = m(x - x_1)$

EXAMPLE 5 Different Viewing Windows

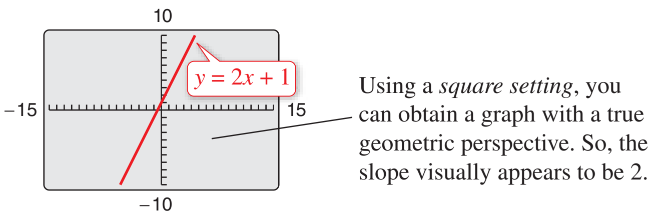
When a graphing utility is used to graph a line, it is important to realize that the line may not visually appear to have the slope indicated by its equation. This occurs because of the viewing window used for the graph. For instance, Figure 1.7 shows graphs of $y = 2x + 1$ produced on a graphing utility using three different viewing windows. Notice that the slopes in Figures 1.7(a) and (b) do not visually appear to be equal to 2. When you use a *square setting*, as in Figure 1.7(c), the slope visually appears to be 2.



(a) ***Nonsquare setting***



(b) ***Nonsquare setting***



(c) ***Square setting***

Figure 1.7

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Use a graphing utility to graph $y = 0.5x - 3$ using each viewing window. Describe the difference in the graphs.

a. Xmin = -5, Xmax = 10, Xscl = 1, Ymin = -1, Ymax = 10, Yscl = 1

b. Xmin = -2, Xmax = 10, Xscl = 1, Ymin = -4, Ymax = 1, Yscl = 1

c. Xmin = -5, Xmax = 10, Xscl = 1, Ymin = -7, Ymax = 3, Yscl = 1

Parallel and Perpendicular Lines

The slope of a line is a convenient tool for determining whether two lines are parallel or perpendicular.

Parallel Lines

Two distinct nonvertical lines are **parallel** if and only if their slopes are equal. That is, $m_1 = m_2$.

EXAMPLE 6 Equations of Parallel Lines

Find the slope-intercept form of the equation of the line that passes through the point $(2, -1)$ and is parallel to the line $2x - 3y = 5$.

Solution

Begin by writing the equation of the line in slope-intercept form.

$$2x - 3y = 5 \qquad \text{Write original equation.}$$

$$-2x + 3y = -5 \qquad \text{Multiply by } -1.$$

$$3y = 2x - 5 \qquad \text{Add } 2x \text{ to each side.}$$

$$y = \frac{2}{3}x - \frac{5}{3} \qquad \text{Write in slope-intercept form.}$$

Therefore, the given line has a slope of

$$m = \tfrac{2}{3}.$$

Any line parallel to the given line must also have a slope of $\frac{2}{3}$. So, the line through $(2, -1)$ has the following equation.

$$y - y_1 = m(x - x_1) \qquad \text{Point-slope form}$$

$$y - (-1) = \frac{2}{3}(x - 2) \qquad \text{Substitute for } y_1, m, \text{ and } x_1.$$

$$y + 1 = \frac{2}{3}x - \frac{4}{3} \qquad \text{Simplify.}$$

$$y = \frac{2}{3}x - \frac{7}{3} \qquad \text{Write in slope-intercept form.}$$

Notice the similarity between the slope-intercept form of the original equation and the slope-intercept form of the parallel equation. The graphs of both equations are shown in Figure 1.8.

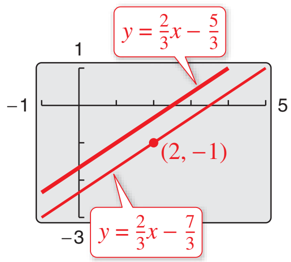

Figure 1.8

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Find the slope-intercept form of the equation of the line that passes through the point $(-4, 1)$ and is parallel to the line $5x - 3y = 8$.

Perpendicular Lines

Two nonvertical lines are **perpendicular** if and only if their slopes are negative reciprocals of each other. That is,

$$m_1 = -\frac{1}{m_2}.$$

Explore the Concept

Graph the lines $y_1 = \frac{1}{2}x + 1$ and $y_2 = -2x + 1$ in the same viewing window. What do you observe?

Graph the lines $y_1 = 2x + 1$, $y_2 = 2x$, and $y_3 = 2x - 1$ in the same viewing window. What do you observe?

EXAMPLE 7 Equations of Perpendicular Lines

Find the slope-intercept form of the equation of the line that passes through the point $(2, -1)$ and is perpendicular to the line $2x - 3y = 5$.

Solution

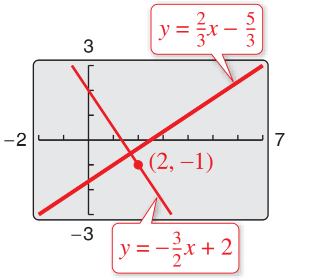
From Example 6, you know that the equation can be written in the slope-intercept form $y = \frac{2}{3}x - \frac{5}{3}$. You can see that the line has a slope of $\frac{2}{3}$. So, any line perpendicular to this line must have a slope of $-\frac{3}{2}$ (because $-\frac{3}{2}$ is the negative reciprocal of $\frac{2}{3}$). So, the line through the point $(2, -1)$ has the following equation.

$$y - (-1) = -\frac{3}{2}(x - 2) \qquad \text{Write in point-slope form.}$$

$$y + 1 = -\frac{3}{2}x + 3 \qquad \text{Simplify.}$$

$$y = -\frac{3}{2}x + 2 \qquad \text{Write in slope-intercept form.}$$

The graphs of both equations are shown in the figure.



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Find the slope-intercept form of the equation of the line that passes through the point $(-4, 1)$ and is perpendicular to the line $5x - 3y = 8$.

EXAMPLE 8 Graphs of Perpendicular Lines

Use a graphing utility to graph the lines $y = x + 1$ and $y = -x + 3$ in the same viewing window. The lines are perpendicular (they have slopes of $m_1 = 1$ and $m_2 = -1$). Do they appear to be perpendicular on the display?

Solution

When the viewing window is nonsquare, as in Figure 1.9, the two lines will not appear perpendicular. When, however, the viewing window is square, as in Figure 1.10, the lines will appear perpendicular.

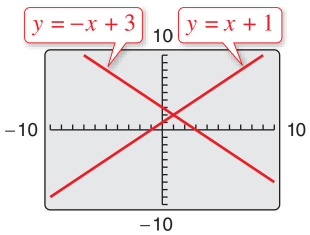

Figure 1.9

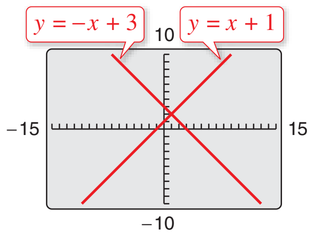

Figure 1.10

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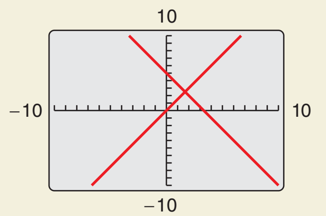
Identify any relationships that exist among the lines $y = 2x$, $y = -2x$, and $y = \frac{1}{2}x$. Then use a graphing utility to graph the three equations in the same viewing window. Adjust the viewing window so that each slope appears visually correct. Use the slopes of the lines to verify your results.

What's Wrong?

You use a graphing utility to graph $y_1 = 1.5x$ and $y_2 = -1.5x + 5$, as shown in the figure. You use the graph to conclude that the lines are perpendicular. What's wrong?



Activities

1. Write an equation of the line that passes through the points $(-2, 1)$ and $(3, 2)$.
   Answer: $x - 5y + 7 = 0$
2. Find the slope of the line that is perpendicular to the line $4x - 7y = 12$.
   Answer: $m = -\frac{7}{4}$
3. Write the equation of the vertical line that passes through the point $(3, 2)$.
   Answer: $x = 3$

1.1 Exercises

See *CalcChat.com* for tutorial help and worked-out solutions to odd-numbered exercises.
For instructions on how to use a graphing utility, see Appendix A.

Vocabulary and Concept Check

1. Match each equation with its form.

(a) $Ax + By + C = 0$	(i) vertical line
(b) $x = a$	(ii) slope-intercept form
(c) $y = b$	(iii) general form
(d) $y = mx + b$	(iv) point-slope form
(e) $y - y_1 = m(x - x_1)$	(v) horizontal line

In Exercises 2 and 3, fill in the blank.

2. For a line, the ratio of the change in y to the change in x is called the _______ of the line.

3. Two lines are _______ if and only if their slopes are equal.

4. What is the relationship between two lines whose slopes are -3 and $\frac{1}{3}$?

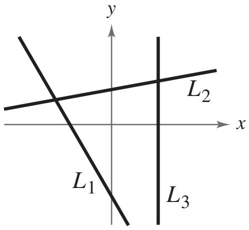
5. What is the slope of a line that is perpendicular to the line represented by $x = 3$?

6. Give the coordinates of a point on the line whose equation in point-slope form is $y - (-1) = \frac{1}{4}(x - 8)$.

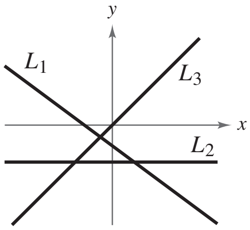
Procedures and Problem Solving

Using Slope **In Exercises 7 and 8, identify the line that has the indicated slope.**

7. (a) $m = \frac{2}{3}$ (b) m is undefined. (c) $m = -2$

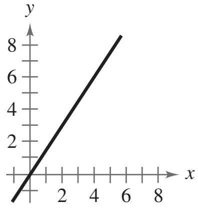


8. (a) $m = 0$ (b) $m = -\frac{3}{4}$ (c) $m = 1$

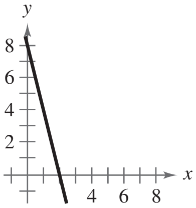


Estimating Slope **In Exercises 9 and 10, estimate the slope of the line.**

9.



10.



Sketching Lines **In Exercises 11 and 12, sketch the lines through the point with the indicated slopes on the same set of coordinate axes.**

	Point	*Slopes*			
11.	(2, 3)	(a) 0	(b) 1	(c) 2	(d) -3
12.	$(-4, 1)$	(a) 4	(b) -2	(c) $\frac{1}{2}$	(d) Undefined

Finding the Slope of a Line **In Exercises 13–16, find the slope of the line passing through the pair of points. Then use a graphing utility to plot the points and use the *draw* feature to graph the line segment connecting the two points. (Use a *square setting*.)**

13. $(0, -10), (-4, 0)$ **14.** $(2, 4), (4, -4)$

15. $(-6, -1), (-6, 4)$ **16.** $(4, 9), (6, 12)$

Using Slope **In Exercises 17–24, use the point on the line and the slope of the line to find three additional points through which the line passes. (There are many correct answers.)**

	Point	*Slope*
17.	(2, 1)	$m = 0$
18.	$(3, -2)$	$m = 0$
19.	(1, 5)	m is undefined.
20.	$(-4, 1)$	m is undefined.
21.	$(0, -9)$	$m = -2$
22.	$(-5, 4)$	$m = 4$
23.	$(7, -2)$	$m = \frac{1}{2}$
24.	$(-1, -6)$	$m = -\frac{1}{3}$

The Point-Slope Form of the Equation of a Line **In Exercises 25–32, find an equation of the line that passes through the given point and has the indicated slope. Sketch the line by hand. Use a graphing utility to verify your sketch, if possible.**

25. $(0, -2), \quad m = 3$
26. $(-3, 6), \quad m = -3$
27. $(2, -3), \quad m = -\frac{1}{2}$
28. $(-2, -5), \quad m = \frac{3}{4}$
29. $(6, -1), \quad m$ is undefined.
30. $(-10, 4), \quad m$ is undefined.
31. $\left(-\frac{1}{2}, \frac{3}{2}\right), \quad m = 0$
32. $(2.3, -8.5), \quad m = 0$

33. Finance The median player salary for the New York Yankees was \$1.5 million in 2007 and \$1.7 million in 2013. Write a linear equation giving the median salary y in terms of the year x. Then use the equation to predict the median salary in 2020.

34. Finance The median player salary for the Dallas Cowboys was \$348,000 in 2004 and \$555,000 in 2013. Write a linear equation giving the median salary y in terms of the year x. Then use the equation to predict the median salary in 2019.

Using the Slope-Intercept Form **In Exercises 35–42, determine the slope and y-intercept (if possible) of the linear equation. Then describe its graph.**

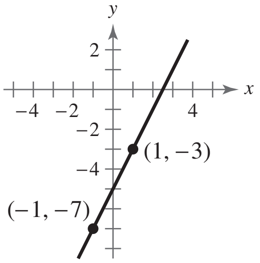
35. $2x - 3y = 9$
36. $3x + 4y = 1$
37. $2x - 5y + 10 = 0$
38. $4x - 3y - 9 = 0$
39. $x = -6$
40. $y = 12$
41. $3y + 2 = 0$
42. $2x - 5 = 0$

Using the Slope-Intercept Form **In Exercises 43–48, (a) find the slope and y-intercept (if possible) of the equation of the line algebraically, and (b) sketch the line by hand. Use a graphing utility to verify your answers to parts (a) and (b).**

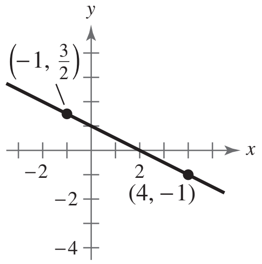
43. $5x - y + 3 = 0$
44. $2x + 3y - 9 = 0$
45. $5x - 2 = 0$
46. $3x + 7 = 0$
47. $3y + 5 = 0$
48. $-11 - 4y = 0$

Finding the Slope-Intercept Form **In Exercises 49 and 50, find the slope-intercept form of the equation of the line shown.**

49.



50.



Finding the Slope-Intercept Form **In Exercises 51–60, write an equation of the line that passes through the points. Use the slope-intercept form (if possible). If not possible, explain why and use the general form. Use a graphing utility to graph the line (if possible).**

51. $(5, -1), (-5, 5)$
52. $(4, 3), (-4, -4)$
53. $(-8, 1), (-8, 7)$
54. $(-1, 6), (5, 6)$
55. $\left(2, \frac{1}{2}\right), \left(\frac{1}{2}, \frac{5}{4}\right)$
56. $(1, 1), \left(6, -\frac{2}{3}\right)$
57. $\left(-\frac{1}{10}, -\frac{3}{5}\right), \left(\frac{9}{10}, -\frac{9}{5}\right)$
58. $\left(\frac{3}{4}, \frac{3}{2}\right), \left(-\frac{4}{3}, \frac{7}{4}\right)$
59. $(1, 0.6), (-2, -0.6)$
60. $(-8, 0.6), (2, -2.4)$

Different Viewing Windows **In Exercises 61 and 62, use a graphing utility to graph the equation using each viewing window. Describe the differences in the graphs.**

61. $y = 0.25x - 2$

Xmin = -1	Xmin = -5	Xmin = -5
Xmax = 9	Xmax = 10	Xmax = 10
Xscl = 1	Xscl = 1	Xscl = 1
Ymin = -5	Ymin = -3	Ymin = -5
Ymax = 4	Ymax = 4	Ymax = 5
Yscl = 1	Yscl = 1	Yscl = 1

62. $y = -8x + 5$

Xmin = -5	Xmin = -5	Xmin = -5
Xmax = 5	Xmax = 10	Xmax = 13
Xscl = 1	Xscl = 1	Xscl = 1
Ymin = -10	Ymin = -80	Ymin = -2
Ymax = 10	Ymax = 80	Ymax = 10
Yscl = 1	Yscl = 20	Yscl = 1

Parallel and Perpendicular Lines **In Exercises 63–66, determine whether the lines L_1 and L_2 passing through the pairs of points are parallel, perpendicular, or neither.**

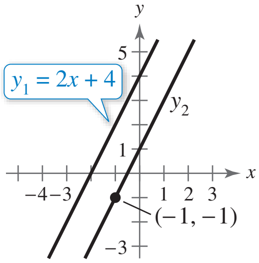
63. L_1: $(0, -1), (5, 9)$
L_2: $(0, 3), (4, 1)$
64. L_1: $(-2, -1), (1, 5)$
L_2: $(1, 3), (5, -5)$
65. L_1: $(3, 6), (-6, 0)$
L_2: $(0, -1), \left(5, \frac{7}{3}\right)$
66. L_1: $(4, 8), (-4, 2)$
L_2: $(3, -5), \left(-1, \frac{1}{3}\right)$

Equations of Parallel and Perpendicular Lines **In Exercises 67–76, write the slope-intercept forms of the equations of the lines through the given point (a) parallel to the given line and (b) perpendicular to the given line.**

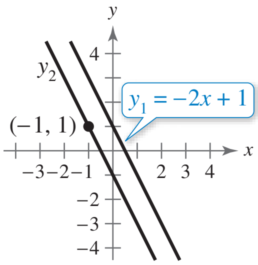
67. $(2, 1), \quad 4x - 2y = 3$
68. $(-3, 2), \quad x + y = 7$
69. $\left(-\frac{2}{3}, \frac{7}{8}\right), \quad 3x + 4y = 7$
70. $\left(\frac{2}{5}, -1\right), \quad 3x - 2y = 6$
71. $(-3.9, -1.4), \quad 6x + 5y = 9$
72. $(-1.2, 2.4), \quad 5x + 4y = 1$
73. $(3, -2), \quad x - 4 = 0$
74. $(3, -1), \quad y - 2 = 0$
75. $(-5, 1), \quad y + 2 = 0$
76. $(-2, 4), \quad x + 5 = 0$

Equations of Parallel Lines **In Exercises 77 and 78, the lines are parallel. Find the slope-intercept form of the equation of line y_2.**

77.

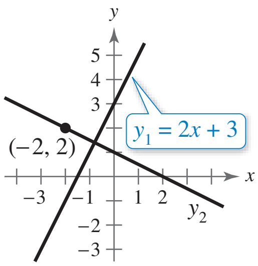


78.

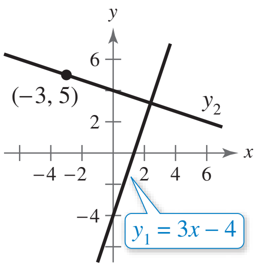


Equations of Perpendicular Lines **In Exercises 79 and 80, the lines are perpendicular. Find the slope-intercept form of the equation of line y_2.**

79.



80.



Graphs of Parallel and Perpendicular Lines **In Exercises 81–84, identify any relationships that exist among the lines, and then use a graphing utility to graph the three equations in the same viewing window. Adjust the viewing window so that each slope appears visually correct. Use the slopes of the lines to verify your results.**

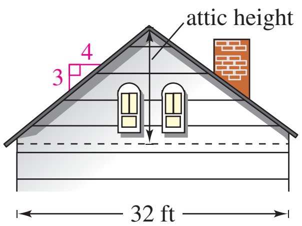
81. (a) $y = 4x$ (b) $y = -4x$ (c) $y = \frac{1}{4}x$

82. (a) $y = \frac{2}{3}x$ (b) $y = -\frac{3}{2}x$ (c) $y = \frac{2}{3}x + 2$

83. (a) $y = -\frac{1}{2}x$ (b) $y = -\frac{1}{2}x + 3$ (c) $y = 2x - 4$

84. (a) $y = x - 8$ (b) $y = x + 1$ (c) $y = -x + 3$

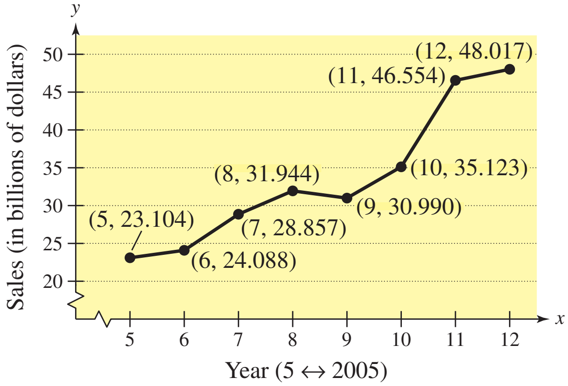
85. Architectural Design The rise-to-run ratio of the roof of a house determines the steepness of the roof. The rise-to-run ratio of the roof in the figure is 3 to 4. Determine the maximum height in the attic of the house if the house is 32 feet wide.



86. Highway Engineering When driving down a mountain road, you notice warning signs indicating that it is a "12% grade." This means that the slope of the road is $-\frac{12}{100}$. Approximate the amount of horizontal change in your position if you note from elevation markers that you have descended 2000 feet vertically.

87. MODELING DATA

The graph shows the sales y (in billions of dollars) of the Coca-Cola Company each year x from 2005 through 2012, where $x = 5$ represents 2005. (Source: Coca-Cola Company)



(a) Use the slopes to determine the years in which the sales showed the greatest increase and greatest decrease.

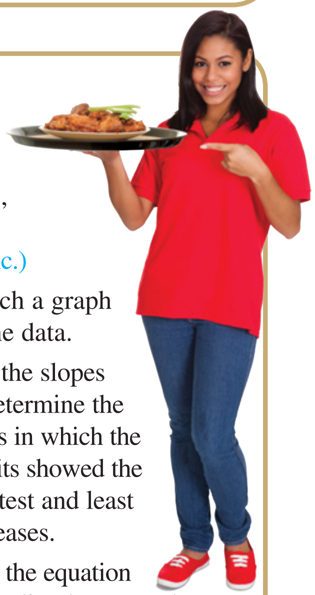
(b) Find the equation of the line between the years 2005 and 2012.

(c) Interpret the meaning of the slope of the line from part (b) in the context of the problem.

(d) Use the equation from part (b) to estimate the sales of the Coca-Cola Company in 2017. Do you think this is an accurate estimate? Explain.

88. MODELING DATA

The table shows the profits y (in millions of dollars) for Buffalo Wild Wings for each year x from 2007 through 2013, where $x = 7$ represents 2007. (Source: Buffalo Wild Wings, Inc.)



DATA

Year, x	Profits, y
7	19.7
8	24.4
9	30.7
10	38.4
11	50.4
12	57.3
13	71.6

Spreadsheet at LarsonPrecalculus.com

(a) Sketch a graph of the data.

(b) Use the slopes to determine the years in which the profits showed the greatest and least increases.

(c) Find the equation of the line between the years 2007 and 2013.

(d) Interpret the meaning of the slope of the line from part (c) in the context of the problem.

(e) Use the equation from part (c) to estimate the profit for Buffalo Wild Wings in 2017. Do you think this is an accurate estimate? Explain.

Using a Rate of Change to Write an Equation In Exercises 89–92, you are given the dollar value of a product in 2015 and the rate at which the value of the product is expected to change during the next 5 years. Write a linear equation that gives the dollar value V of the product in terms of the year t. (Let $t = 15$ represent 2015.)

	2015 Value	*Rate*
89.	\$2540	\$125 increase per year
90.	\$156	\$5.50 increase per year
91.	\$20,400	\$2000 decrease per year
92.	\$245,000	\$5600 decrease per year

93. Accounting A school district purchases a high-volume printer, copier, and scanner for \$25,000. After 10 years, the equipment will have to be replaced. Its value at that time is expected to be \$2000.

(a) Write a linear equation giving the value V of the equipment for each year t during its 10 years of use.

(b) Use a graphing utility to graph the linear equation representing the depreciation of the equipment, and use the *value* or *trace* feature to complete the table. Verify your answers algebraically by using the equation you found in part (a).

t	0	1	2	3	4	5	6	7	8	9	10
V											

94. Meterology Recall that water freezes at 0°C (32°F) and boils at 100°C (212°F).

(a) Find an equation of the line that shows the relationship between the temperature in degrees Celsius C and degrees Fahrenheit F.

(b) Use the result of part (a) to complete the table.

C		$-10°$	$10°$			$177°$
F	$0°$			$68°$	$90°$	

95. Business A contractor purchases a bulldozer for \$36,500. The bulldozer requires an average expenditure of \$11.25 per hour for fuel and maintenance, and the operator is paid \$19.50 per hour.

(a) Write a linear equation giving the total cost C of operating the bulldozer for t hours. (Include the purchase cost of the bulldozer.)

(b) Assuming that customers are charged \$80 per hour of bulldozer use, write an equation for the revenue R derived from t hours of use.

(c) Use the profit formula $(P = R - C)$ to write an equation for the profit gained from t hours of use.

(d) Use the result of part (c) to find the break-even point (the number of hours the bulldozer must be used to gain a profit of 0 dollars).

96. Real Estate A real estate office handles an apartment complex with 50 units. When the rent per unit is \$580 per month, all 50 units are occupied. However, when the rent is \$625 per month, the average number of occupied units drops to 47. Assume that the relationship between the monthly rent p and the demand x is linear.

(a) Write an equation of the line giving the demand x in terms of the rent p.

(b) Use a graphing utility to graph the demand equation and use the *trace* feature to estimate the number of units occupied when the rent is \$655. Verify your answer algebraically.

(c) Use the demand equation to predict the number of units occupied when the rent is lowered to \$595. Verify your answer graphically.

97. *Why you should learn it* *(p. 3)* In 1994, Penn State University had an enrollment of 73,500 students. By 2013, the enrollment had increased to 98,097. (Source: Penn State Fact Book)

(a) What was the average annual change in enrollment from 1994 to 2013?

(b) Use the average annual change in enrollment to estimate the enrollments in 1996, 2006, and 2011.

(c) Write an equation of a line that represents the given data. What is its slope? Interpret the slope in the context of the problem.

98. Writing Using the results of Exercise 97, write a short paragraph discussing the concepts of *slope* and *average rate of change.*

Conclusions

True or False? In Exercises 99 and 100, determine whether the statement is true or false. Justify your answer.

99. The line through $(-8, 2)$ and $(-1, 4)$ and the line through $(0, -4)$ and $(-7, 7)$ are parallel.

100. If the points $(10, -3)$ and $(2, -9)$ lie on the same line, then the point $\left(-12, -\frac{37}{2}\right)$ also lies on that line.

Exploration In Exercises 101–104, use a graphing utility to graph the equation of the line in the form

$$\frac{x}{a} + \frac{y}{b} = 1, \quad a \neq 0, b \neq 0.$$

Use the graphs to make a conjecture about what a and b represent. Verify your conjecture.

101. $\dfrac{x}{7} + \dfrac{y}{-3} = 1$

102. $\dfrac{x}{-6} + \dfrac{y}{2} = 1$

103. $\dfrac{x}{4} + \dfrac{y}{-\frac{2}{3}} = 1$

104. $\dfrac{x}{\frac{1}{2}} + \dfrac{y}{5} = 1$

Using Intercepts **In Exercises 105–108, use the results of Exercises 101–104 to write an equation of the line that passes through the points.**

105. x-intercept: $(2, 0)$
y-intercept: $(0, 9)$

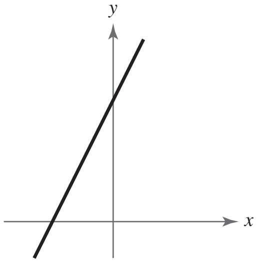
106. x-intercept: $(-5, 0)$
y-intercept: $(0, -4)$

107. x-intercept: $\left(-\frac{1}{6}, 0\right)$
y-intercept: $\left(0, -\frac{2}{3}\right)$

108. x-intercept: $\left(\frac{3}{4}, 0\right)$
y-intercept: $\left(0, \frac{4}{3}\right)$

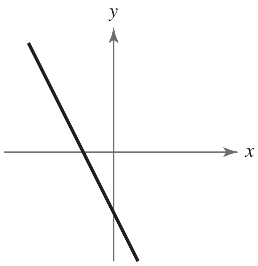
Think About It **In Exercises 109 and 110, determine which equation(s) may be represented by the graphs shown. (There may be more than one correct answer.)**

109.



(a) $2x - y = -10$
(b) $2x + y = 10$
(c) $x - 2y = 10$
(d) $x + 2y = 10$

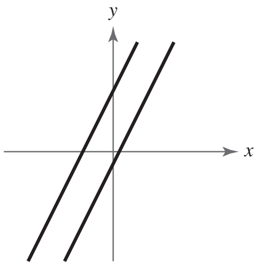
110.



(a) $2x + y = 5$
(b) $2x + y = -5$
(c) $x - 2y = 5$
(d) $x - 2y = -5$

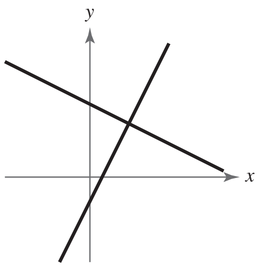
Think About It **In Exercises 111 and 112, determine which pair of equations may be represented by the graphs shown.**

111.



(a) $2x - y = 5$
$2x - y = 1$
(b) $2x + y = -5$
$2x + y = 1$
(c) $2x - y = -5$
$2x - y = 1$
(d) $x - 2y = -5$
$x - 2y = -1$

112.



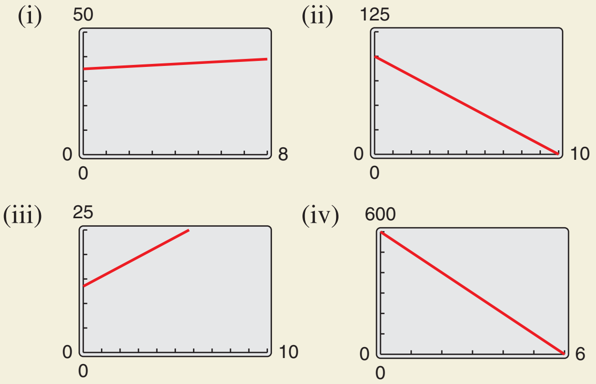
(a) $2x - y = 2$
$x + 2y = 12$
(b) $x - y = 1$
$x + y = 6$
(c) $2x + y = 2$
$x - 2y = 12$
(d) $x - 2y = 2$
$x + 2y = 12$

113. **Think About It** Does every line have both an x-intercept and a y-intercept? Explain.

114. **Think About It** Can every line be written in slope-intercept form? Explain.

115. **Think About It** Does every line have an infinite number of lines that are parallel to it? Explain.

116. **HOW DO YOU SEE IT?** Match the description with its graph. Determine the slope and y-intercept of each graph and interpret their meaning in the context of the problem. [The graphs are labeled (i), (ii), (iii), and (iv).]



(a) You are paying \$10 per week to repay a \$100 loan.
(b) An employee is paid \$13.50 per hour plus \$2 for each unit produced per hour.
(c) A sales representative receives \$35 per day for food plus \$0.50 for each mile traveled.
(d) A tablet computer that was purchased for \$600 depreciates \$100 per year.

Cumulative Mixed Review

Identifying Polynomials **In Exercises 117–122, determine whether the expression is a polynomial. If it is, write the polynomial in standard form.**

117. $x + 20$

118. $3x - 10x^2 + 1$

119. $4x^2 + x^{-1} - 3$

120. $2x^2 - 2x^4 - x^3 + \sqrt{2}$

121. $\dfrac{x^2 + 3x + 4}{x^2 - 9}$

122. $\sqrt{x^2 + 7x + 6}$

Factoring Trinomials **In Exercises 123–126, factor the trinomial.**

123. $x^2 - 6x - 27$

124. $x^2 + 11x + 28$

125. $2x^2 + 11x - 40$

126. $3x^2 - 16x + 5$

127. ***Make a Decision*** To work an extended application analyzing the numbers of bachelor's degrees earned by women in the United States from 2001 through 2012, visit this textbook's website at *LarsonPrecalculus.com.* (Data Source: National Center for Education Statistics)

The *Make a Decision* exercise indicates a multipart exercise using large data sets. Visit this textbook's website at *LarsonPrecalculus.com.*

1.2 Functions

Introduction to Functions

Many everyday phenomena involve two quantities that are related to each other by some rule of correspondence. The mathematical term for such a rule of correspondence is a **relation.** Here are two examples.

1. The simple interest I earned on an investment of \$1000 for 1 year is related to the annual interest rate r by the formula $I = 1000r$.
2. The area A of a circle is related to its radius r by the formula $A = \pi r^2$.

Not all relations have simple mathematical formulas. For instance, you can match NFL starting quarterbacks with touchdown passes and time of day with temperature. In each of these cases, there is some relation that matches each item from one set with exactly one item from a different set. Such a relation is called a **function.**

Definition of a Function

A **function** f from a set A to a set B is a relation that assigns to each element x in the set A exactly one element y in the set B. The set A is the **domain** (or set of inputs) of the function f, and the set B contains the **range** (or set of outputs).

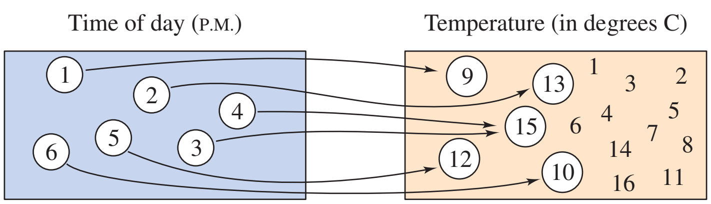
What you should learn

- Determine whether a relation between two variables represents a function.
- Use function notation and evaluate functions.
- Find the domains of functions.
- Use functions to model and solve real-life problems.
- Evaluate difference quotients.

Why you should learn it

Many natural phenomena can be modeled by functions, such as the force of water against the face of a dam, explored in Exercise 80 on page 27.

To help understand this definition, look at the function that relates the time of day to the temperature in Figure 1.11.



Set A is the domain.
Inputs: 1, 2, 3, 4, 5, 6

Set B contains the range.
Outputs: 9, 10, 12, 13, 15

Figure 1.11

This function can be represented by the ordered pairs

$$\{(1, 9^\circ), (2, 13^\circ), (3, 15^\circ), (4, 15^\circ), (5, 12^\circ), (6, 10^\circ)\}.$$

In each ordered pair, the first coordinate (x-value) is the **input** and the second coordinate (y-value) is the **output.**

Characteristics of a Function from Set A to Set B

1. Each element of A must be matched with an element of B.
2. Some elements of B may not be matched with any element of A.
3. Two or more elements of A may be matched with the same element of B.
4. An element of A (the domain) cannot be matched with two different elements of B.

To determine whether or not a relation is a function, you must decide whether each input value is matched with exactly one output value. When any input value is matched with two or more output values, the relation is not a function.

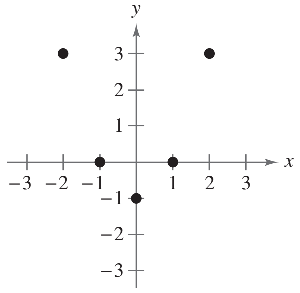
EXAMPLE 1 Testing for Functions

Determine whether each relation represents y as a function of x.

a.

Input, x	2	2	3	4	5
Output, y	11	10	8	5	1

b.



Solution

a. This table *does not* describe y as a function of x. The input value 2 is matched with two different y-values.

b. The graph *does* describe y as a function of x. Each input value is matched with exactly one output value.

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Determine whether the relation represents y as a function of x.

Input, x	0	1	2	3	4
Output, y	-4	-2	0	2	4

In algebra, it is common to represent functions by equations or formulas involving two variables. For instance, $y = x^2$ represents the variable y as a function of the variable x. In this equation, x is the **independent variable** and y is the **dependent variable.** The domain of the function is the set of all values taken on by the independent variable x, and the range of the function is the set of all values taken on by the dependent variable y.

EXAMPLE 2 Testing for Functions Represented Algebraically

See LarsonPrecalculus.com for an interactive version of this type of example.

Determine whether each equation represents y as a function of x.

a. $x^2 + y = 1$ **b.** $-x + y^2 = 1$

Solution

To determine whether y is a function of x, try to solve for y in terms of x.

a. $x^2 + y = 1$ Write original equation.

$y = 1 - x^2$ Solve for y.

Each value of x corresponds to exactly one value of y. So, y is a function of x.

b. $-x + y^2 = 1$ Write original equation.

$y^2 = 1 + x$ Add x to each side.

$y = \pm\sqrt{1 + x}$ Solve for y.

The $\pm$ indicates that for a given value of x, there correspond two values of y. For instance, when $x = 3$, $y = 2$ or $y = -2$. So, y is not a function of x.

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Determine whether each equation represents y as a function of x.

a. $x^2 + y^2 = 8$ **b.** $y - 4x^2 = 36$

Explore the Concept

Use a graphing utility to graph $x^2 + y = 1$. Then use the graph to write a convincing argument that each x-value corresponds to at most one y-value.

Use a graphing utility to graph $-x + y^2 = 1$. (*Hint:* You will need to use two equations.) Does the graph represent y as a function of x? Explain.

Function Notation

When an equation is used to represent a function, it is convenient to name the function so that it can be referenced easily. For example, you know that the equation $y = 1 - x^2$ describes y as a function of x. Suppose you give this function the name "f." Then you can use the following **function notation.**

Understanding the concept of functions is essential. Be sure students understand function notation. Frequently, $f(x)$ is misinterpreted as "f times x" rather than "f of x."

Input	*Output*	*Equation*
x	$f(x)$	$f(x) = 1 - x^2$

The symbol $f(x)$ is read as the *value of f at x* or simply *f of x*. The symbol $f(x)$ corresponds to the y-value for a given x. So, you can write $y = f(x)$. Keep in mind that f is the *name* of the function, whereas $f(x)$ is the *output value* of the function at the *input value* x. In function notation, the *input* is the independent variable and the *output* is the dependent variable. For instance, the function $f(x) = 3 - 2x$ has *function values* denoted by $f(-1)$, $f(0)$, and so on. To find these values, substitute the specified input values into the given equation.

For $x = -1$, $\quad f(-1) = 3 - 2(-1) = 3 + 2 = 5.$

For $x = 0$, $\quad f(0) = 3 - 2(0) = 3 - 0 = 3.$

Although f is often used as a convenient function name and x is often used as the independent variable, you can use other letters. For instance,

$$f(x) = x^2 - 4x + 7, \quad f(t) = t^2 - 4t + 7, \quad \text{and} \quad g(s) = s^2 - 4s + 7$$

all define the same function. In fact, the role of the independent variable is that of a "placeholder." Consequently, the function could be written as

$$f(\boxed{}) = (\boxed{})^2 - 4(\boxed{}) + 7.$$

EXAMPLE 3 Evaluating a Function

Let $g(x) = -x^2 + 4x + 1$. Find each value of the function.

a. $g(2)$ **b.** $g(t)$ **c.** $g(x + 2)$

Solution

a. Replacing x with 2 in $g(x) = -x^2 + 4x + 1$ yields the following.

$$g(2) = -(2)^2 + 4(2) + 1 = -4 + 8 + 1 = 5$$

b. Replacing x with t yields the following.

$$g(t) = -(t)^2 + 4(t) + 1 = -t^2 + 4t + 1$$

c. Replacing x with $x + 2$ yields the following.

$$\begin{aligned} g(x+2) &= -(x+2)^2 + 4(x+2) + 1 && \text{Substitute } x + 2 \text{ for } x. \\ &= -(x^2 + 4x + 4) + 4x + 8 + 1 && \text{Multiply.} \\ &= -x^2 - 4x - 4 + 4x + 8 + 1 && \text{Distributive Property} \\ &= -x^2 + 5 && \text{Simplify.} \end{aligned}$$

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Let $f(x) = 10 - 3x^2$. Find each function value.

a. $f(2)$ **b.** $f(-4)$ **c.** $f(x - 1)$

In part (c) of Example 3, note that $g(x + 2)$ is not equal to $g(x) + g(2)$. In general, $g(u + v) \neq g(u) + g(v)$.

Library of Parent Functions: Absolute Value Function

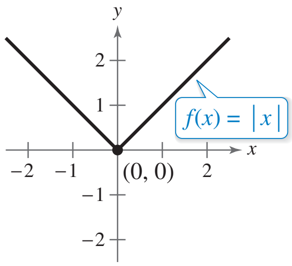
The *parent absolute value function* given by

$$f(x) = |x|$$

can be written as a piecewise-defined function. The basic characteristics of the parent absolute value function are summarized below and on the inside cover of this text.

Graph of $f(x) = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

Domain: $(-\infty, \infty)$
Range: $[0, \infty)$
Intercept: $(0, 0)$
Decreasing on $(-\infty, 0)$
Increasing on $(0, \infty)$



Additional Example
Evaluate at $x = 0, 1, 3.$

$$f(x) = \begin{cases} \dfrac{x}{2} + 1, & x \leq 1 \\ 3x + 2, & x > 1 \end{cases}$$

Solution

Because $x = 0$ is less than or equal to 1, use $f(x) = (x/2) + 1$ to obtain

$$f(0) = \frac{0}{2} + 1 = 1.$$

For $x = 1$, use $f(x) = (x/2) + 1$ to obtain

$$f(1) = \frac{1}{2} + 1 = 1\frac{1}{2}.$$

For $x = 3$, use $f(x) = 3x + 2$ to obtain

$$f(3) = 3(3) + 2 = 11.$$

A function defined by two or more equations over a specified domain is called a **piecewise-defined function.**

EXAMPLE 4 A Piecewise-Defined Function

Evaluate the function when $x = -1$, 0, and 1.

$$f(x) = \begin{cases} x^2 + 1, & x < 0 \\ x - 1, & x \geq 0 \end{cases}$$

Solution

Because $x = -1$ is less than 0, use $f(x) = x^2 + 1$ to obtain

$$f(-1) = (-1)^2 + 1 \qquad \text{Substitute } -1 \text{ for } x.$$
$$= 2. \qquad \text{Simplify.}$$

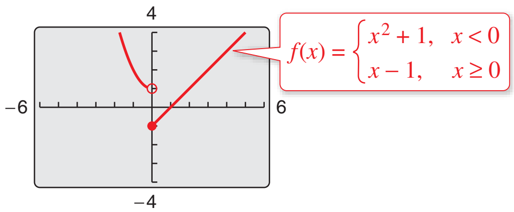
For $x = 0$, use $f(x) = x - 1$ to obtain

$$f(0) = 0 - 1 \qquad \text{Substitute 0 for } x.$$
$$= -1. \qquad \text{Simplify.}$$

For $x = 1$, use $f(x) = x - 1$ to obtain

$$f(1) = 1 - 1 \qquad \text{Substitute 1 for } x.$$
$$= 0. \qquad \text{Simplify.}$$

The graph of f is shown in the figure.



Technology Tip

Most graphing utilities can graph piecewise-defined functions. For instructions on how to enter a piecewise-defined function into your graphing utility, consult your user's manual. You may find it helpful to set your graphing utility to *dot mode* before graphing such functions.

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Evaluate the function given in Example 4 when $x = -2$, 2, and 3.

The Domain of a Function

The domain of a function can be described explicitly or it can be *implied* by the expression used to define the function. The **implied domain** is the set of all real numbers for which the expression is defined. For instance, the function

$$f(x) = \frac{1}{x^2 - 4}$$ Domain excludes x-values that result in division by zero.

has an implied domain that consists of all real numbers x other than $x = \pm 2$. These two values are excluded from the domain because division by zero is undefined. Another common type of implied domain is that used to avoid even roots of negative numbers. For example, the function

$$(x) = \sqrt{x}$$ Domain excludes x-values that result in even roots of negative numbers.

is defined only for $x \geq 0$. So, its implied domain is the interval $[0, \infty)$. In general, the domain of a function *excludes* values that would cause division by zero *or* result in the even root of a negative number.

Explore the Concept

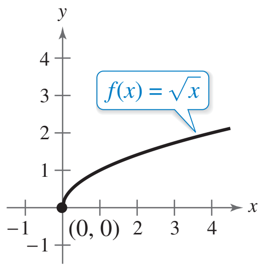
Use a graphing utility to graph $y = \sqrt{4 - x^2}$. What is the domain of this function? Then graph $y = \sqrt{x^2 - 4}$. What is the domain of this function? Do the domains of these two functions overlap? If so, for what values?

Library of Parent Functions: Square Root Function

Radical functions arise from the use of rational exponents. The most common radical function is the *parent square root* function given by $f(x) = \sqrt{x}$. The basic characteristics of the parent square root function are summarized below and on the inside cover of this text.

Graph of $f(x) = \sqrt{x}$

Domain: $[0, \infty)$
Range: $[0, \infty)$
Intercept: $(0, 0)$
Increasing on $(0, \infty)$



Remark

Because the square root function is not defined for $x < 0$, you must be careful when analyzing the domains of complicated functions involving the square root symbol.

EXAMPLE 5 Finding the Domain of a Function

Find the domain of each function.

a. f: $\{(-3, 0), (-1, 4), (0, 2), (2, 2), (4, -1)\}$

b. $g(x) = -3x^2 + 4x + 5$

c. $h(x) = \dfrac{1}{x + 5}$

Solution

a. The domain of f consists of all first coordinates in the set of ordered pairs.

$$\text{Domain} = \{-3, -1, 0, 2, 4\}$$

b. The domain of g is the set of all *real* numbers.

c. Excluding x-values that yield zero in the denominator, the domain of h is the set of all real numbers x except $x = -5$.

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Find the domain of each function.

a. f: $\{(-2, 2), (-1, 1), (0, 3), (1, 1), (2, 2)\}$ **b.** $g(x) = \dfrac{1}{3 - x}$

EXAMPLE 6 Finding the Domain of a Function

Find the domain of each function.

a. Volume of a sphere: $V = \frac{4}{3}\pi r^3$ **b.** $k(x) = \sqrt{4 - 3x}$

Solution

a. Because this function represents the volume of a sphere, the values of the radius r must be positive (see Figure 1.12). So, the domain is the set of all real numbers r such that $r > 0$.

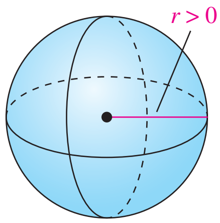

Figure 1.12

b. This function is defined only for x-values for which $4 - 3x \geq 0$. By solving this inequality, you will find that the domain of k is all real numbers that are less than or equal to $\frac{4}{3}$.

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Find the domain of each function.

a. Circumference of a circle: $C = 2\pi r$ **b.** $h(x) = \sqrt{x - 16}$

In Example 6(a), note that the *domain of a function may be implied by the physical context.* For instance, from the equation $V = \frac{4}{3}\pi r^3$, you would have no reason to restrict r to positive values, but the physical context implies that a sphere cannot have a negative or zero radius.

For some functions, it may be easier to find the domain and range of the function by examining its graph.

EXAMPLE 7 Finding the Domain and Range of a Function

Use a graphing utility to find the domain and range of the function $f(x) = \sqrt{9 - x^2}$.

Solution

Graph the function as $y = \sqrt{9 - x^2}$, as shown in Figure 1.13. Using the *trace* feature of a graphing utility, you can determine that the x-values extend from -3 to 3 and the y-values extend from 0 to 3. So, the domain of the function f is all real numbers such that $-3 \leq x \leq 3$, and the range of f is all real numbers such that $0 \leq y \leq 3$.

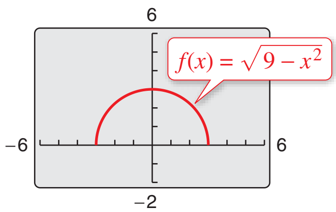

Figure 1.13

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Use a graphing utility to find the domain and range of the function $f(x) = \sqrt{4 - x^2}$.

Activities

1. Evaluate $f(x) = 2 + 3x - x^2$ for
   a. $f(-3)$
   b. $f(x + 1)$
   c. $f(x + h) - f(x)$

   Answers:
   a. -16
   b. $-x^2 + x + 4$
   c. $3h - 2xh - h^2$
2. Determine whether y is a function of x: $2x^3 + 3x^2y^2 + 1 = 0$.

   Answer: No
3. Find the domain: $f(x) = \dfrac{3}{x + 1}$.

   Answer: All real numbers x except $x = -1$

Remark

Be sure you see that the *range* of a function is not the same as the use of *range* relating to the viewing window of a graphing utility.

Applications

EXAMPLE 8 Interior Design Services Employees

The number N (in thousands) of employees in the interior design services industry in the United States decreased in a linear pattern from 2007 through 2010 (see Figure 1.14). In 2011, the number rose and increased through 2012 in a different linear pattern. These two patterns can be approximated by the function

$$N(t) = \begin{cases} -4.64t + 76.2, & 7 \le t \le 10 \\ 0.90t + 20.0, & 11 \le t \le 12 \end{cases}$$

where t represents the year, with $t = 7$ corresponding to 2007. Use this function to approximate the number of employees for the years 2007 and 2011. (Source: U.S. Bureau of Labor Statistics)

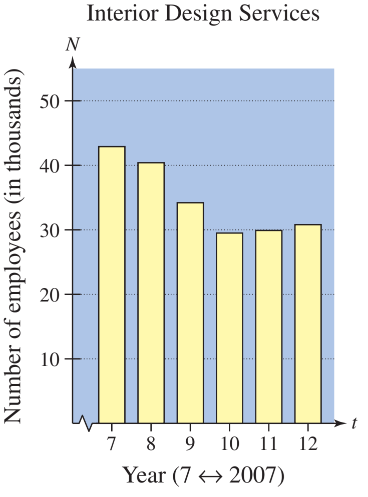

Figure 1.14

Solution

For 2007, $t = 7$, so use $N(t) = -4.64t + 76.2$.

$$N(7) = -4.64(7) + 76.2 = -32.48 + 76.2 = 43.72 \text{ thousand employees}$$

For 2011, $t = 11$, so use $N(t) = 0.90t + 20.0$.

$$N(11) = 0.90(11) + 20.0 = 9.9 + 20.0 = 29.9 \text{ thousand employees}$$

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Use the function in Example 8 to approximate the number of employees for the years 2010 and 2012.

EXAMPLE 9 The Path of a Baseball

A baseball is hit at a point 3 feet above the ground at a velocity of 100 feet per second and an angle of 45°. The path of the baseball is given by $f(x) = -0.0032x^2 + x + 3$, where $f(x)$ is the height of the baseball (in feet) and x is the horizontal distance from home plate (in feet). Will the baseball clear a 10-foot fence located 300 feet from home plate?

Algebraic Solution

The height of the baseball is a function of the horizontal distance from home plate. When $x = 300$, you can find the height of the baseball as follows.

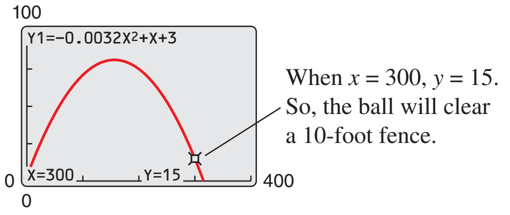
$$f(x) = -0.0032x^2 + x + 3 \qquad \text{Write original function.}$$

$$f(300) = -0.0032(300)^2 + 300 + 3 \qquad \text{Substitute 300 for } x.$$

$$= 15 \qquad \text{Simplify.}$$

When $x = 300$, the height of the baseball is 15 feet. So, the baseball will clear a 10-foot fence.

Graphical Solution



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A second baseman throws a baseball toward the first baseman 60 feet away. The path of the baseball is given by $f(x) = -0.004x^2 + 0.3x + 6$, where $f(x)$ is the height of the baseball (in feet) and x is the horizontal distance from the second baseman (in feet). The first baseman can reach 8 feet high. Can the first baseman catch the baseball without jumping?

Difference Quotients

One of the basic definitions in calculus employs the ratio

$$\frac{f(x+h)-f(x)}{h}, \quad h \neq 0.$$

This ratio is called a **difference quotient,** as illustrated in Example 10.

EXAMPLE 10 Evaluating a Difference Quotient

For $f(x) = x^2 - 4x + 7$, find $\dfrac{f(x+h)-f(x)}{h}$.

Solution

$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{[(x+h)^2 - 4(x+h) + 7] - (x^2 - 4x + 7)}{h}\\ &= \frac{x^2 + 2xh + h^2 - 4x - 4h + 7 - x^2 + 4x - 7}{h}\\ &= \frac{2xh + h^2 - 4h}{h}\\ &= \frac{h(2x + h - 4)}{h}\\ &= 2x + h - 4, \quad h \neq 0\end{aligned}$$

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For $f(x) = x^2 + 2x - 3$, evaluate the difference quotient in Example 10.

Remark

Notice in Example 10 that h cannot be zero in the original expression. Therefore, you must restrict the domain of the simplified expression by listing $h \neq 0$ so that the simplified expression is equivalent to the original expression.

Summary of Function Terminology

Function: A **function** is a relationship between two variables such that to each value of the independent variable there corresponds exactly one value of the dependent variable.

Function Notation: $y = f(x)$

f is the *name* of the function.
y is the **dependent variable,** or output value.
x is the **independent variable,** or input value.
$f(x)$ is the *value of the function at* x.

Domain: The **domain** of a function is the set of all values (inputs) of the independent variable for which the function is defined. If x is in the domain of f, then f is said to be *defined* at x. If x is not in the domain of f, then f is said to be *undefined* at x.

Range: The **range** of a function is the set of all values (outputs) assumed by the dependent variable (that is, the set of all function values).

Implied Domain: If f is defined by an algebraic expression and the domain is not specified, then the **implied domain** consists of all real numbers for which the expression is defined.

The symbol $\int$ indicates an example or exercise that highlights algebraic techniques specifically used in calculus.

1.2 Exercises

See *CalcChat.com* for tutorial help and worked-out solutions to odd-numbered exercises.
For instructions on how to use a graphing utility, see Appendix A.

Vocabulary and Concept Check

In Exercises 1 and 2, fill in the blanks.

1. A relation that assigns to each element x from a set of inputs, or ________ , exactly one element y in a set of outputs, or ________ , is called a ________ .

2. For an equation that represents y as a function of x, the ________ variable is the set of all x in the domain, and the ________ variable is the set of all y in the range.

3. Can the ordered pairs (3, 0) and (3, 5) represent a function?

4. To find $g(x + 1)$, what do you substitute for x in the function $g(x) = 3x - 2$?

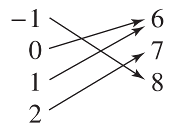
5. Does the domain of the function $f(x) = \sqrt{1 + x}$ include $x = -2$?

6. Is the domain of a piecewise-defined function *implied* or *explicitly described*?

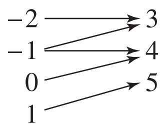
Procedures and Problem Solving

Testing for Functions **In Exercises 7–10, does the relation describe a function? Explain your reasoning.**

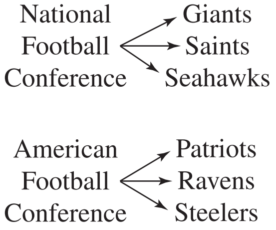
7. *Domain* *Range*



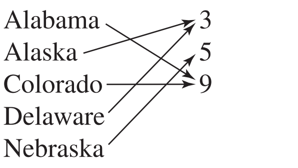
8. *Domain* *Range*



9. *Domain* *Range*



10. *Domain* (State) *Range* (Electoral votes 2010–2012)



Testing for Functions **In Exercises 11–14, determine whether the relation represents y as a function of x. Explain your reasoning.**

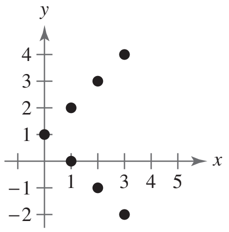
11.

Input, x	-3	-1	0	1	3
Output, y	-9	-1	0	1	-9

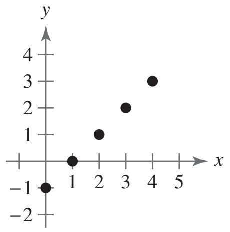
12.

Input, x	0	1	2	1	0
Output, y	-4	-2	0	2	4

13.



14.



Testing for Functions **In Exercises 15 and 16, which sets of ordered pairs represent functions from A to B? Explain.**

15. $A = \{0, 1, 2, 3\}$ and $B = \{-2, -1, 0, 1, 2\}$

(a) $\{(0, 1), (1, -2), (2, 0), (3, 2)\}$

(b) $\{(0, -1), (2, 2), (1, -2), (3, 0), (1, 1)\}$

(c) $\{(1, 0), (-2, 3), (-1, 3), (0, 0)\}$

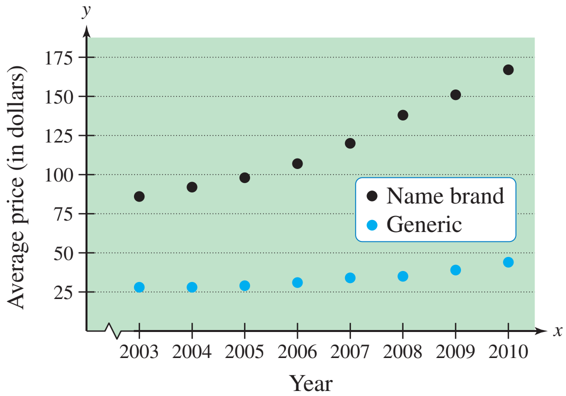
16. $A = \{a, b, c\}$ and $B = \{0, 1, 2, 3\}$

(a) $\{(a, 1), (c, 2), (c, 3), (b, 3)\}$

(b) $\{(a, 1), (b, 2), (c, 3)\}$

(c) $\{(1, a), (0, a), (2, c), (3, b)\}$

Pharmacology **In Exercises 17 and 18, use the graph, which shows the average prices of name brand and generic drug prescriptions in the United States.** (Source: National Association of Chain Drug Stores)



17. Is the average price of a name brand prescription a function of the year? Is the average price of a generic prescription a function of the year? Explain.

18. Let $b(t)$ and $g(t)$ represent the average prices of name brand and generic prescriptions, respectively, in year t. Find $b(2009)$ and $g(2006)$.

Testing for Functions Represented Algebraically **In Exercises 19–30, determine whether the equation represents y as a function of x.**

19. $x^2 + y^2 = 4$

20. $x = y^2 + 1$

21. $y = \sqrt{x^2 - 1}$

22. $y = \sqrt{x + 5}$

23. $2x + 3y = 4$

24. $x = -y + 5$

25. $y^2 = x^2 - 1$

26. $x + y^2 = 3$

27. $y = |4 - x|$

28. $|y| = 3 - 2x$

29. $x = -7$

30. $y = 8$

Evaluating a Function **In Exercises 31–46, evaluate the function at each specified value of the independent variable and simplify.**

31. $f(t) = 3t + 1$

(a) $f(2)$ (b) $f(-4)$ (c) $f(t + 2)$

32. $g(y) = 7 - 3y$

(a) $g(0)$ (b) $g\left(\frac{7}{3}\right)$ (c) $g(s + 5)$

33. $h(t) = t^2 - 2t$

(a) $h(2)$ (b) $h(1.5)$ (c) $h(x - 4)$

34. $V(r) = \frac{4}{3}\pi r^3$

(a) $V(3)$ (b) $V\left(\frac{3}{2}\right)$ (c) $V(2r)$

35. $f(y) = 3 - \sqrt{y}$

(a) $f(4)$ (b) $f(0.25)$ (c) $f(4x^2)$

36. $f(x) = \sqrt{x + 8} + 2$

(a) $f(-4)$ (b) $f(8)$ (c) $f(x - 8)$

37. $q(x) = \dfrac{1}{x^2 - 9}$

(a) $q(-3)$ (b) $q(2)$ (c) $q(y + 3)$

38. $q(t) = \dfrac{2t^2 + 3}{t^2}$

(a) $q(2)$ (b) $q(0)$ (c) $q(-x)$

39. $f(x) = \dfrac{|x|}{x}$

(a) $f(9)$ (b) $f(-9)$ (c) $f(t)$

40. $f(x) = |x| + 4$

(a) $f(5)$ (b) $f(-5)$ (c) $f(t)$

41. $f(x) = \begin{cases} 2x + 1, & x < 0 \\ 2x + 2, & x \geq 0 \end{cases}$

(a) $f(-1)$ (b) $f(0)$ (c) $f(2)$

42. $f(x) = \begin{cases} 2x + 5, & x \leq 0 \\ 2 - x, & x > 0 \end{cases}$

(a) $f(-2)$ (b) $f(0)$ (c) $f(1)$

43. $f(x) = \begin{cases} x^2 + 2, & x \leq 1 \\ 2x^2 + 2, & x > 1 \end{cases}$

(a) $f(-2)$ (b) $f(1)$ (c) $f(2)$

44. $f(x) = \begin{cases} x^2 - 4, & x \leq 0 \\ 1 - 2x^2, & x > 0 \end{cases}$

(a) $f(-2)$ (b) $f(0)$ (c) $f(1)$

45. $f(x) = \begin{cases} x + 2, & x < 0 \\ 4, & 0 \leq x < 2 \\ x^2 + 1, & x \geq 2 \end{cases}$

(a) $f(-2)$ (b) $f(0)$ (c) $f(2)$

46. $f(x) = \begin{cases} 5 - 2x, & x < 0 \\ 5, & 0 \leq x < 1 \\ 4x + 1, & x \geq 1 \end{cases}$

(a) $f(-4)$ (b) $f(0)$ (c) $f(1)$

Evaluating a Function **In Exercises 47–50, assume that the domain of f is the set $A = \{-2, -1, 0, 1, 2\}$. Determine the set of ordered pairs representing the function f.**

47. $f(x) = (x - 1)^2$

48. $f(x) = x^2 - 3$

49. $f(x) = |x| + 2$

50. $f(x) = |x + 1|$

Evaluating a Function **In Exercises 51 and 52, complete the table.**

51. $h(t) = \frac{1}{2}|t + 3|$

t	-5	-4	-3	-2	-1
$h(t)$					

52. $f(s) = \dfrac{|s - 2|}{s - 2}$

s	0	1	$\frac{3}{2}$	$\frac{5}{2}$	4
$f(s)$					

Finding the Inputs That Have Outputs of Zero **In Exercises 53–56, find all values of x such that $f(x) = 0$.**

53. $f(x) = 15 - 3x$

54. $f(x) = 5x + 1$

55. $f(x) = \dfrac{9x - 4}{5}$

56. $f(x) = \dfrac{2x - 3}{7}$

Finding the Domain of a Function **In Exercises 57–66, find the domain of the function.**

57. $f(x) = 5x^2 + 2x - 1$

58. $g(x) = 1 - 2x^2$

59. $h(t) = \dfrac{4}{t}$

60. $s(y) = \dfrac{3y}{y + 5}$

61. $f(x) = \sqrt[3]{x - 4}$

62. $f(x) = \sqrt[4]{x^2 + 3x}$

63. $g(x) = \dfrac{1}{x} - \dfrac{3}{x + 2}$

64. $h(x) = \dfrac{10}{x^2 - 2x}$

65. $g(y) = \dfrac{y + 2}{\sqrt{y - 10}}$

66. $f(x) = \dfrac{\sqrt{x + 6}}{6 + x}$

Finding the Domain and Range of a Function In Exercises 67–70, use a graphing utility to graph the function. Find the domain and range of the function.

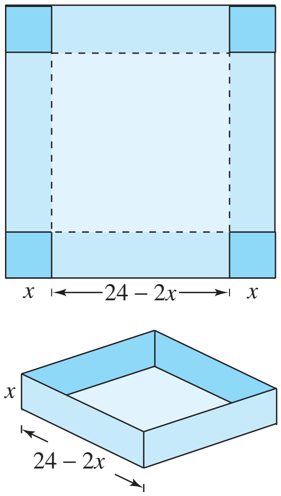
67. $f(x) = \sqrt{16 - x^2}$ **68.** $f(x) = \sqrt{x^2 + 1}$

69. $g(x) = |2x + 3|$ **70.** $g(x) = |3x - 5|$

71. Geometry Write the area A of a circle as a function of its circumference C.

72. Geometry Write the area A of an equilateral triangle as a function of the length s of its sides.

73. Exploration An open box of maximum volume is to be made from a square piece of material, 24 centimeters on a side, by cutting equal squares from the corners and turning up the sides. (See figure.)



(a) The table shows the volume V (in cubic centimeters) of the box for various heights x (in centimeters). Use the table to estimate the maximum volume.

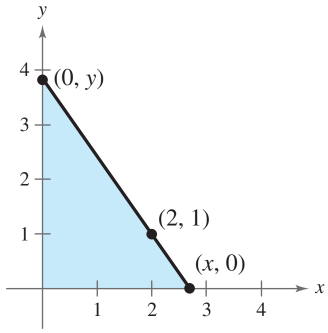
Height, x	Volume, V
1	484
2	800
3	972
4	1024
5	980
6	864

(b) Plot the points (x, V) from the table in part (a). Does the relation defined by the ordered pairs represent V as a function of x?

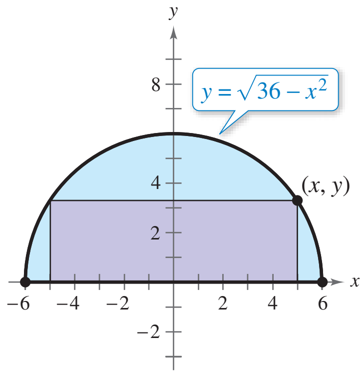
(c) If V is a function of x, write the function and determine its domain.

(d) Use a graphing utility to plot the points from the table in part (a) with the function from part (c). How closely does the function represent the data? Explain.

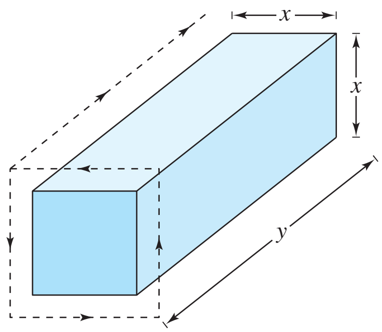
74. Geometry A right triangle is formed in the first quadrant by the x- and y-axes and a line through the point (2, 1), as shown in the figure. Write the area A of the triangle as a function of x and determine the domain of the function.



75. Geometry A rectangle is bounded by the x-axis and the semicircle $y = \sqrt{36 - x^2}$, as shown in the figure. Write the area A of the rectangle as a function of x and determine the domain of the function.



76. Geometry A rectangular package to be sent by the U.S. Postal Service can have a maximum combined length and girth (perimeter of a cross section) of 108 inches. (See figure.)



(a) Write the volume V of the package as a function of x. What is the domain of the function?

(b) Use a graphing utility to graph the function. Be sure to use an appropriate viewing window.

(c) What dimensions will maximize the volume of the package? Explain.

77. Business A company produces a product for which the variable cost is \$68.75 per unit and the fixed costs are \$248,000. The product sells for \$99.99. Let x be the number of units produced and sold.

(a) The total cost for a business is the sum of the variable cost and the fixed costs. Write the total cost C as a function of the number of units produced.

(b) Write the revenue R as a function of the number of units sold.

(c) Write the profit P as a function of the number of units sold. (*Note*: $P = R - C$.)

(d) Use the model in part (c) to find $P(20{,}000)$. Interpret your result in the context of the situation.

(e) Use the model in part (c) to find $P(0)$. Interpret your result in the context of the situation.

78. MODELING DATA

The table shows the revenue y (in thousands of dollars) of a landscaping business for each month of 2015, with $x = 1$ representing January.

Month, x	Revenue, y
1	5.2
2	5.6
3	6.6
4	8.3
5	11.5
6	15.8
7	12.8
8	10.1
9	8.6
10	6.9
11	4.5
12	2.7

The mathematical model below represents the data.

$$f(x) = \begin{cases} -1.97x + 26.3 \\ 0.505x^2 - 1.47x + 6.3 \end{cases}$$

(a) Identify the independent and dependent variables and explain what they represent in the context of the problem.

(b) What is the domain of each part of the piecewise-defined function? Explain your reasoning.

(c) Use the mathematical model to find $f(5)$. Interpret your result in the context of the problem.

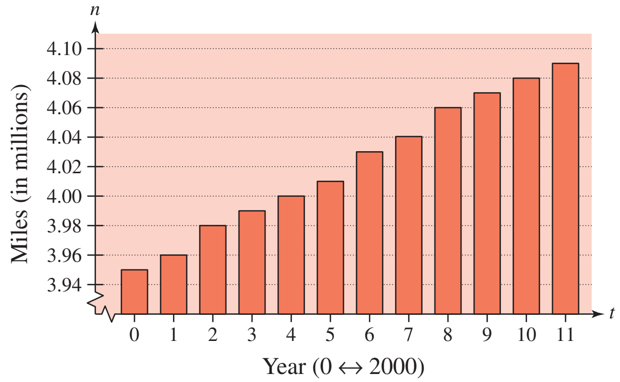
(d) Use the mathematical model to find $f(11)$. Interpret your result in the context of the problem.

(e) How do the values obtained from the models in parts (c) and (d) compare with the actual data values?

79. Civil Engineering The total numbers n (in millions) of miles for all public roadways in the United States from 2000 through 2011 can be approximated by the model

$$n(t) = \begin{cases} 0.0050t^2 + 0.005t + 3.95, & 0 \le t \le 2 \\ 0.013t + 3.95, & 2 < t \le 11 \end{cases}$$

where t represents the year, with $t = 0$ corresponding to 2000. The actual numbers are shown in the bar graph. (Source: U.S. Federal Highway Administration)



(a) Identify the independent and dependent variables and explain what they represent in the context of the problem.

(b) Use the *table* feature of a graphing utility to approximate the total number of miles for all public roadways each year from 2000 through 2011.

(c) Compare the values in part (b) with the actual values shown in the bar graph. How well does the model fit the data?

(d) Do you think the piecewise-defined function could be used to predict the total number of miles for all public roadways for years outside the domain? Explain your reasoning.

80. ***Why you should learn it*** *(p. 16)* The force F (in tons) of water against the face of a dam is estimated by the function

$$F(y) = 149.76\sqrt{10}\,y^{5/2}$$

where y is the depth of the water (in feet).

(a) Complete the table. What can you conclude?

y	5	10	20	30	40
$F(y)$					

(b) Use a graphing utility to graph the function. Describe your viewing window.

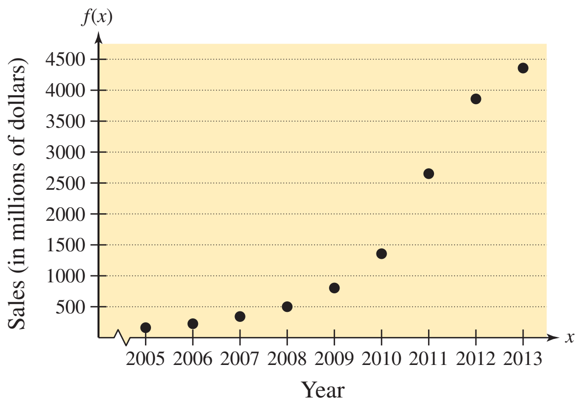
(c) Use the table to approximate the depth at which the force against the dam is 1,000,000 tons. Verify your answer graphically. How could you find a better estimate?

81. Projectile Motion The height y (in feet) of a baseball thrown by a child is

$$y = -0.1x^2 + 3x + 6$$

where x is the horizontal distance (in feet) from where the ball was thrown. Will the ball fly over the glove of another child 30 feet away trying to catch the ball? Explain. (Assume that the child who is trying to catch the ball holds a baseball glove at a height of 5 feet.)

82. Business The graph shows the sales (in millions of dollars) of Green Mountain Coffee Roasters from 2005 through 2013. Let $f(x)$ represent the sales in year x. (Source: Green Mountain Coffee Roasters, Inc.)



(a) Find $\dfrac{f(2013) - f(2005)}{2013 - 2005}$ and interpret the result in the context of the problem.

(b) An approximate model for the function is

$S(t) = 90.442t^2 - 1075.25t + 3332.5, \quad 5 \le t \le 13$

where S is the sales (in millions of dollars) and $t = 5$ represents 2005. Complete the table and compare the results with the data in the graph.

t	5	6	7	8	9	10	11	12	13
$S(t)$									

Evaluating a Difference Quotient **In Exercises 83–86, find the difference quotient and simplify your answer.**

83. $f(x) = 2x, \quad \dfrac{f(x + c) - f(x)}{c}, \quad c \ne 0$

84. $g(x) = 3x - 1, \quad \dfrac{g(x + h) - g(x)}{h}, \quad h \ne 0$

85. $f(x) = x^2 - x + 1, \quad \dfrac{f(2 + h) - f(2)}{h}, \quad h \ne 0$

86. $f(x) = x^3 + x, \quad \dfrac{f(x + h) - f(x)}{h}, \quad h \ne 0$

Conclusions

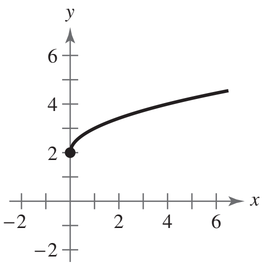
True or False? **In Exercises 87 and 88, determine whether the statement is true or false. Justify your answer.**

87. The domain of the function $f(x) = x^4 - 1$ is $(-\infty, \infty)$, and the range of f is $(0, \infty)$.

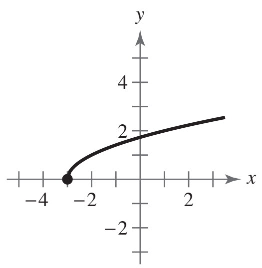
88. The set of ordered pairs $\{(-8, -2), (-6, 0), (-4, 0), (-2, 2), (0, 4), (2, -2)\}$ represents a function.

Think About It **In Exercises 89 and 90, write a square root function for the graph shown. Then identify the domain and range of the function.**

89.

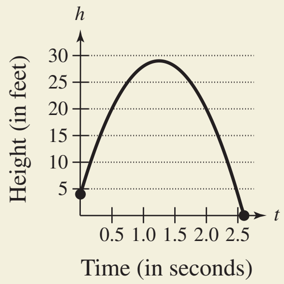


90.



91. Think About It Given $f(x) = x^2$, is f the independent variable? Why or why not?

92. HOW DO YOU SEE IT? The graph represents the height h of a projectile after t seconds.



(a) Explain why h is a function of t.

(b) Approximate the height of the projectile after 0.5 second and after 1.25 seconds.

(c) Approximate the domain of h.

(d) Is t a function of h? Explain.

Cumulative Mixed Review

Operations with Rational Expressions **In Exercises 93–96, perform the operation and simplify.**

93. $12 - \dfrac{4}{x + 2}$

94. $\dfrac{3}{x^2 + x - 20} + \dfrac{2x}{x^2 + 4x - 5}$

95. $\dfrac{x^5}{2x^3 + 4x^2} \cdot \dfrac{4x + 8}{3x}$

96. $\dfrac{x + 7}{2(x - 9)} \div \dfrac{x - 7}{2(x - 9)}$

The symbol ∫ indicates an example or exercise that highlights algebraic techniques specifically used in calculus.

1.3 Graphs of Functions

The Graph of a Function

In Section 1.2, some functions were represented graphically by points on a graph in a coordinate plane in which the input values are represented by the horizontal axis and the output values are represented by the vertical axis. The **graph of a function** f is the collection of ordered pairs $(x, f(x))$ such that x is in the domain of f. As you study this section, remember the geometric interpretations of x and $f(x)$.

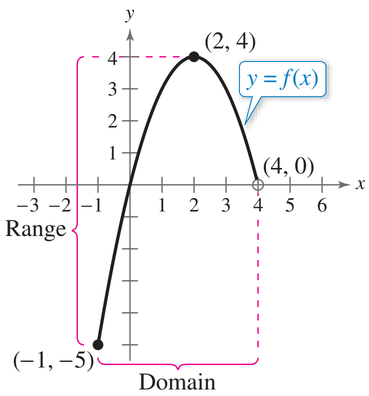
$$x = \text{the directed distance from the } y\text{-axis}$$

$$f(x) = \text{the directed distance from the } x\text{-axis}$$

Example 1 shows how to use the graph of a function to find the domain and range of the function.

EXAMPLE 1 Finding the Domain and Range of a Function

Use the graph of the function f to find (a) the domain of f, (b) the function values $f(-1)$ and $f(2)$, and (c) the range of f.



Solution

a. The closed dot at $(-1, -5)$ indicates that $x = -1$ is in the domain of f, whereas the open dot at $(4, 0)$ indicates that $x = 4$ is not in the domain. So, the domain of f is all x in the interval $[-1, 4)$.

b. Because $(-1, -5)$ is a point on the graph of f, it follows that

$$f(-1) = -5.$$

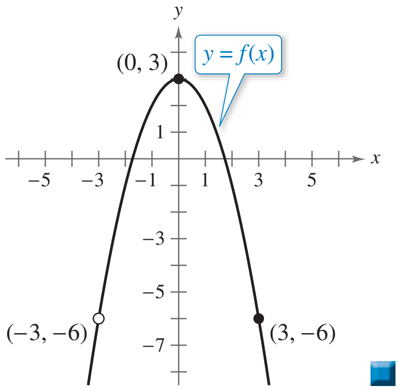
Similarly, because $(2, 4)$ is a point on the graph of f, it follows that

$$f(2) = 4.$$

c. Because the graph does not extend below $f(-1) = -5$ or above $f(2) = 4$, the range of f is the interval $[-5, 4]$.

Checkpoint *Audio-video solution in English & Spanish at LarsonPrecalculus.com.*

Use the graph of the function f to find (a) the domain of f, (b) the function values $f(0)$ and $f(3)$, and (c) the range of f.



The use of dots (open or closed) at the extreme left and right points of a graph indicates that the graph does not extend beyond these points. When no such dots are shown, assume that the graph extends beyond these points.

What you should learn

- Find the domains and ranges of functions and use the Vertical Line Test for functions.
- Determine intervals on which functions are increasing, decreasing, or constant.
- Determine relative maximum and relative minimum values of functions.
- Identify and graph step functions and other piecewise-defined functions.
- Identify even and odd functions.

Why you should learn it

Graphs of functions provide visual relationships between two variables. For example, in Exercise 92 on page 39, you will use the graph of a step function to model the cost of sending a package.

EXAMPLE 2 Finding the Domain and Range of a Function

Find the domain and range of $f(x) = \sqrt{x - 4}$.

Algebraic Solution

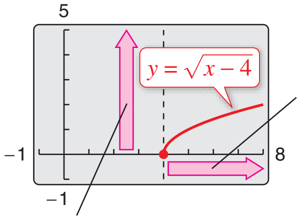
Because the expression under a radical cannot be negative, the domain of $f(x) = \sqrt{x - 4}$ is the set of all real numbers such that $x - 4 \geq 0$. Solve this linear inequality for x as follows. (For help with solving linear inequalities, see Appendix E at this textbook's *Companion Website*.)

$$x - 4 \geq 0 \quad \text{Write original inequality.}$$

$$x \geq 4 \quad \text{Add 4 to each side.}$$

So, the domain is the set of all real numbers greater than or equal to 4. Because the value of a radical expression is never negative, the range of $f(x) = \sqrt{x - 4}$ is the set of all nonnegative real numbers.

Graphical Solution



The x-coordinates of points on the graph extend from 4 to the right. So, the domain is the set of all real numbers greater than or equal to 4.

The y-coordinates of points on the graph extend from 0 upwards. So, the range is the set of all nonnegative real numbers.

Checkpoint *Audio-video solution in English & Spanish at LarsonPrecalculus.com.*

Find the domain and range of $f(x) = \sqrt{x - 1}$.

By the definition of a function, at most one y-value corresponds to a given x-value. It follows, then, that a vertical line can intersect the graph of a function at most once. This leads to the **Vertical Line Test** for functions.

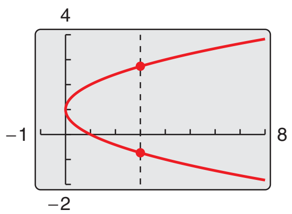
Vertical Line Test for Functions

A set of points in a coordinate plane is the graph of y as a function of x if and only if no vertical line intersects the graph at more than one point.

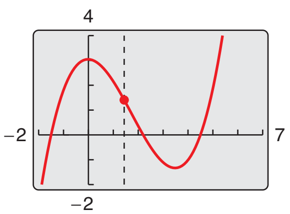
EXAMPLE 3 Vertical Line Test for Functions

Use the Vertical Line Test to decide whether each graph represents y as a function of x.

a.



b.



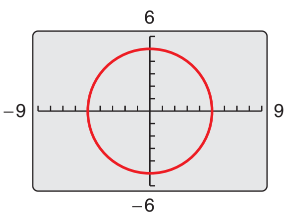
Solution

a. This is *not* a graph of y as a function of x because you can find a vertical line that intersects the graph twice.

b. This *is* a graph of y as a function of x because every vertical line intersects the graph at most once.

Checkpoint *Audio-video solution in English & Spanish at LarsonPrecalculus.com.*

Use the Vertical Line Test to decide whether the graph at the right represents y as a function of x.



Technology Tip

Most graphing utilities are designed to graph functions of x more easily than other types of equations. For instance, the graph shown in Example 3(a) represents the equation $x - (y - 1)^2 = 0$. To use a graphing utility to duplicate this graph, you must first solve the equation for y to obtain $y = 1 \pm \sqrt{x}$, and then graph the two equations $y_1 = 1 + \sqrt{x}$ and $y_2 = 1 - \sqrt{x}$ in the same viewing window.

Increasing and Decreasing Functions

The more you know about the graph of a function, the more you know about the function itself. Consider the graph shown in Figure 1.15. Moving from *left to right,* this graph falls from $x = -2$ to $x = 0$, is constant from $x = 0$ to $x = 2$, and rises from $x = 2$ to $x = 4$.

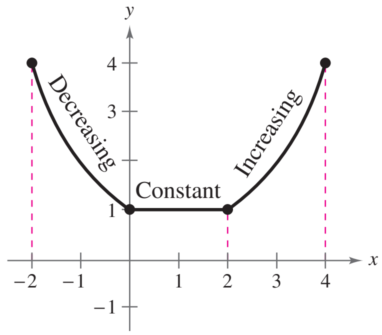


Figure 1.15

Increasing, Decreasing, and Constant Functions

A function f is **increasing** on an interval when, for any x_1 and x_2 in the interval,

$$x_1 < x_2 \text{ implies } f(x_1) < f(x_2).$$

A function f is **decreasing** on an interval when, for any x_1 and x_2 in the interval,

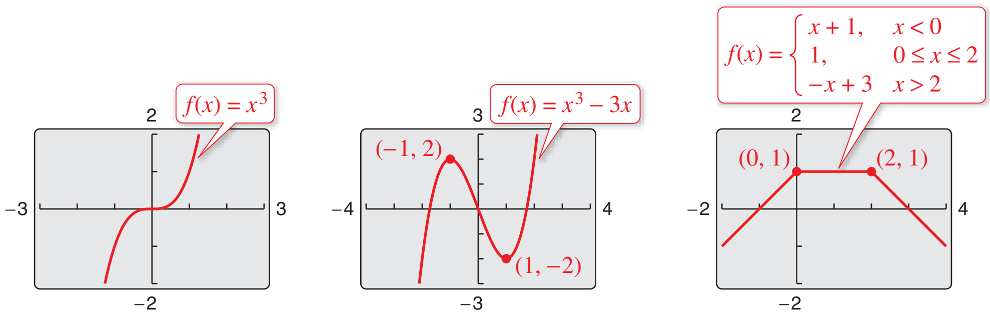
$$x_1 < x_2 \text{ implies } f(x_1) > f(x_2).$$

A function f is **constant** on an interval when, for any x_1 and x_2 in the interval,

$$f(x_1) = f(x_2).$$

EXAMPLE 4 Increasing and Decreasing Functions

In Figure 1.16, determine the open intervals on which each function is increasing, decreasing, or constant.



(a) **(b)** **(c)**

Figure 1.16

Solution

a. Although it might appear that there is an interval in which this function is constant, you can see that if $x_1 < x_2$, then $(x_1)^3 < (x_2)^3$, which implies that $f(x_1) < f(x_2)$. So, the function is increasing over the entire real line.

b. This function is increasing on the interval $(-\infty, -1)$, decreasing on the interval $(-1, 1)$, and increasing on the interval $(1, \infty)$.

c. This function is increasing on the interval $(-\infty, 0)$, constant on the interval $(0, 2)$, and decreasing on the interval $(2, \infty)$.

Checkpoint *Audio-video solution in English & Spanish at LarsonPrecalculus.com.*

Graph the function $f(x) = x^3 + 3x^2 - 1$. Then use the graph to describe the increasing and decreasing behavior of the function.

Relative Minimum and Maximum Values

The points at which a function changes its increasing, decreasing, or constant behavior are helpful in determining the relative maximum or relative minimum values of the function.

Definition of Relative Minimum and Relative Maximum

A function value $f(a)$ is called a **relative minimum** of f when there exists an interval (x_1, x_2) that contains a such that

$$x_1 < x < x_2 \quad \text{implies} \quad f(a) \le f(x).$$

A function value $f(a)$ is called a **relative maximum** of f when there exists an interval (x_1, x_2) that contains a such that

$$x_1 < x < x_2 \quad \text{implies} \quad f(a) \ge f(x).$$

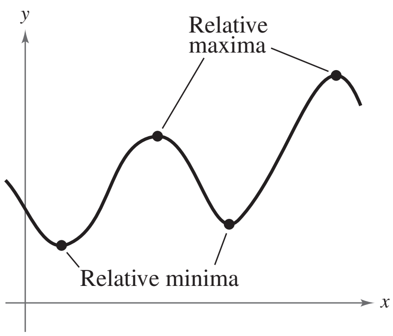


Figure 1.17

Figure 1.17 shows several different examples of relative minima and relative maxima. In Section 2.1, you will study a technique for finding the *exact points* at which a second-degree polynomial function has a relative minimum or relative maximum. For the time being, however, you can use a graphing utility to find reasonable approximations of these points.

EXAMPLE 5 Approximating a Relative Minimum

Use a graphing utility to approximate the relative minimum of the function given by $f(x) = 3x^2 - 4x - 2$.

Solution

The graph of f is shown in Figure 1.18. By using the *zoom* and *trace* features of a graphing utility, you can estimate that the function has a relative minimum at the point

$(0.67, -3.33)$. See Figure 1.19.

Later, in Section 2.1, you will be able to determine that the exact point at which the relative minimum occurs is $\left(\frac{2}{3}, -\frac{10}{3}\right)$.

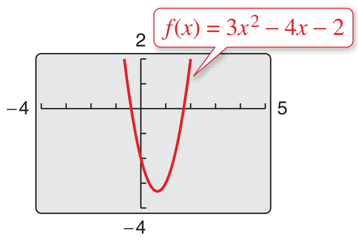


Figure 1.18

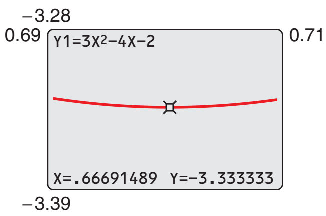


Figure 1.19

Technology Tip

When you use a graphing utility to estimate the x- and y-values of a relative minimum or relative maximum, the *zoom* feature will often produce graphs that are nearly flat, as shown in Figure 1.19. To overcome this problem, you can manually change the vertical setting of the viewing window. The graph will stretch vertically when the values of Ymin and Ymax are closer together.

✔ ***Checkpoint*** *Audio-video solution in English & Spanish at LarsonPrecalculus.com.*

Use a graphing utility to approximate the relative maximum of the function

$f(x) = -4x^2 - 7x + 3$.

You can also use the *table* feature of a graphing utility to numerically approximate the relative minimum of the function in Example 5. Using a table that begins at 0.6 and increments the value of x by 0.01, you can approximate that the minimum of f occurs at the point $(0.67, -3.33)$. Some graphing utilities have built-in programs that will find minimum or maximum values, as shown in Example 6.

EXAMPLE 6 Approximating Relative Minima and Maxima

Use a graphing utility to approximate the relative minimum and relative maximum of the function $f(x) = -x^3 + x$.

Solution

By using the *minimum* and *maximum* features of the graphing utility, you can estimate that the function has a relative minimum at the point

$(-0.58, -0.38)$ See Figure 1.20.

and a relative maximum at the point

$(0.58, 0.38)$. See Figure 1.21.

If you take a course in calculus, you will learn a technique for finding the exact points at which this function has a relative minimum and a relative maximum.

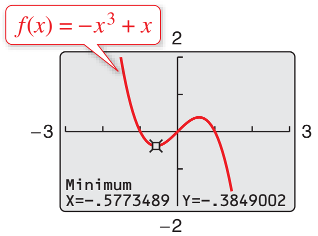


Figure 1.20

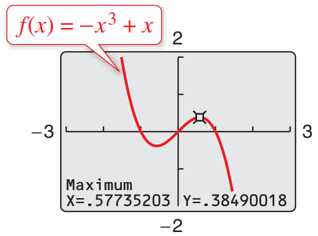


Figure 1.21

Checkpoint *Audio-video solution in English & Spanish at LarsonPrecalculus.com.*

Use a graphing utility to approximate the relative minimum and relative maximum of the function $f(x) = 2x^3 + 3x^2 - 12x$.

EXAMPLE 7 Temperature

During a 24-hour period, the temperature y (in degrees Fahrenheit) of a certain city can be approximated by the model

$$y = 0.026x^3 - 1.03x^2 + 10.2x + 34, \quad 0 \le x \le 24$$

where x represents the time of day, with $x = 0$ corresponding to 6 A.M. Approximate the maximum temperature during this 24-hour period.

Solution

Using the *maximum* feature of a graphing utility, you can determine that the maximum temperature during the 24-hour period was approximately 64°F. This temperature occurred at about 12:36 P.M. ($x \approx 6.6$), as shown in Figure 1.22.

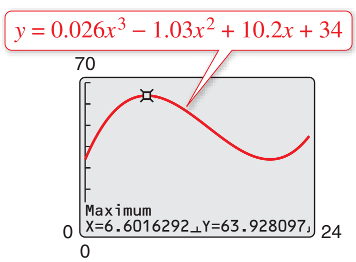


Figure 1.22

Checkpoint *Audio-video solution in English & Spanish at LarsonPrecalculus.com.*

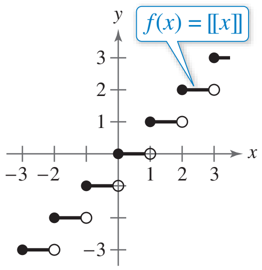
In Example 7, approximate the minimum temperature during the 24-hour period.

Step Functions and Piecewise-Defined Functions

Library of Parent Functions: Greatest Integer Function

The *greatest integer function*, denoted by $[\![x]\!]$ and defined as the greatest integer less than or equal to x, has an infinite number of breaks or steps—one at each integer value in its domain. The basic characteristics of the greatest integer function are summarized below.

Graph of $f(x) = [\![x]\!]$
Domain: $(-\infty, \infty)$
Range: the set of integers
x-intercepts: in the interval $[0, 1)$
y-intercept: $(0, 0)$
Constant between each pair of consecutive integers
Jumps vertically one unit at each integer value



Technology Tip

Most graphing utilities display graphs in *connected mode*, which works well for graphs that do not have breaks. For graphs that do have breaks, such as the greatest integer function, it is better to use *dot mode*. Graph the greatest integer function [often called Int (x)] in *connected* and *dot modes*, and compare the two results.

Demonstrate the real-life nature of step functions by discussing Exercises 91 and 92 in this section. If writing is a part of your course, this section provides a good opportunity for students to find other examples of step functions and write brief essays on their applications.

Because of the vertical jumps described above, the greatest integer function is an example of a **step function** whose graph resembles a set of stairsteps. Some values of the greatest integer function are as follows.

$$[\![-1]\!] = (\text{greatest integer} \le -1) = -1$$
$$[\![-\tfrac{1}{2}]\!] = \left(\text{greatest integer} \le -\tfrac{1}{2}\right) = -1$$
$$[\![\tfrac{1}{10}]\!] = \left(\text{greatest integer} \le \tfrac{1}{10}\right) = 0$$
$$[\![1.5]\!] = (\text{greatest integer} \le 1.5) = 1$$

In Section 1.2, you learned that a piecewise-defined function is a function that is defined by two or more equations over a specified domain. To sketch the graph of a piecewise-defined function, you need to sketch the graph of each equation on the appropriate portion of the domain.

EXAMPLE 8 Sketching a Piecewise-Defined Function

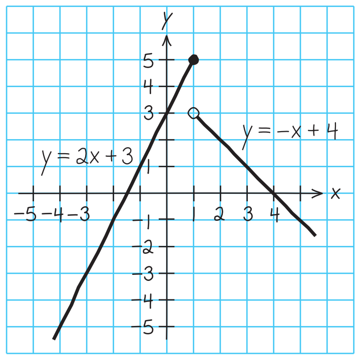
Sketch the graph of

$$f(x) = \begin{cases} 2x + 3, & x \le 1 \\ -x + 4, & x > 1 \end{cases}$$

by hand.

Solution

This piecewise-defined function is composed of two linear functions. At and to the left of $x = 1$, the graph is the line $y = 2x + 3$. To the right of $x = 1$, the graph is the line $y = -x + 4$, as shown in the figure. Notice that the point $(1, 5)$ is a solid dot and the point $(1, 3)$ is an open dot. This is because $f(1) = 5$.



Checkpoint *Audio-video solution in English & Spanish at LarsonPrecalculus.com.*

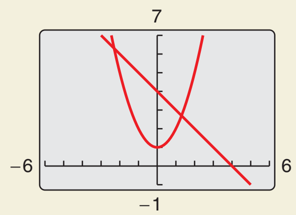
Sketch the graph of $f(x) = \begin{cases} -\frac{1}{2}x - 6, & x \le -4 \\ x + 5, & x > -4 \end{cases}$ by hand.

What's Wrong?

You use a graphing utility to graph

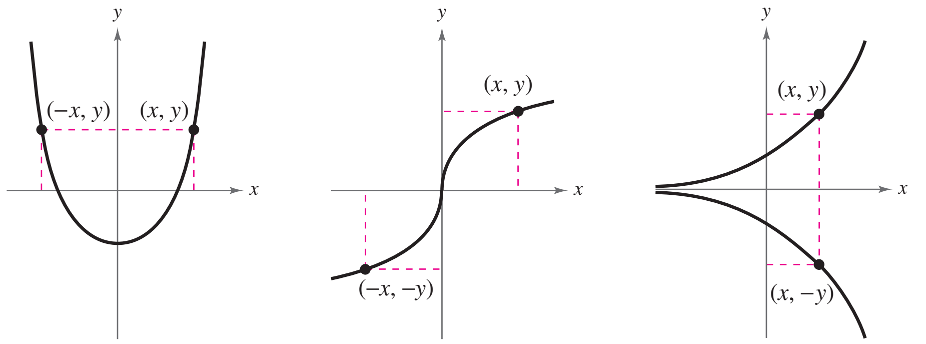
$$f(x) = \begin{cases} x^2 + 1, & x \le 0 \\ 4 - x, & x > 0 \end{cases}$$

by letting $y_1 = x^2 + 1$ and $y_2 = 4 - x$, as shown in the figure. You conclude that this is the graph of f. What's wrong?



Even and Odd Functions

A graph has *symmetry with respect to the y-axis* if whenever (x, y) is on the graph, then so is the point $(-x, y)$. A graph has *symmetry with respect to the origin* if whenever (x, y) is on the graph, then so is the point $(-x, -y)$. A graph has *symmetry with respect to the x-axis* if whenever (x, y) is on the graph, then so is the point $(x, -y)$. A function whose graph is symmetric with respect to the y-axis is an **even function.** A function whose graph is symmetric with respect to the origin is an **odd function.** A graph that is symmetric with respect to the x-axis is not the graph of a function (except for the graph of $y = 0$). These three types of symmetry are illustrated in Figure 1.23.



Symmetric to y-axis
Even function
Figure 1.23

Symmetric to origin
Odd function

Symmetric to x-axis
Not a function

Explore the Concept

Graph each function with a graphing utility. Determine whether the function is *even*, *odd*, or *neither*.

$f(x) = x^2 - x^4$

$g(x) = 2x^3 + 1$

$h(x) = x^5 - 2x^3 + x$

$j(x) = 2 - x^6 - x^8$

$k(x) = x^5 - 2x^4 + x - 2$

$p(x) = x^9 + 3x^5 - x^3 + x$

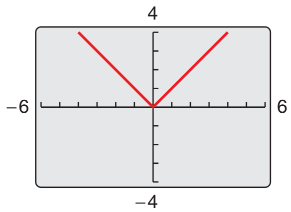
What do you notice about the equations of functions that are (a) odd and (b) even? Describe a way to identify a function as (c) odd, (d) even, or (e) neither odd nor even by inspecting the equation.

EXAMPLE 9 Even and Odd Functions

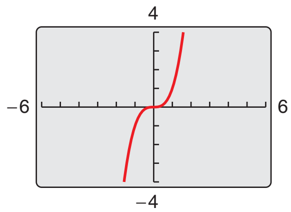
See LarsonPrecalculus.com for an interactive version of this type of example.

For each graph, determine whether the function is even, odd, or neither.

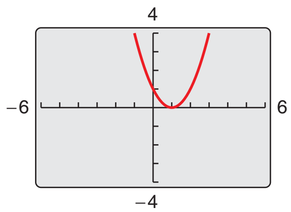
a.



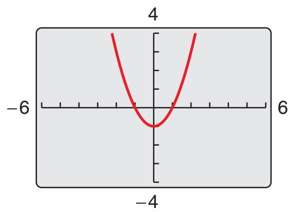
b.



c.



d.



Solution

a. The graph is symmetric with respect to the y-axis. So, the function is even.

b. The graph is symmetric with respect to the origin. So, the function is odd.

c. The graph is neither symmetric with respect to the origin nor with respect to the y-axis. So, the function is neither even nor odd.

d. The graph is symmetric with respect to the y-axis. So, the function is even.

Checkpoint *Audio-video solution in English & Spanish at LarsonPrecalculus.com.*

Use a graphing utility to graph $f(x) = x^2 - 4$ and determine whether it is even, odd, or neither.

Test for Even and Odd Functions

A function f is **even** when, for each x in the domain of f, $f(-x) = f(x)$.
A function f is **odd** when, for each x in the domain of f, $f(-x) = -f(x)$.

EXAMPLE 10 Even and Odd Functions

Determine whether each function is even, odd, or neither.

a. $g(x) = x^3 - x$

b. $h(x) = x^2 + 1$

c. $f(x) = x^3 - 1$

Algebraic Solution

a. This function is odd because

$$\begin{aligned} g(-x) &= (-x)^3 - (-x) \\ &= -x^3 + x \\ &= -(x^3 - x) \\ &= -g(x). \end{aligned}$$

b. This function is even because

$$\begin{aligned} h(-x) &= (-x)^2 + 1 \\ &= x^2 + 1 \\ &= h(x). \end{aligned}$$

c. Substituting $-x$ for x produces

$$\begin{aligned} f(-x) &= (-x)^3 - 1 \\ &= -x^3 - 1. \end{aligned}$$

Because

$$f(x) = x^3 - 1$$

and

$$-f(x) = -x^3 + 1$$

you can conclude that

$$f(-x) \neq f(x)$$

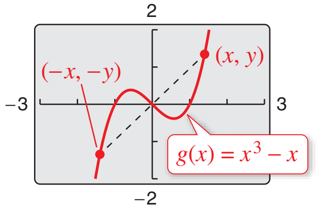
and

$$f(-x) \neq -f(x).$$

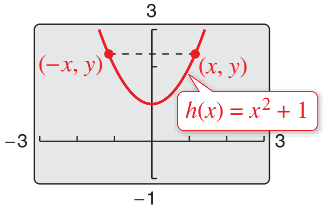
So, the function is neither even nor odd.

Graphical Solution

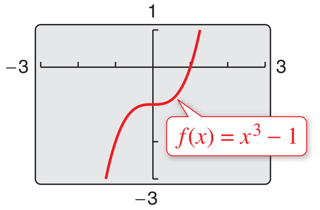
a. The graph is symmetric with respect to the origin. So, this function is odd.



b. The graph is symmetric with respect to the y-axis. So, this function is even.



c. The graph is neither symmetric with respect to the origin nor with respect to the y-axis. So, this function is neither even nor odd.



Checkpoint Audio-video solution in English & Spanish at LarsonPrecalculus.com.

Determine whether the function is even, odd, or neither. Then describe the symmetry.

a. $f(x) = 5 - 3x$ **b.** $g(x) = x^4 - x^2 - 1$ **c.** $h(x) = 2x^3 + 3x$

To help visualize symmetry with respect to the origin, place a pin at the origin of a graph and rotate the graph 180°. If the result after rotation coincides with the original graph, then the graph is symmetric with respect to the origin.

1.3 Exercises

See *CalcChat.com* for tutorial help and worked-out solutions to odd-numbered exercises. For instructions on how to use a graphing utility, see Appendix A.

Vocabulary and Concept Check

In Exercises 1 and 2, fill in the blank.

1. A function f is ________ on an interval when, for any x_1 and x_2 in the interval, $x_1 < x_2$ implies $f(x_1) > f(x_2)$.

2. A function f is ________ when, for each x in the domain of f, $f(-x) = f(x)$.

3. The graph of a function f is the segment from $(1, 2)$ to $(4, 5)$, including the endpoints. What is the domain of f?

4. A vertical line intersects a graph twice. Does the graph represent a function?

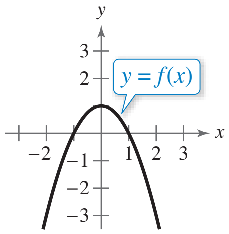
5. Let f be a function such that $f(2) \geq f(x)$ for all values of x in the interval $(0, 3)$. Does $f(2)$ represent a relative minimum or a relative maximum?

6. Given $f(x) = [\![x]\!]$, in what interval does $f(x) = 5$?

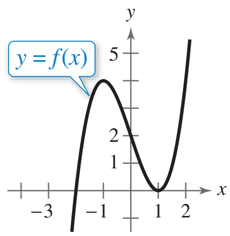
Procedures and Problem Solving

Finding the Domain and Range of a Function **In Exercises 7–10, use the graph of the function to find the domain and range of f. Then find $f(0)$.**

7.



8.



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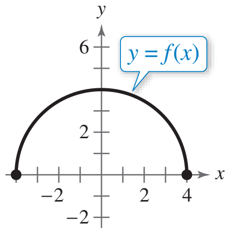

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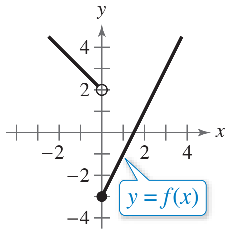


Finding the Domain and Range of a Function **In Exercises 11–16, use a graphing utility to graph the function and estimate its domain and range. Then find the domain and range algebraically.**

11. $f(x) = -2x^2 + 3$

12. $f(x) = x^2 - 1$

13. $f(x) = \sqrt{x + 2}$

14. $h(t) = \sqrt{4 - t^2}$

15. $f(x) = |x + 3|$

16. $f(x) = -\frac{1}{4}|x - 5|$

Analyzing a Graph **In Exercises 17 and 18, use the graph of the function to answer the questions.**

(a) **Determine the domain of the function.**

(b) **Determine the range of the function.**

(c) **Find the value(s) of x for which $f(x) = 0$.**

(d) **What are the values of x from part (c) referred to graphically?**

(e) **Find $f(0)$, if possible.**

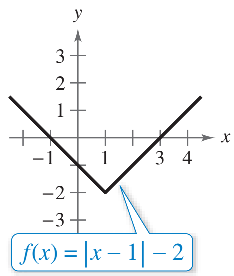
(f) **What is the value from part (e) referred to graphically?**

(g) **What is the value of f at $x = 1$? What are the coordinates of the point?**

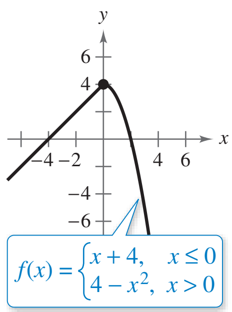
(h) **What is the value of f at $x = -1$? What are the coordinates of the point?**

(i) **The coordinates of the point on the graph of f at which $x = -3$ can be labeled $(-3, f(-3))$, or $(-3, \quad)$.**

17.

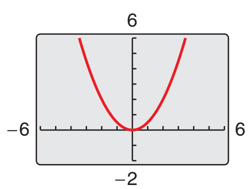


18.

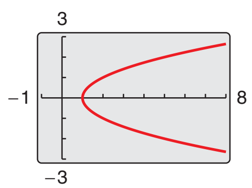


Vertical Line Test for Functions **In Exercises 19–22, use the Vertical Line Test to determine whether y is a function of x. Describe how you can use a graphing utility to produce the given graph.**

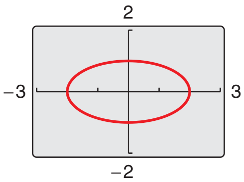
19. $y = \frac{1}{2}x^2$



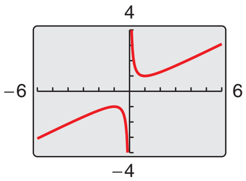
20. $x - y^2 = 1$



21. $0.25x^2 + y^2 = 1$

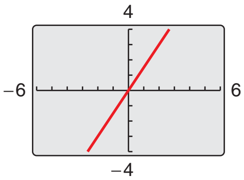


22. $x^2 = 2xy - 1$

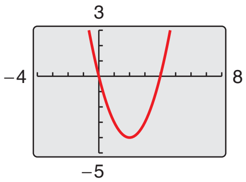


Increasing and Decreasing Functions **In Exercises 23–26, determine the open intervals on which the function is increasing, decreasing, or constant.**

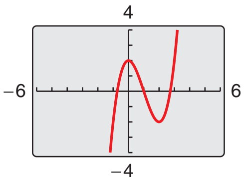
23. $f(x) = \frac{3}{2}x$



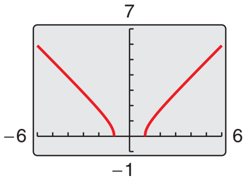
24. $f(x) = x^2 - 4x$



25. $f(x) = x^3 - 3x^2 + 2$



26. $f(x) = \sqrt{x^2 - 1}$



Increasing and Decreasing Functions **In Exercises 27–34, (a) use a graphing utility to graph the function and (b) determine the open intervals on which the function is increasing, decreasing, or constant.**

27. $f(x) = 3$ **28.** $f(x) = x$

29. $f(x) = x^{2/3}$ **30.** $f(x) = -x^{3/4}$

31. $f(x) = x\sqrt{x + 3}$ **32.** $f(x) = x\sqrt{3 - x}$

33. $f(x) = |x + 1| + |x - 1|$

34. $f(x) = -|x + 4| - |x + 1|$

Approximating Relative Minima and Maxima **In Exercises 35–46, use a graphing utility to graph the function and to approximate any relative minimum or relative maximum values of the function.**

35. $f(x) = x^2 - 6x$ **36.** $f(x) = 3x^2 - 2x - 5$

37. $y = -2x^3 - x^2 + 14x$ **38.** $y = x^3 - 6x^2 + 15$

39. $h(x) = (x - 1)\sqrt{x}$ **40.** $g(x) = x\sqrt{4 - x}$

41. $f(x) = x^2 - 4x - 5$ **42.** $f(x) = 3x^2 - 12x$

43. $f(x) = x^3 - 3x$ **44.** $f(x) = -x^3 + 3x^2$

45. $f(x) = 3x^2 - 6x + 1$ **46.** $f(x) = 8x - 4x^2$

Library of Parent Functions **In Exercises 47–52, sketch the graph of the function by hand. Then use a graphing utility to verify the graph.**

47. $f(x) = [\![x]\!] + 2$ **48.** $f(x) = [\![x]\!] - 3$

49. $f(x) = [\![x - 1]\!] - 2$ **50.** $f(x) = [\![x + 2]\!] + 1$

51. $f(x) = 2[\![x]\!]$ **52.** $f(x) = [\![4x]\!]$

Describing a Step Function **In Exercises 53 and 54, use a graphing utility to graph the function. State the domain and range of the function. Describe the pattern of the graph.**

53. $s(x) = 2\left(\frac{1}{4}x - [\![\frac{1}{4}x]\!]\right)$

54. $g(x) = 2\left(\frac{1}{4}x - [\![\frac{1}{4}x]\!]\right)^2$

Sketching a Piecewise-Defined Function **In Exercises 55–62, sketch the graph of the piecewise-defined function by hand.**

55. $f(x) = \begin{cases} 2x + 3, & x < 0 \\ 3 - x, & x \ge 0 \end{cases}$

56. $f(x) = \begin{cases} x + 6, & x \le -4 \\ 3x - 4, & x > -4 \end{cases}$

57. $f(x) = \begin{cases} \sqrt{4 + x}, & x < 0 \\ \sqrt{4 - x}, & x \ge 0 \end{cases}$

58. $f(x) = \begin{cases} 1 - (x - 1)^2, & x \le 2 \\ \sqrt{x - 2}, & x > 2 \end{cases}$

59. $f(x) = \begin{cases} x + 3, & x \le 0 \\ 3, & 0 < x \le 2 \\ 2x - 1, & x > 2 \end{cases}$

60. $g(x) = \begin{cases} x + 5, & x \le -3 \\ 5, & -3 < x < 1 \\ 5x - 4, & x \ge 1 \end{cases}$

61. $f(x) = \begin{cases} 2x + 1, & x \le -1 \\ x^2 - 2, & x > -1 \end{cases}$

62. $h(x) = \begin{cases} 3 + x, & x < 0 \\ x^2 + 1, & x \ge 0 \end{cases}$

Even and Odd Functions **In Exercises 63–72, use a graphing utility to graph the function and determine whether it is even, odd, or neither.**

63. $f(x) = 5$ **64.** $f(x) = -9$

65. $f(x) = 3x - 2$ **66.** $f(x) = 4 - 5x$

67. $h(x) = x^2 + 6$ **68.** $f(x) = -x^2 - 8$

69. $f(x) = \sqrt{1 - x}$ **70.** $g(t) = \sqrt[3]{t - 1}$

71. $f(x) = |x + 2|$ **72.** $f(x) = -|x - 5|$

Think About It **In Exercises 73–78, find the coordinates of a second point on the graph of a function f if the given point is on the graph and the function is (a) even and (b) odd.**

73. $\left(\frac{3}{2}, 4\right)$ **74.** $\left(-\frac{5}{3}, -7\right)$

75. $(-2, -9)$ **76.** $(5, -1)$

77. $(x, -y)$ **78.** $(2a, 2c)$

Algebraic-Graphical-Numerical **In Exercises 79–86, determine whether the function is even, odd, or neither (a) algebraically, (b) graphically by using a graphing utility to graph the function, and (c) numerically by using the *table* feature of the graphing utility to compare $f(x)$ and $f(-x)$ for several values of x.**

79. $f(t) = t^2 + 2t - 3$

80. $f(x) = x^6 - 2x^2 + 3$

81. $g(x) = x^3 - 5x$

82. $h(x) = x^5 - 4x^3$

83. $f(x) = x\sqrt{1 - x^2}$

84. $f(x) = x\sqrt{x + 5}$

85. $g(s) = 4s^{2/3}$

86. $f(s) = 4s^{3/5}$

Finding the Intervals Where a Function is Positive **In Exercises 87–90, graph the function and determine the interval(s) (if any) on the real axis for which $f(x) \geq 0$. Use a graphing utility to verify your results.**

87. $f(x) = 4 - x$

88. $f(x) = 4x + 8$

89. $f(x) = x^2 - 9$

90. $f(x) = x^2 - 4x$

91. Business The cost of parking in a metered lot is \$1.00 for the first hour and \$0.50 for each additional hour or portion of an hour.

(a) A customer needs a model for the cost C of parking in the metered lot for t hours. Which of the following is the appropriate model?

$C_1(t) = 1 + 0.50[\![t - 1]\!]$

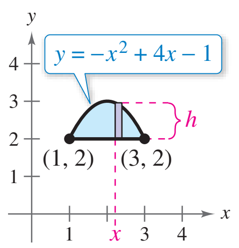
$C_2(t) = 1 - 0.50[\![-(t - 1)]\!]$

(b) Use a graphing utility to graph the appropriate model. Estimate the cost of parking in the metered lot for 7 hours and 10 minutes.

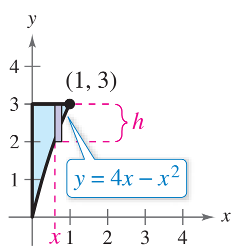
92. ***Why you should learn it*** *(p. 29)* The cost of sending an overnight package from New York to Atlanta is \$23.20 for a package weighing up to but not including 1 pound and \$2.00 for each additional pound or portion of a pound. Use the greatest integer function to create a model for the cost C of overnight delivery of a package weighing x pounds, where $x > 0$. Sketch the graph of the function.

Using the Graph of a Function **In Exercises 93 and 94, write the height h of the rectangle as a function of x.**

93.



94.



95. MODELING DATA

The number N (in thousands) of existing condominiums and cooperative homes sold each year from 2010 through 2013 in the United States is approximated by the model

$N = -24.83t^3 + 906t^2 - 10{,}928.2t + 44{,}114,$

$10 \leq t \leq 13$

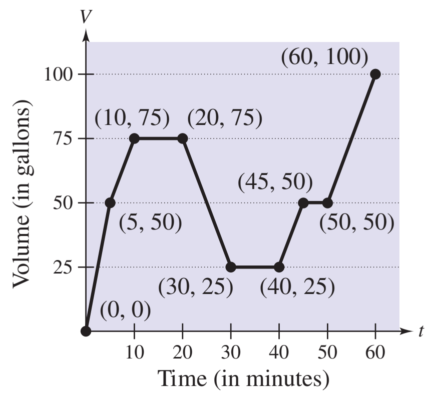
where t represents the year, with $t = 10$ corresponding to 2010. (Source: National Association of Realtors)

(a) Use a graphing utility to graph the model over the appropriate domain.

(b) Use the graph from part (a) to determine during which years the number of cooperative homes and condos was increasing. During which years was the number decreasing?

(c) Approximate the minimum number of cooperative homes and condos sold from 2010 through 2013.

96. Mechanical Engineering The intake pipe of a 100-gallon tank has a flow rate of 10 gallons per minute, and two drain pipes have a flow rate of 5 gallons per minute each. The graph shows the volume V of fluid in the tank as a function of time t. Determine in which pipes the fluid is flowing in specific subintervals of the one-hour interval of time shown on the graph. (There are many correct answers.)



Conclusions

True or False? **In Exercises 97 and 98, determine whether the statement is true or false. Justify your answer.**

97. A function with a square root cannot have a domain that is the set of all real numbers.

98. It is possible for an odd function to have the interval $[0, \infty)$ as its domain.

Think About It **In Exercises 99–104, match the graph of the function with the description that best fits the situation.**

(a) The air temperature at a beach on a sunny day

(b) The height of a football kicked in a field goal attempt

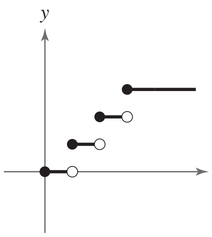
(c) The number of children in a family over time

(d) The population of California as a function of time

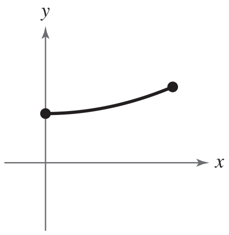
(e) The depth of the tide at a beach over a 24-hour period

(f) The number of cupcakes on a tray at a party

99.



100.



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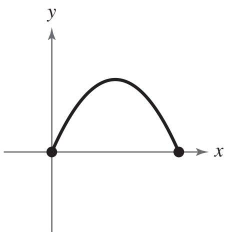

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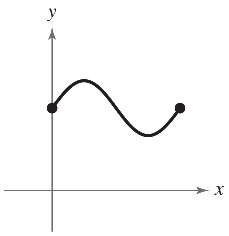

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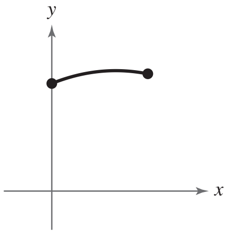

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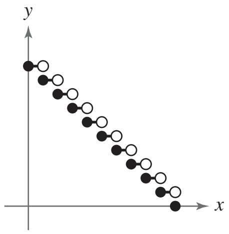


105. **Think About It** Does the graph in Exercise 19 represent x as a function of y? Explain.

106. **Think About It** Does the graph in Exercise 21 represent x as a function of y? Explain.

107. **Think About It** Can you represent the greatest integer function using a piecewise-defined function?

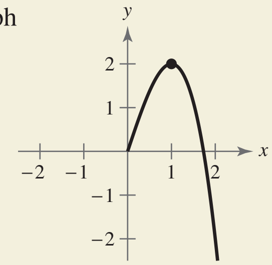
108. **Think About It** How does the graph of the greatest integer function differ from the graph of a line with a slope of zero?

109. **Think About It** Let f be an even function. Determine whether g is even, odd, or neither. Explain.

(a) $g(x) = -f(x)$ (b) $g(x) = f(-x)$

(c) $g(x) = f(x) - 2$ (d) $g(x) = -f(x + 3)$

110. **HOW DO YOU SEE IT?** Half of the graph of an odd function is shown.



(a) Sketch a complete graph of the function.

(b) Find the domain and range of the function.

(c) Identify the open intervals on which the function is increasing, decreasing, or constant.

(d) Find any relative minimum and relative maximum values of the function.

111. **Proof** Prove that a function of the following form is odd.

$$y = a_{2n+1}x^{2n+1} + a_{2n-1}x^{2n-1} + \cdots + a_3x^3 + a_1x$$

112. **Proof** Prove that a function of the following form is even.

$$y = a_{2n}x^{2n} + a_{2n-2}x^{2n-2} + \cdots + a_2x^2 + a_0$$

Cumulative Mixed Review

Identifying Terms and Coefficients **In Exercises 113–116, identify the terms. Then identify the coefficients of the variable terms of the expression.**

113. $-2x^2 + 11x + 3$ **114.** $10 + 3x$

115. $\dfrac{x}{3} - 5x^2 + x^3$ **116.** $7x^4 + \sqrt{2}x^2 - x$

Evaluating a Function **In Exercises 117 and 118, evaluate the function at each specified value of the independent variable and simplify.**

117. $f(x) = -x^2 - x + 3$

(a) $f(4)$ (b) $f(-5)$ (c) $f(x - 2)$

118. $f(x) = x\sqrt{x - 3}$

(a) $f(3)$ (b) $f(12)$ (c) $f(6)$

Evaluating a Difference Quotient **In Exercises 119 and 120, find the difference quotient and simplify your answer.**

119. $f(x) = x^2 - 2x + 9, \dfrac{f(3 + h) - f(3)}{h}, h \neq 0$

120. $f(x) = 5 + 6x - x^2, \dfrac{f(6 + h) - f(6)}{h}, h \neq 0$