AP CALCULUS BC SUMMER ASSIGNMENT – DO NOT SHOW YOUR WORK ON THIS!

- **Complete these problems during the last two weeks of August. SHOW ALL WORK. Know how to do ALL of these problems, so do them well.**
- **Items marked with a * denote that a graphing calculator may be used.**
- **You will be tested on Calculus A material (derivatives; Ch. 1-4) within the first few weeks of school. You must do well to continue in the course.**

1. If $f(x)=\dfrac{x^2-9}{x+3}$ is continuous at $x=-3$, then $f(-3)=$

a. 3 b. –3 c. 0
d. 6 e. –6

2. If $f(x)=e^{\sin x}$, how many zeros does $f'(x)$ have on the closed interval $[0, 2\pi]$?

a. 1 b. 2 c. 3
d. 4 e. 5

3. $\displaystyle\lim_{x\to\infty}\frac{10^8x^5+10^6x^4+10^4x^2}{10^9x^6+10^7x^5+10^5x^3}=$

a. 0 b. 1 c. –1
d. $\frac{1}{10}$ e. $-\frac{1}{10}$

4. The graph of which function has $y=-1$ as an asymptote?

a. $y=e^{-x}$ b. $y=\dfrac{-x}{1-x}$ c. $y=\ln(x+1)$
d. $y=\dfrac{x}{x+1}$ e. $y=\dfrac{x}{1-x}$

5. If $f(x)=\sqrt{4\sin x+2}$, then $f'(0)=$

a. –2 b. 0 c. $\sqrt{2}$
d. $\dfrac{\sqrt{2}}{2}$ e. 1

6. The equation of the tangent line to the curve $x^2+y^2=169$ at the point (5,–12) is

a. $5y-12x=-120$
b. $5x-12y=119$
c. $5x-12y=169$
d. $12x+5y=0$
e. $12x+5y=169$

7. If $f(x)=x-1$ and $g(x)=x^2+1$ then $f(g(x))=g(f(x))$ when $x=$

a. –½ b. ½ c. –1
d. 1 e. 0

8. If the graph of $f(x)=2x^2+\dfrac{k}{x}$ has a point of inflection at $x=-1$, then the value of k is

a. 1 b. –1 c. 2
d. –2 e. 0

9. The graph of $y=\sqrt[3]{x^2+1}$ is symmetric with respect to which of the following?

I The x – axis
II The y – axis
III The origin

a. I only b. II only
c. III only d. II and III only
e. I, II, and III

10. $\dfrac{d}{dx}\left(e^{3\ln x}\right)=$

a. $e^{3\ln x}$ b. $\dfrac{e^{3\ln x}}{x}$ c. x^3
d. $3x^2$ e. 3

11. For what values of x is the graph of $y=\dfrac{2}{4-x}$ concave downward?

a. No values of x b. $x<4$
c. $x>-4$ d. $x<-4$
e. $x>4$

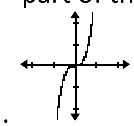
12. A particle moves along the x – axis in such a way that its position at time t is given by $x(t)=\dfrac{1-t}{1+t}$. What is the acceleration of the particle at time $t=0$?

a. $-\frac{3}{5}$ b. –4 c. 4
d. 2 e. –2

13. If $y=x^{(x^3)}$ for $x>0$, then $\dfrac{dy}{dx}=$

a. $x^3\cdot x^{(x^3-1)}$ b. $4x^3$
c. $x^2+3x^2\ln x$ d. $x^{(x^3+2)}(1+3\ln x)$
e. $3x^{(x^3+2)}\ln x$

14. If, for all values of x, $f'(x)<0$ and $f''(x)>0$ which one of the following curves could be a part of the graph of f?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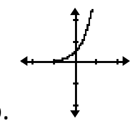
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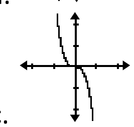
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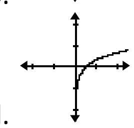
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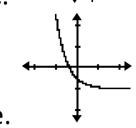

e.

15. The maximum value of $f(x)=2x^3-9x^2+12x-1$ on [–1, 2] is

a. 0 b. 1 c. 2
d. 3 e. 4

16. *The $\displaystyle\lim_{x\to-3}\frac{x^2+3x}{\sqrt{x^2+6x+9}}$ is

a. –3 b. –1 c. 1
d. 3 e. nonexistent

17. *Let f and g be differentiable functions such that $f(1)=4,\ g(1)=3,\ f'(3)=-5$
$f'(1)=-4,\ g'(1)=-3,\ g'(3)=2.$
If $h(x)=f(g(x))$, then $h'(1)=$

a. –9 b. 15 c. 0
d. –5 e. –12

18. *The shortest distance from the curve $xy = 4$ to the origin is

a. 2 b. 4 c. $\sqrt{2}$
d. $2\sqrt{2}$ e. $\frac{1}{2}\sqrt{2}$

19. *If $f(x)=3x^2-8x^{-2}$, then $\lim_{h\to 0}\dfrac{f(2+h)-f(2)}{h}$ =

a. 10 b. 14 c. 20
d. –14 e. –20

20. *How many real solutions does the equation $\sin(6x)=2e^x$ have?

a. None b. One
c. Six d. Eight
e. Infinitely many

21. *If $f(x)=|(x^2-12)(x^2+4)|$, how many numbers in the interval $-2\le x\le 3$ satisfy the conclusion of the Mean Value Theorem?

a. None b. One c. Two
d. Three e. Four

22. *If $f'(x)=e^x+\sin x$ then $f(x)$ may be

a. $\dfrac{e^{x+1}}{x+1}+\cos x$ b. $e^x+\cos x$
c. $e^x-\cos x-1$ d. $xe^{-x}+\cos x$
e. $e^{2x}-\cos x$

23. If $f(x)=(2+3x)^4$, then the 4th derivative of f is

a. 0 b. 4! (3) c. $4!(3^4)$
d. $4!(3^5)$ e. 4!(2+3x)

24. At what value(s) of x does $f(x)=x^4-8x^2$ have a relative minimum?

a. 0 and –2 only b. 0 and 2 only
c. 0 only d. –2 and 2 only
e. –2, 0, and 2

25. The $\lim_{h\to 0}\dfrac{|x+h|-|x|}{h}$ at $x = 3$ is

a. 0 b. 1 c. 3
d. –1 e. nonexistent

26. Let f be the function given by $f(x)=x^3$. What are all values of c that satisfy the conclusion of the Mean Value Theorem on the closed interval [–1,2]?

a. 0 only b. 1 only c. $\sqrt{3}$ only
d. –1 and 1 e. $-\sqrt{3}$ and $\sqrt{3}$

27. If $x+y=xy$, then $\dfrac{dy}{dx}$ is

a. $\dfrac{1}{x-1}$ b. $\dfrac{y-1}{x-1}$ c. $\dfrac{1-y}{x-1}$
d. $x+y-1$ e. $\dfrac{2-xy}{y}$

28. If $g(x) = \ln(\ln 2x)$, then $g'(x)$ =

a. $\dfrac{x}{\ln 2x}$ b. $\dfrac{1}{x(\ln 2x)}$ c. $\dfrac{1}{\ln 2x}$
d. $\dfrac{2}{\ln 2x}$ e. $\dfrac{1}{\ln(\ln 2x)}$

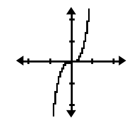
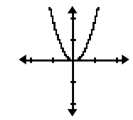
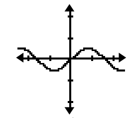
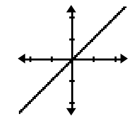
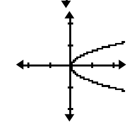
29. If $f(x)=\dfrac{x}{x+1}$ for $x\ne -1$ then the range of f is

a. All real numbers b. $y\ne -1$ c. $y\ne 1$
d. $y\le 1$ e. $y<1$

30. In which interval is the function $f(x)=x^3+6x^2+9x+1$ increasing?

a. $(-\infty,-3)$ only b. $(-3,-1)$ only
c. $(-1,\infty)$ only d. $(-\infty,-3)\cup(-1,\infty)$
e. $(-\infty,-3)\cup(1,\infty)$

31. Which of the following graphs represents an even function?

a. 
b. 
c. 
d. 
e. 

32. For $|x|<1$, the derivative of $y=\ln\sqrt{1-x^2}$ is

a. $\dfrac{x}{1-x^2}$ b. $\dfrac{x}{x^2-1}$ c. $\dfrac{-x}{x^2-1}$
d. $\dfrac{1}{2(1-x^2)}$ e. $\dfrac{1}{\sqrt{1-x^2}}$

33. If $f(x)=2e^{2x}$, then $f'(\ln 3)=$

a. 9 b. 18 c. 24
d. 32 e. 36

34. Consider the function $f(x)=\begin{cases}\dfrac{\sin x}{x}, & x\ne 0\\ k, & x=0\end{cases}$. In order for $f(x)$ to be continuous at $x = 0$ the value of k must be

a. 0 b. 1
c. –1 d. π
e. A number greater than 1

35. What are all values of x for which the graph of $y=x^3-6x^2$ is concave downward?

a. $0<x<4$ b. $x>2$ c. $x<2$
d. $x<0$ e. $x>4$

36. If $\dfrac{dy}{dx}=e^{3x}$, then y could be

a. $3e^{3x}$ b. e^{x^3} c. $\frac{1}{3}e^{x^3}$
d. $3e^{x^3}$ e. $\frac{1}{3}e^{3x}$

37. If the fundamental period of the function $f(x)=3\cos\left(\dfrac{kx}{2}\right)$ is $\dfrac{2\pi}{3}$, then k may be

a. 2 b. 3 c. 4
d. 6 e. 8

38. If $y=xe^x$, then $\dfrac{d^ny}{dx^n}=$

a. e^x b. e^{nx}
c. $(x+n)e^x$ d. x^ne^x
e. $(x+n^2)e^x$

39. A particle moves on the x – axis in such a way that its position at time t is given by $x(t)=3t^5-25t^3+60t$. For what values of t is the particle moving to the left?

a. $-2<t<1$ only
b. $-2<t<-1$ and $1<t<2$
c. $-1<t<-1$ and $t>2$
d. $1<t<2$ only
e. $t<-2, -1<t<1$, and $t>2$

40. The equation of the normal line to the curve $y=\sqrt[3]{x^2-1}$ at the point where $x = 3$ is

a. $y + 12x = 38$ b. $y - 4x = 10$
c. $y + 2x = 4$ d. $y + 2x = 8$
e. $y - 2x = -4$

41. *f is a function such that $\lim\limits_{x\to a}\dfrac{f(x)-f(a)}{x-a}=0$. Which of the following must be true?

a. $\lim\limits_{x\to a}f(x)$ does not exist
b. $f(a)$ does not exist
c. $f'(a) = 0$
d. $f(a) = 0$
e. $f(x)$ is continuous at $x = 0$

42. *If $f(x)=\sqrt{(x^2+2)^3}$, then $f'(x)=$

a. $\dfrac{3\sqrt{x^2+2}}{2}$ b. $3x\sqrt{x^2+2}$
c. $\sqrt{6x(x^2+2)^2}$ d. $\dfrac{3x}{\sqrt{x^2+2}}$
e. $\dfrac{4x}{3\sqrt[3]{x^2+2}}$

43. *Of the choices given, which value is NOT in the domain of the function $f(x)=(\cos x)^x$?

a. 1 b. $\dfrac{\pi}{2}$ c. $\dfrac{4\pi}{3}$
d. 4 e. 2π

44. *If f is a function which is everywhere increasing and concave upwards, which statement is true about f^{-1}, the inverse of f?

a. f^{-1} is not a function.
b. f^{-1} is increasing and concave upwards
c. f^{-1} is increasing and concave downwards
d. f^{-1} is decreasing and concave upwards
e. f^{-1} is decreasing and concave downwards

45. *A function whose derivative is a constant multiple of itself must be

a. Periodic b. Linear
c. Exponential d. Quadratic
e. Logarithmic

46. *For how many real numbers x is it true that $\sin x=\dfrac{x}{10}$?

a. Three b. Five c. Six
d. Seven e. Infinitely many

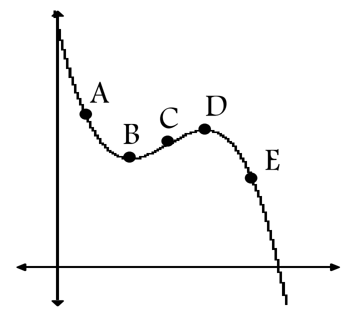
47. *What is the 50th derivative of $\cos x$?

a. $-\cos x$ b. $\cos x$ c. $\sin x$
d. $-\sin x$ e. 0

48. *Suppose that f is a continuous function defined for all real numbers x and $f(-5) = 3$ and $f(-1) = -2$. If $f(x) = 0$ for one and only one value of x then which of the following could be x?

a. −7 b. −2 c. 0
d. 1 e. 2

49. The graph shows the distance $s(t)$ from a reference point of a particle moving on a number line, as a function of time. Which of the points marked is the closest to the point where the acceleration first becomes negative?



a. A b. B c. C d. D e. E

50. *The function $f(x)=\tan(3^x)$ has one zero in the interval [0,1.4]. The derivative at this point is

a. 0.411 b. 1.042
c. 3.451 d. 3.763
e. undefined

51. The $\lim\limits_{x\to\frac{\pi}{2}}\dfrac{\sin\left(x-\frac{\pi}{2}\right)}{x-\frac{\pi}{2}}$ is

a. $\dfrac{\pi}{2}$ b. 0 c. 1
d. −1 e. nonexistent

52. If $y=2xe^{-x}$, then y has a point of inflection at x =

a. 0 b. 1 c. 2 d. −2 e. 4

53. If the radius of a sphere is increasing at the rate of 2 inches per second, how fast, in cubic inches per second, is the volume increasing when the radius is 10 inches?

a. 800π b. 800 c. 3200π
d. 40π e. 80π

54. What are all values of x for which $\ln(x^2-1)>0$?

a. $|x|>\sqrt{2}$ b. $|x|>1$ c. $|x|\geq\sqrt{2}$
d. $|x|\geq 1$ e. $|x|<\sqrt{2}$

55. The $\lim_{x\to 3}\dfrac{(3-x)^2}{(x-3)}$ is

a. 0 b. –2 c. 1
d. –1 e. nonexistent

56. If $f(x) = \ln x$ and $g(x)=e^x$, then $f(g(4))$ =

a. ln 4 b. e^4 c. 4
d. e^{-4} e. –4

57. If $f(x)=\arctan\left(\dfrac{1}{x}\right)$, then $f'(x)$ =

a. $\dfrac{-1}{x^2+x}$ b. $\dfrac{x}{\sqrt{x^2-1}}$ c. $\dfrac{x^2}{x^2+1}$
d. $\dfrac{1}{x^2+1}$ e. $\dfrac{-1}{x^2+1}$

58. If $f(x) = \cos(\arcsin x)$, what is the range of f?

a. $\{x \mid -1\leq x\leq 0\}$
b. $\{x \mid -1\leq x\leq 1\}$
c. $\{x \mid 0\leq x\leq \frac{\pi}{2}\}$
d. $\{x \mid -\frac{\pi}{2}\leq x\leq \frac{\pi}{2}\}$
e. $\{x \mid 0\leq x\leq 1\}$

59. Which of the following is true about the graph of $y=e^x-x$?

a. It is asymptotic to the x-axis.
b. It has a relative maximum at $x = 0$.
c. It is always concave upward.
d. It is decreasing for all x greater than 0.
e. It has a point of inflection at $x = 0$.

60. If $f(x)=\sqrt{e^{2x}+1}$ then $f'(0)=$

a. $\dfrac{\sqrt{2}}{4}$ b. $\sqrt{2}$ c. $\dfrac{\sqrt{2}}{2}$
d. 1 e. $-\dfrac{\sqrt{2}}{2}$

61. The coordinates of the point on the curve $y=x^2+1$ which is closest to (3,1) is

a. (1,2) b. (2,5) c. (3,10)
d. $\left(\frac{3}{2},\frac{13}{4}\right)$ e. $\left(\frac{1}{2},\frac{5}{4}\right)$

62. $\lim_{x\to+\infty}\left(1+\dfrac{1}{x}\right)^{3x}$ =

a. $3e$ b. 1 c. –1
d. $3e^3$ e. e^3

63. A particle moves along the x – axis so that at any time t its position is given by $x(t)=(t+1)(t-3)^3$. For what values of t is the <u>velocity</u> of the particle increasing?

a. $t>3$ only b. $0<t<3$ only
c. $1<t<3$ only d. $t<1$ or $t>3$
e. $0<t<3$ or $t>3$

64. * The equations of the tangent line to the curve $y=x^3-6x^2$ at its point of inflection is

a. $y = -12x + 8$ b. $y = -12x + 40$
c. $y = 12x - 8$ d. $y = -12x + 12$
e. $y = 12x - 40$

65. *If $\lim_{x\to 3}\dfrac{g(3)-g(x)}{3-x}=-0.628$, then at the point x = 3, the graph of $g(x)$

a. is decreasing
b. is increasing
c. is concave upwards
d. is concave downwards
e. attains a relative minimum point

66. *For $0\leq x\leq\frac{\pi}{2}$, an antiderivative of $2\tan x$ is

a. $\ln(\sec 2x)$ b. $2\sec^2 x$
c. $\ln(\sec^2 x)$ d. $2\ln(\cos x)$
e. $\ln(2\sec x)$

67. * If the derivative of a function f is given by $f'(x)=\sin(x^x)$, then how many critical points does the function $f(x)$ have on the interval [0.2, 2.6]?

a. 0 b. 1 c. 2 d. 3 e. 4

68. *The derivative of $(4x)^3\cdot(2x)^6$ is

a. $72x^8$ b. $124x^{17}$
c. $30x(4x)^2(2x)^5$ d. $72x(4x)^2(2x)^5$
e. $144(4x)^2(2x)^5$

69. *The second derivative of a function is given by $f''(x)=0.5+\cos x-e^{-x}$. How many points of inflection does the function $f(x)$ have on the interval $0\leq x\leq 20$?

a. None b. Three c. Six
d. Seven e. Ten

70. *The equation of the line tangent to the curve $y=\dfrac{kx+8}{k+x}$ at $x = -2$ is $y = x + 4$. What is the value of k?

a. –3 b. –1 c. 1
d. 3 e. 4

71. *How many zeros does the function $y = \sin(\ln x)$ have for $0 < x \le 1$?

a. One b. Two c. Three d. Four e. More than four

72. *For all $x > 0$, if $f(\ln x) = x^2$, then $f(x) =$

a. $\sqrt{e^x}$ b. $2\ln x$ c. $e^{\sqrt{x}}$ d. $\sqrt{\ln x}$ e. e^{2x}

73. What is the domain of the function $f(x) = \sqrt{\dfrac{x+2}{x-1}}$?

a. $\{x: x \neq 1\}$ b. $\{x: x \le -2\}$ c. $\{x: x \le -2 \text{ or } x > 1\}$ d. $\{x: x > 1\}$ e. $\{x: -2 \le x < 1\}$

74. The position of a particle on the x – axis at time t, $t > 0$, is $\ln t$. The average velocity of the particle for $1 \le t \le e$ is

a. 1 b. $\dfrac{1}{e} - 1$ c. $\dfrac{1}{e-1}$ d. e e. $e - 1$

75. If $f(n+1) = \dfrac{2f(n)+1}{2}$ and $f(1) = 2$, then $f(37) =$

a. 18 b. 19 c. 20 d. 21 e. 22

76. If $f(x) = \dfrac{x+1}{x}$, $x \neq 0$, and $f(g(x)) = x$, then $g(x) =$

a. $x(x-1)$ b. $1 - x$ c. $\dfrac{x^2}{x+1}$ d. $\dfrac{1}{x-1}$ e. $x(x+1)$

77. The slope of the line <u>normal</u> to the graph of $y = \ln\frac{2}{x}$ at $x = 2$ is

a. 2 b. –1 c. –2 d. –½ e. undefined

78. The minimum value of $f(x) = e^x - 2x$ is

a. $\ln 2$ b. $e^2 - 4$ c. $\sqrt{e} - 1$ d. $2(1 - \ln 2)$ e. 2

79. If $f(x) = 3 + |x - 2|$, then $f'(2)$ is

a. 3 b. 1 c. –1 d. 2 e. nonexistent

80. The volume of an expanding sphere is increasing at a rate of 12 cubic feet per second. When the volume of the sphere is 36π cubic feet, how fast, in square feet per second, is the surface area increasing?

a. 8 b. 6 c. 8π d. $\frac{8\pi}{3}$ e. 10

81. If $y = 5^{(x^3-2)}$, then $\dfrac{dy}{dx} =$

a. $(x^3-2)5^{(x^3-2)}$

b. $3x^2(\ln 5)5^{(x^3-2)}$

c. $(3x^2)5^{(x^3-2)}$

d. $(\ln 5)5^{(x^3-2)}$

e. $x^3(\ln 5)5^{(x^3-2)}$

82. If $y = \dfrac{1-x}{x-1}$, then $\dfrac{dy}{dx} =$

a. –1 b. 0 c. $\dfrac{-1}{x-1}$ d. $\dfrac{-2}{x-1}$ e. $\dfrac{-2x}{(x-1)^2}$

83. The fundamental period for the graph of $y = 1 - 2\sin^2(2x)$ is

a. 4 b. $\frac{\pi}{4}$ c. $\frac{\pi}{2}$ d. π e. 2π

84. $\dfrac{d}{dx}\left(\dfrac{\sin x}{1+\cos x}\right) =$

a. 1 b. $\dfrac{1}{1+\cos x}$ c. $\dfrac{-1}{1+\cos x}$ d. $-\cot x$ e. $\dfrac{2\cos^2 x}{(1+\cos x)^2}$

85. The $\lim\limits_{x\to 0}\dfrac{\cos(\frac{\pi}{2}+x) - \cos(\frac{\pi}{2}-x)}{x}$ is

a. 1 b. –2 c. –1 d. 0 e. 2

86. If $y = \arcsin\left(\dfrac{3x}{4}\right)$, then $\dfrac{dy}{dx} =$

a. $\dfrac{-3}{\sqrt{16-9x^2}}$ b. $\dfrac{12}{16+9x^2}$ c. $\dfrac{4}{\sqrt{16-9x^2}}$ d. $\dfrac{12}{\sqrt{16-9x^2}}$ e. $\dfrac{3}{\sqrt{16-9x^2}}$

87. If the graph of a function f is symmetric about the y-axis, and contains the point (–2,1), which point is also on f ?

a. (–2 ,–1) b. (1,–2) c. (0,0) d. (1,2) e. (2,1)

88. *If $e^{xy} = 2$, then at the point (1,ln2), $\dfrac{dy}{dx} =$

a. –ln2 b. 2ln2 c. ln2 d. $-2e$ e. –4ln2

89. *At the point of intersection of $f(x) = \cos x$ and $g(x) = 1 - x^2$, the tangent lines are

a. the same line
b. parallel lines
c. perpendicular lines
d. intersecting but not perpendicular lines
e. none of the above

90. *The $\lim_{h\to 0}\frac{\tan 2(x+h)-\tan(2x)}{h}$ is

a. 0 b. $2\cot(2x)$ c. $\sec^2(2x)$
d. $2\sec^2(2x)$ e. nonexistent

91. *A particle moves along the x-axis so that its position at any time $t > 0$ is given by $x(t)=t^4-10t^3+29t^2-36t+2$. For which value of t is the speed the greatest?

a. $t = 1$ b. $t = 2$ c. $t = 3$
d. $t = 4$ e. $t = 5$

92. *A particle moves along the x-axis so that at any time t its position is given by $x(t)=\frac{1}{2}\sin t+\cos(2t)$. What is the acceleration of the particle at $t=\frac{\pi}{2}$?

a. 0 b. $\frac{1}{2}$ c. $\frac{3}{2}$ d. $\frac{5}{2}$ e. $\frac{7}{2}$

93. *The derivative of a function if given by $f'(x)=(\sin x)(\cos^2(3x))$. Which of the following is true about the function $f(x)$ for $-\pi\le x\le\pi$?

a. $f(x)$ is an odd function
b. $f(x)$ is increasing for all values in the interval
c. $f(x)$ has exactly one relative minimum in the interval
d. $f(x)$ has no points of inflection in the interval
e. $f(-\pi)$ is the absolute minimum value

94. *How many points of inflection does the function $f(x)=\left(\frac{\pi}{3}\right)^{x^3-8}$ have?

a. None b. One c. Two
d. Three e. Infinitely many

95. *The function $y=x^4+bx^2+8x+1$ has a horizontal tangent and a point of inflection for the same value of x. What must be the value of b?

a. –1 b. 4 c. 1 d. 6 e. –6

96. If $y=(2x^2+1)^4$, then $\frac{dy}{dx}=$

a. $16x^3$ b. $4(2x^2+1)^3$
c. $4x(2x^2+1)^3$ d. $16(2x^2+1)^3$
e. $16x(2x^2+1)^3$

97. If the graph of a function f has a horizontal tangent at the point (1,2), what is the equation of the normal line at this point?

a. $y = 2$ b. $y = 1$ c. $y = -1$
d. $x = 1$ e. $x = 0$

98. If $f(x)=x^3-x+3$ and if c is the only real number such that $f(c) = 0$, then c is between

a. –2 and –1 b. –1 and 0
c. 0 and 1 d. 1 and 2
e. 2 and 3

99. The graph of $y=2x^3+24x-18$ is

a. Increasing for all x
b. Decreasing for all x
c. Only decreasing for all x such that $|x| > 2$
d. Only increasing for all x such that $|x| < 2$
e. Only decreasing for all x such that $|x| < -2$

100. For how many real numbers x does $e^x=\ln|x|$?

a. 0 b. 1 c. 2
d. 3 e. Infinitely many

101. If $f(x)=x\sqrt[3]{x}$, then $f'(x)=$

a. $4x^3$ b. $\frac{3}{7}x^{\frac{7}{3}}$ c. $\frac{4}{3}x^{\frac{1}{3}}$
d. $\frac{1}{3}x^{\frac{1}{3}}$ e. $\frac{1}{3}x^{-\frac{2}{3}}$

102. $\sin(xy)=x^2$, then $\frac{dy}{dx}=$

a. $2x\sec(xy)$ b. $\frac{\sec(xy)}{x^2}$
c. $2x\sec(xy)-y$ d. $\frac{2x\sec(xy)}{y}$
e. $\frac{2x\sec(xy)-y}{x}$

103. How many points of inflection does the graph of $y=2x^6+9x^5+10x^4-x+2$ have?

a. None b. One c. Two
d. Three e. Four

104. $\lim_{x\to 2}\frac{2^{\frac{x}{2}}-2}{2^x-4}$ is

a. 0 b. ¼ c. ½
d. ln2 e. nonexistent

105. The graph of $y=\frac{x}{1-|x|}$ has

a. No horizontal asymptotes and one vertical asymptote
b. One horizontal asymptote and one vertical asymptote
c. Two horizontal asymptotes and one vertical asymptote
d. One horizontal asymptote and two vertical asymptotes
e. Two horizontal asymptotes and two vertical asymptotes

106. If $f(x)=\begin{cases}x^2+2, x\le 1\\ 2x+1, x>1\end{cases}$, then $f'(1)$ is

a. ½ b. 1 c. 2
d. 3 e. nonexistent

107. For what value of k will $\frac{8x+k}{x^2}$ have a relative minimum at $x = 4$?
a. –32 b. –16 c. 0 d. 16 e. 32

108. If f is a function such that $f(0) = 1$, $f(1) = 2$, and $f(n)=\frac{f(n-2)}{f(n-1)}$ for all integers $n \geq 0$, what is the value of $f(4)$?
a. $\frac{1}{8}$ b. $\frac{2}{3}$
c. 1 d. 2
e. It cannot be determined from the given information given.

109. What is the maximum value of the derivative of $f(x)=3x^2-x^3$?
a. 0 b. 1 c. 2 d. 3 e. 4

110. If $f(x)=x^{-\frac{1}{3}}$, what is the derivative of the inverse of $f(x)$?
a. $x^{\frac{1}{3}}$ b. $-\frac{1}{3}x^{-\frac{4}{3}}$ c. $\frac{1}{3}x^{-\frac{2}{3}}$
d. $-3x^{-2}$ e. $-3x^{-4}$

111. * $\lim\limits_{h\to 0}\frac{2(x+h)^5-5(x+h)^3-2x^5+5x^3}{h}$ is
a. 0 b. $10x^3-15x$
c. $10x^4+15x^2$ d. $10x^4-15x^2$
e. $-10x^4+15x^2$

112. *What is the 20th derivative of $y = \sin(2x)$?
a. $-2^{20}\sin(2x)$ b. $2^{20}\sin(2x)$
c. $-2^{19}\cos(2x)$ d. $2^{20}\cos(2x)$
e. $2^{21}\cos(2x)$

113. *Suppose that $f(x)$ is a twice-differentiable function on the closed interval $[a,b]$. If $f'(c) = 0$ for $a < c < b$, which of the following statements must be true?
I. $f(a) = f(b)$
II. f has a relative extremum at $x = c$
III. f has a point of inflection at $x = c$
a. None
b. I only
c. II only
d. I and II
e. II and III

114. * $\frac{d}{dx}\left(x^{\frac{1}{\ln x}}\right)=$
a. 0 b. 1 c. $\ln x$ d. $x\ln x$ e. $x^{\ln x}$

115. *Let f be a function which is continuous on [2,10] and whose derivative is given by $f'(x)=\frac{\cos x}{\ln(x+1)}$. Which of the following is true about $f(x)$ on the interval [2,10]?
I. $f(x)$ is monotonic
II. $f(x)$ has a relative minimum
III. $f(x)$ has three points of inflection
a. I only
b. II only
c. III only
d. II and III only
e. I, II, and III

116. *If f is a continuous function on the closed interval $[a,b]$, which of the following is NOT necessarily true?
I. f has a minimum on $[a,b]$
II. f has a maximum on $[a,b]$
III. $f'(c) = 0$ for $a < c < b$
a. I only d. I and II only
b. II only e. I, II, and III
c. III only

117. *A particle moves along the x-axis so that its position at any time t > 0 is given by $x(t)=t^3+22t+3-6\cos(\pi t)$. For what value of t is the velocity negative?
a. $t = \frac{1}{2}$ b. $t = 1$ c. $t=\frac{3}{2}$
d. $t = 2$ e. The velocity is never negative

118. Which of the following functions is symmetric with respect to the origin?
a. $y = |x|$ b. $y=e^x$
c. $y=x^3+1$ d. $y = \sin x$
e. $y = \cos x$

119. The $\lim\limits_{h\to 0}\frac{(x+h)^3-x^3}{h}$ at the point $x = 2$ is
a. 36 b. 12 c. 8 d. 2 e. 0

120. If $f(x)=(x-1)^2\cos x$, then $f'(0)=$
a. –2 b. –1 c. 0 d. 1 e. 2

121. What is the domain of the function f given by $f(x)=\ln\sqrt{\frac{x+2}{x-4}}$?
a. $\{x: x < -2\}$ b. $\{x: x \neq 4\}$
c. $\{x: x > 4\}$ d. $\{x: -2 < x < 4\}$
e. $\{x: x < -2 \text{ or } x > 4\}$

122. $\lim\limits_{x\to 2}\frac{x-2}{2-x}$ is
a. –1 b. 0 c. 1
d. 2 e. nonexistent

123. If the line $y = 4x + 3$ is tangent to the curve $y=x^2+c$, then c is
a. 2 b. 4 c. 7 d. 11 e. 15

124. If $f(x)=\frac{\sin^2 x}{1-\cos x}$, then $f'(x)$ =
a. $\cos x$ b. $\sin x$
c. $-\sin x$ d. $-\cos x$
e. $2\cos x$

125. The equation of the horizontal asymptote for the graph of $y=\dfrac{2-e^{\frac{1}{x}}}{2+e^{\frac{1}{x}}}$ is

a. $y = -1$ b. $y = -½$ c. $y=\frac{1}{3}$
d. $y = ½$ e. $y = 1$

126. Let $f(x)=\begin{cases}\dfrac{x^2-1}{x-1}, x\neq 1\\ 1, \quad x=1\end{cases}$. Which of the following statements is correct?

a. $f(x)$ is continuous at 1 since $f(x)$ is defined at $x = 1$.
b. $f(x)$ is continuous at 1 since $\lim\limits_{x\to 1} f(x)$ exists.
c. $f(x)$ is not continuous at 1 since $f(x)$ is not defined at $x = 1$.
d. $f(x)$ is not continuous at 1 since $\lim\limits_{x\to 1} f(x)$ does not exist.
e. $f(x)$ is not continuous at 1 since $\lim\limits_{x\to 1} f(x)\neq f(1)$.

127. The equation of the tangent line to the curve $y=\dfrac{3x+4}{4x-3}$ at the point (1,7) is

a. $y + 25x = 32$ b. $y - 31x = -24$
c. $y - 7x = 0$ d. $y + 5x = 12$
e. $y - 25x = -18$

128. If $y=\ln(3x+5)$, then $\dfrac{d^2y}{dx^2}=$

a. $\dfrac{3}{3x+5}$ b. $\dfrac{3}{(3x+5)^2}$ c. $\dfrac{9}{(3x+5)^2}$
d. $\dfrac{-9}{(3x+5)^2}$ e. $\dfrac{-3}{(3x+5)^2}$

129. The derivative of $e^{\left(e^x\right)}$ is

a. e^x b. $e^{\left(e^x\right)}$ c. $e^{\left(e^{x^2}\right)}$
d. $e^{\left(x+e^x\right)}$ e. $e^{\left(xe^x\right)}$

130. If $f(x)=e^{2x}$ and $g(x)$ is the inverse function of $f(x)$, then $f(g(\ln 2))$ =

a. ¼ b. ½ ln2 c. ln2
d. 2 e. 4

131. What is the area of the largest rectangle with lower base on the x – axis and upper vertices on the curve $y=12-x^2$?

a. 8 b. 12 c. 16 d. 32 e. 48

132. $\lim\limits_{h\to 0}\dfrac{3^{2+h}-9}{h}$ is

a. 0 b. 1 c. 9 d. 9ln3 e. $9e^3$

133. If $y^2-2xy=21$, then $\dfrac{dy}{dx}$ at (2,–3) is

a. $-\frac{6}{5}$ b. $-\frac{3}{5}$ c. $-\frac{2}{5}$ d. $\frac{3}{8}$ e. $\frac{3}{5}$

134. *The maximum value of $\dfrac{k-\ln x}{x}$ occurs when x =

a. k b. $k + 1$ c. e^k
d. e^{k+1} e. $1+e^k$

135. *How many extrema (maximum and minimum) does the function $f(x)=(x+2)^3(x-5)^2$ have on the interval $-3\le x\le 6$?

a. None b. One c. Two
d. Three e. Four

136. *The tangent line to the graph of $y = \sin x$ at the point $\left(\frac{2\pi}{3},\frac{\sqrt{3}}{2}\right)$ crosses the sine graph at the point where x =

a. –0.781 b. 4.712 c. 5.388
d. 5.760 e. 6.283

137. *If $f(x)=2x^3$, then the average rate of change of f on the interval [0,2] is

a. 4 b. 8 c. 12 d. 16 e. 24

138. *Which statement is true for the function $f(x) = \ln(\tan x)$ on the interval $\pi<x<\frac{5\pi}{4}$?

a. $f(x)$ is increasing at an increasing rate
b. $f(x)$ is increasing at a decreasing rate
c. $f(x)$ has an absolute maximum in the open interval
d. $f(x)$ has a point of inflection in the open interval
e. $f(x)$ has a point of symmetry in the open interval

139. *A company must manufacture x calculators weekly that can be sold for $75-0.01x$ dollars each, at a cost of $1850+28x-x^2+0.001x^3$ dollars for manufacturing x calculators. The number of calculators the company should manufacture weekly in order to maximize its weekly profit is

a. 611 b. 652 c. 683
d. 749 e. 754

140. *A missile rises vertically from a point on the ground 75,000 feet from a radar station. If the missile is rising at a rate of 16,500 feet per minute at the instant when it is 38,000 feet high, what is the rate of change, in radians per minute, of the missile's angle of elevation from the radar station at this instant?

a. 0.175 b. 0.219 c. 0.227
d. 0.469 e. 0.507