

Occasionally you may be asked to solve an equation in one variable where the variable appears on both sides of the equation.

Example 3 : Solve

$$3x + 5 = 2x + 1 \quad \text{for } x$$

We will undo the operations in a way that puts all the x parts of the expression on one side of the equation.

$$\begin{aligned} 3x + 5 &= 2x + 1 \\ 3x + 5 - 2x &= 2x + 1 - 2x && \text{(subtract } 2x \text{ from both sides)} \\ x + 5 &= 1 \\ x + 5 - 5 &= 1 - 5 && \text{(subtract } 5 \text{ from both sides)} \\ x &= -4 \end{aligned}$$

Sometimes the expressions will be more complex and could involve brackets. In these cases we expand out the brackets and proceed.

Example 4 :

$$\begin{aligned} 3(5 - x) - 2(5 + x) &= 3(x + 1) \\ 15 - 3x - 10 - 2x &= 3x + 3 && \text{(collect like terms)} \\ 5 - 5x &= 3x + 3 \\ 5 - 5x + 5x &= 3x + 3 + 5x && \text{(add } 5x \text{ to both sides)} \\ 5 &= 8x + 3 \\ 5 - 3 &= 8x + 3 - 3 && \text{(subtract } 3 \text{ from both sides)} \\ 2 &= 8x \\ \frac{2}{8} &= \frac{8x}{8} && \text{(divide both sides by } 8) \\ \frac{1}{4} &= x \end{aligned}$$

Once you feel confident in these processes there is no need to put in the intermediate steps illustrated in these examples.

Exercises:

1. Solve the following equations:

(1) $2x - 1 = 9$

(2) $\frac{y}{3} + 4 = 12$

(3) $2(x + 1) - 7 = 5$

(4) $4(y + 3) - 2y = 7$

(5) $5(y + 2) - 4(y - 1) = 6$

(6) $5(2 - x) - 3(4 - 2x) = 20$

(7) $2m + 4 - 3m = 8(m - 1)$

(8) $3m + 12 = 2(m - 3) + 4$

(9) $\frac{x+1}{4} = 5$

(10) $\frac{x}{5} + \frac{x}{3} = 10$

Exercises 2.2 Solving Equations in One Variable

1. Solve

(a) $x + 4 = -7$

(b) $2 - x = 13$

(c) $15y = 45$

(d) $-\frac{t}{2} = -9$

(e) $3y - 20 = \frac{1}{2}$

(f) $\frac{x+3}{2} = -1$

(g) $3x + 2 = 4x - 7$

(h) $\frac{x}{2} + 7 = \frac{3x}{4}$

(i) $2x(x + 3) = 2x^2 + 15$

(j) $(y + 7)(y + 7) = y^2$

(k) $2x + 7 + 8x = 13$

(l) $3(x + 1) + 4x = 26$

(m) $8(m - 3) - 2(m - 2) = 20$

(n) $\frac{y+3}{2} = \frac{y-4}{3}$

(o) $3(4 - y) = 2(y + 5)$

(p) $\frac{x}{7} = 3\frac{1}{7}$

(q) $\frac{x+1}{2} = \frac{3}{4}$

(r) $16t - 7 + 4t = 12t - 1$

(s) $\frac{t}{4} + 3 = \frac{t}{8} - 1$

(t) $8 = \frac{1}{3}T + 2$

2. (a) Three times a number is equal to the number decreased by two. What is the number?
- (b) The sum of two consecutive numbers is 93. What are the numbers?
- (c) The sum of two consecutive *even* numbers is 46. Find the numbers.
- (d) Two times a number is equal to six less than three times the number. What is the number?

Answers 2.2

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|--------|---------------------|--------|-------------|---------------------|
| (1) 5 | (3) 5 | (5) -8 | (7) 1^1 | (9) 19 |
| (2) 24 | (4) $-2\frac{1}{2}$ | (6) 22 | (8) -14^3 | (10) $\frac{75}{4}$ |

Exercises 2.2

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|------------------------|------------------------|------------------------|-----------------------|
| 1. (a) $x = -11$ | (f) $x = -5$ | (k) $x = \frac{3}{5}$ | (p) $x = 22$ |
| (b) $x = -11$ | (g) $x = 9$ | (l) $x = 3\frac{2}{7}$ | (q) $x = \frac{1}{2}$ |
| (c) $y = 3$ | (h) $x = 28$ | (m) $m = 6\frac{2}{3}$ | (r) $t = \frac{3}{4}$ |
| (d) $t = 18$ | (i) $x = \frac{5}{2}$ | (n) $y = -17$ | (s) $t = -32$ |
| (e) $y = \frac{41}{6}$ | (j) $y = \frac{-7}{2}$ | (o) $y = \frac{2}{5}$ | (t) $T = 18$ |
2. (a) -1
(b) 46,47
(c) 22,24
(d) 6