AP Calculus Semester I Non Calculator Exam (1 point each) **DO NOT mark / write on this test.**

1. Find the limit, if it exists: $\lim_{x\to\pi} \tan\left(\frac{2}{3}x\right)$

a. $\frac{\sqrt{3}}{3}$ b. $-\sqrt{3}$ c. $\sqrt{3}$ d. $-\frac{\sqrt{3}}{3}$ e. none of these

2. Find the limit, if it exists: $\lim_{x\to\frac{5\pi}{6}} \frac{\sin x}{x}$

a. 0 b. $\frac{5\pi}{12}$ c. 1 d. $\frac{3}{5\pi}$ e. none of these

3. Find the limit, if it exists: $\lim_{x\to5} \frac{x^3-125}{x-5}$

a. DNE b. 1 c. 75 d. 25 e. none of these

4. Find the limit, if it exists: $\lim_{x\to0} \frac{\sqrt{3+x}-\sqrt{3}}{x}$

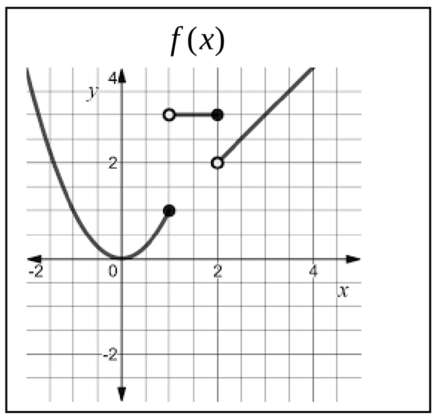
a. DNE b. 0 c. $\frac{1}{\sqrt{3+x}+\sqrt{3}}$ d. $\frac{3}{2\sqrt{3}}$ e. none of these

5. Find the limit: $\lim_{x\to4^-} \frac{x^2}{x^2-16}$

a. ∞ b. $-\infty$ c. 0 d. 1 e. none of these

6. Find the limit: $\lim_{x\to0^+} \left(x^5+\frac{1}{x}\right)$

a. ∞ b. $-\infty$ c. 0 d. 1 e. none of these



7. Refer to the figure of $f(x)$ at the right. Find $\lim_{x\to2} f(x)$

a. DNE b. 0 c. 2

d. 1 e. none of these

8. Refer to the figure of $f(x)$ at the right. Find $\lim_{x\to1^+} f(x)$

a. DNE b. 1 c. 2

d. 3 e. none of these

9. Given: $f(x)=\begin{cases} x^2+2; & x\neq1 \\ 1 \quad ; & x=1 \end{cases}$. Find $\lim_{x\to1} f(x)$

a. DNE b. 1 c. 3 d. $-\frac{2}{5}$ e. none of these

10. Find the value of ***a*** that makes f(x) continuous: $f(x)=\begin{cases} ax+2, & x\leq2 \\ 3-x, & x>2 \end{cases}$

a. 0 b. 1 c. 2 d. -2 e. none of these

11. Determine the location of the removable discontinuity, if any: $f(x)=\dfrac{x+8}{x^2+4x-32}$

a. $x=4$ b. $x=-8$ c. $x=4\ and\ -8$ d. $x=8$ e. none of these

12. Find the vertical asymptotes, if any, of the function: $h(x)=\dfrac{x}{x^3-4x}$

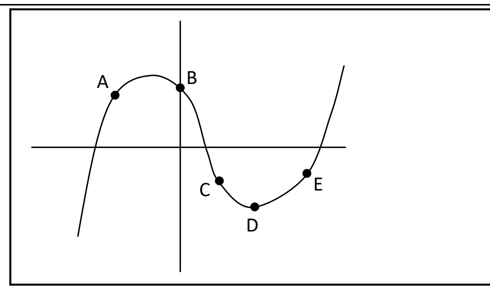
a. $x=0$ b. $x=0,\ 2$ c. $x=0,\ \pm 2$ d. $x=\pm 2$ e. none of these

13. Which choice uses the definition of the derivative to find f'(x) given $f(x)=2x^2+4$?

a. $\dfrac{\left(2x^2+4+\Delta x\right)-\left(2x^2+4\right)}{\Delta x}$ b. $\lim\limits_{\Delta x\to 0}\dfrac{\left(2x^2+4+\Delta x\right)-\left(2x^2+4\right)}{\Delta x}$ c. $\lim\limits_{\Delta x\to 0}\dfrac{\left[2\left(x+\Delta x\right)^2+4\right]-\left(2x^2+4\right)}{\Delta x}$

d. $\lim\limits_{\Delta x\to \infty}\dfrac{\left[2\left(x+\Delta x\right)^2+4\right]-\left(2x^2+4\right)}{\Delta x}$ e. none of these

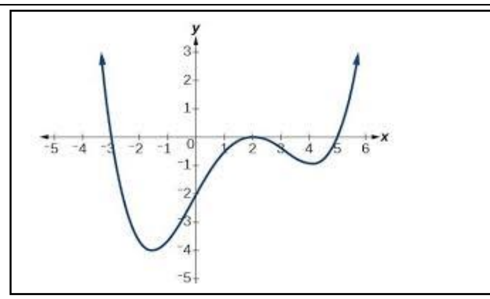
14. Given the graph of $f\,'(x)$ below, at which point is it true that $f\,'(x)<0$ and $f\,''(x)>0$?



a. A b. B c. C

d. D e. E

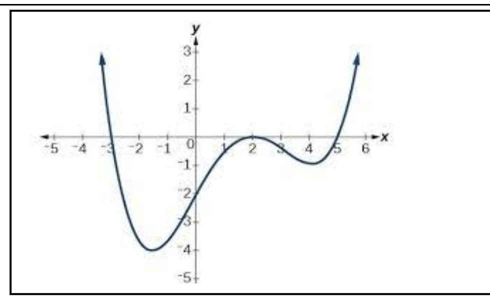
15. The graph of ***f'***, the derivative of ***f***, is shown on the right. On what interval(s) is ***f*** increasing?



a. $(-\infty,-3)\cup(5,\infty)$ b. $(-1.5,2)\cup(4.1,\infty)$

c. $(-3,2)\cup(2,5)$ d. $(-\infty,0)\cup(3,\infty)$

16. The graph of ***f'***, the derivative of ***f***, is shown on the right. On what interval(s) is ***f*** concave up?



a. $(-\infty,-3)\cup(5,\infty)$ b. $(-1.5,2)\cup(4.1,\infty)$

c. $(-3,2)\cup(2,5)$ d. $(-\infty,0)\cup(3,\infty)$

17. Find $\dfrac{dy}{dx}$ for $y=(7-x^6)\sqrt{x}$

a. $\dfrac{-7x^7+7}{2\sqrt{x}}$ b. $\dfrac{-6x^5}{2\sqrt{x}}$ c. $7x^{\frac{1}{2}}-x^{\frac{13}{2}}$ d. $\dfrac{-13x^7+7}{2\sqrt{x}}$ e. none of these

18. Find $f\,'(x)$ for $f(x)=\left(5-x\right)^{\frac{7}{2}}$

a. $\dfrac{7}{2}\left(5-x\right)$ b. $\dfrac{7}{2}\left(5-x\right)^{\frac{5}{2}}$ c. $-\dfrac{7}{2}\left(5-x\right)^{\frac{5}{2}}$ d. $\dfrac{7}{2}\left(5-x\right)^{\frac{2}{5}}$ e. none of these

19. Find the first derivative of the function: $y = \sin\sqrt{x} + \sqrt{\cos x}$ a. $y' = \cos\sqrt{x} + \frac{1}{2\sqrt{\cos x}}$ b. $y' = \frac{\cos\sqrt{x}}{2\sqrt{x}} - \frac{\sin x}{2\sqrt{\cos x}}$ c. $y' = \frac{\cos\sqrt{x}}{2\sqrt{x}} - \frac{\cos x}{2\sqrt{\sin x}}$ d. $y' = \cos\left(\frac{1}{2}\right) - \frac{\sin x}{2\sqrt{\cos x}}$ e. none of these
20. Find the tangent line equation to the graph of ***f*** at the given point: $f(x) = -3x^2 + 2\sqrt{x}; \quad (4,-44)$ a. $y+44 = \frac{49}{2}(x-4)$ b. $y+44 = -\frac{95}{4}(x-4)$ c. $y+44 = \frac{97}{4}(x-4)$ d. $y+44 = -\frac{47}{2}(x-4)$ e. none of these
21. Find the slope of the graph of $-16 = x^2y - 2x$ at $(2,-3)$. a. $-\frac{5}{2}$ b. $-\frac{1}{2}$ c. $\frac{1}{2}$ d. $-\frac{7}{2}$ e. none of these
22. Find $\frac{dy}{dx}$ by implicit differentiation: $38 = x^3 - 2x^2y + 3xy^2$ a. $\frac{dy}{dx} = 3x^2 - 4xy + 6xy$ b. $\frac{dy}{dx} = \frac{-3x^2 - 4xy - 3y^2}{-2x^2 + 6xy}$ c. $\frac{dy}{dx} = \frac{3x^2}{4x - 6xy}$ d. $\frac{dy}{dx} = \frac{-3x^2 + 4xy - 3y^2}{-2x^2 + 6xy}$ e. none of these
23. Find $\frac{d^2y}{dx^2}$ given: $2y^2 - x^3 = 0$ a. $\frac{d^2y}{dx^2} = \frac{24xy^2 - 9x^4}{16y^3}$ b. $\frac{d^2y}{dx^2} = \frac{3xy^2 - 9x^4}{2y^3}$ c. $\frac{d^2y}{dx^2} = \frac{6xy^2 - 9x^4}{16y^2}$ d. $\frac{d^2y}{dx^2} = \frac{24xy - 3x^2}{16y^2}$ e. none of these
24. Determine the x-value(s) at which the absolute minima is occurring given: $f(x) = x^3 - 3x^2, [-1,3]$. a. $x = -1$ *only* b. $x = 2$ *only* c. $x = -1, 2$ d. $x = 0, -1$ e. none of these
25. Find all the critical numbers for the function $f(x) = \frac{x-5}{x+3}$ a. $x = 1$ b. $x = -3$ c. $x = 1, -3$ d. $x = 4$ e. none of these
26. Let $f''(x) = 4x^3 - 2x$ and let $f(x)$ have critical numbers -1, 0, and 1. Use the Second Derivative Test to determine if any of the critical numbers gives a relative maximum. a. -1 b. 0 c. 1 d. -1 *and* 1 e. none of these
27. The function $s(t) = t^3 - 12t^2 + 36t + 6$ describes the motion of a particle moving along a line. On which interval is the particle moving to the left? a. $[2,6]$ b. $(2,6)$ c. $[0,12]$ d. $(0,12)$ e. none of these

28. Determine the x values of where the points of inflection occur for: $f(x)=\frac{x^6}{30}-\frac{x^4}{12}+\frac{15x}{2}$.

a. $x=1,-1$ b. $x=1,0,-1$ c. $x=0,-1$ d. $x=0,1$ e. none of these

29. Apply the Mean Value Th. to the function on the indicated interval, and find all values of ***c*** in ***(a, b)***.

$f(x)=x^{\frac{2}{3}};\quad [0,1]$

a. 1 b. $\frac{8}{27}$ c. $\frac{3}{2}$ d. $\frac{1}{9}$ e. none of these

30. State why Rolle's Theorem does not apply to the function $f(x)=\frac{2}{(x+1)^2}$ on the interval [-2, 0].

a. f *is not continuous on* $[-2,0]$ b. $f(-2)\neq f(0)$ c. f *is not differentiable at* $x=-1$

d. *both* a and c e. none of these

31. Given: $f(x)=x^3-6x^2+1$. Find the open intervals on which the function is increasing.

a. $(-\infty,0)\cup(4,\infty)$ b. $(-\infty,\infty)$ c. $(0,4)$ d. $(4,\infty)$ e. none of these

32. If the tangent line to $y=f(x)$ at (5, 1) also passes through the point (2, 7), then $f'(5)=$

a. 0 b. 1 c. -2 d. 2 e. none of these

33. Find the interval where the function is concave down: $f(x)=-x^3+3x+2$

a. $(-\infty,0)$ b. $(-\infty,\infty)$ c. $(0,4)$ d. $(0,\infty)$ e. none of these

34. Given that $f(x)=x^3+ax^2+bx$ has critical number at $x=1$ and $x=3$, find a and b.

a. $a=4;\ b=-3$ b. $a=-2;\ b=-3$ c. $a=-6;\ b=-3$ d. $a=6;\ b=-6$ e. none of these

35. Two positive numbers, a and b, multiply to 18. If we wish to minimize $2a+b$, what must the value of a be?

a. 2 b. 3 c. -2 d. -4 e. none of these

36. If $h(x)=f(g(x))$, $g(3)=6$, $g'(3)=-8$, $f'(3)=2$, and $f'(6)=8$, find $h'(3)$.

a. 22 b. -46 c. -28 d. -64 e. none of these

37. The radius r of a sphere is increasing at the rate of 0.3 in/sec. At the instant when the surface area S becomes 100π square inches, what is the rate of increase, in cubic inches per second, in the volume V? $\left(S=4\pi r^2 \;\; and \;\; V=\frac{4}{3}\pi r^3\right)$

a. 30π b. 25π c. 22.5π d. 12π e. none of these