① reflection across x-axis
vertical Stretch by factor of 3
shift right 2, up 1

② Domain $-4 \le x \le -1$ $[-4, -1]$
Range $-1 \le y \le 4$ $[-1, 4]$

③ Factoring

$$\frac{2x^2-x-3}{x+1} = \frac{(2x-3)\cancel{(x+1)}}{\cancel{(x+1)}} = \boxed{2x-3}$$

Synthetic Division

-1	2	-1	-3
	↓	-2	3
	2	-3	$\boxed{0}$

$\boxed{2x-3}$

Long division

$$\begin{array}{r} 2x-3 \\ x+1 \overline{)\, 2x^2-x-3} \\ +(\overline{2}x^2 + \overline{2}x) \\ \hline -3x-3 \\ +(+3x+3) \\ \hline 0 \end{array}$$

④ Solve: $\dfrac{x+10}{x^2+16x+63} = \dfrac{4}{x+9} + \dfrac{11}{x+7}$

$$\frac{x+10}{(x+9)(x+7)} = \frac{4}{x+9}\left(\frac{x+7}{x+7}\right) + \frac{11}{x+7}\left(\frac{x+9}{x+9}\right)$$

$$\frac{x+10}{\cancel{(x+9)(x+7)}} = \frac{4x+28 + 11x+99}{\cancel{(x+9)(x+7)}}$$

$$x+10 = 15x+127$$

$$-127 = 14x$$

$$\boxed{x = \frac{-127}{14}}$$

(5) Factor:

$$\frac{5x^2-17x+6}{x-3} = \frac{(5x-2)\cancel{(x-3)}}{\cancel{x-3}} = \boxed{5x-2}$$

Synthetic division:

$$\begin{array}{r|rrr} & 5 & -17 & 6 \\ 3 & \downarrow & 15 & -6 \\ \hline & 5 & -2 & \boxed{0} \end{array}$$

$\boxed{5x-2}$

long division:

$$\begin{array}{r} 5x-2 \\ x-3\,\overline{)\,5x^2-17x+6} \\ +(-5x^2+15x) \downarrow \\ \hline -2x+6 \\ +(+2x\mp 6) \\ \hline 0 \end{array}$$

(6) $25x^2-16y^2$

$(5x-4y)(5x+4y)$

(7) $\left(\frac{x-2}{x-2}\right)\frac{1}{(3x-1)(x+1)} + \frac{2}{(x-2)(x+1)}\left(\frac{3x-1}{3x-1}\right)$

$$= \frac{x-2 \quad +2(3x-1)}{(x-2)(3x-1)(x+1)} = \frac{7x-4}{(x-2)(3x-1)(x+1)}$$

(8) $$\frac{3\cancel{(x+1)}}{\cancel{(x+2)}\cancel{(x+1)}} \cdot \frac{\cancel{(x+3)}\cancel{(x+2)}}{(2x-3)\cancel{(x-3)}}$$

$= \frac{3}{2x-3}$

copy dot flip

(9) $\dfrac{2x^2-x-3}{4x^2-9} \cdot \dfrac{4x^2-2x-6}{x^2-9x+14}$

$\dfrac{(\cancel{2x-3})(x+1)}{(2x+3)(\cancel{2x-3})} \cdot \dfrac{2(2x^2-x-3)}{(x-7)(x-2)}$

$\dfrac{(x+1)}{2x+3} \cdot \dfrac{2(2x-3)(x+1)}{(x-7)(x-2)}$

$\dfrac{2(x+1)^2(2x-3)}{(2x+3)(x-7)(x-2)}$

(10) $f(x) = \dfrac{x^2-2x+4}{x^2-1}$ * check to see if you can simplify

$= \dfrac{x^2-2x+4}{(x-1)(x+1)}$ ← no holes

V.A : set denominator equal to zero

$x-1=0$ $x+1=0$

$x=1$ $x=-1$

H.A: $f(x) = \dfrac{ax^n + \dots}{bx^m + \dots}$

if $n < m$ $y=0$ is h.A

if $n = m$ $y = \frac{a}{b}$

if $n > m$ slant, no horizonta

h.A: $y=1$ ← $\dfrac{1}{1}$ $\dfrac{x^2-2x+4}{x^2-1}$

(11) $\dfrac{1}{(x+5)(x+3)}$ all real #'s except $x=-5,-3$

(12) $\dfrac{4x^2-36x+56}{x^3-9x^2+14x} =$

$= \dfrac{4\,(x^2-9x+14)}{x\,(x^2-9x+14)} = \dfrac{4}{x}$

(13) opens up, same direction (+even)

$f(x) = (x+3)(x-2)^2(x-5)$

(14) $(x-3)(x+\sqrt{3})(x-\sqrt{3}) = x^{3}$

like i, radical zeros come in pairs

(15) $g(x) = \dfrac{2}{x+4} + 1$

V.A $= x+4=0$
$x=-4$

H.A : $y=1$

D: $(-\infty,-4) \cup (-4,+\infty)$

R: $(-\infty,1) \cup (1,+\infty)$

(16) inverse $f(x) = 3(x-4)^2+1$

$y = 3(x-4)^2+1$

$x = 3(y-4)^2+1$

$-1 \qquad -1$

switch x & y
solve for y

$\dfrac{x-1}{3} = \dfrac{3(y-4)^2}{3}$

$$\sqrt{\frac{x-1}{3}} = \sqrt{(y-4)^2}$$

$$\sqrt{\frac{x-1}{3}} = y-4$$

$$+4 \qquad +4$$

$$y = \sqrt{\frac{x-1}{3}} + 4$$

(17) $\sqrt[3]{4x^2-4x+1} - \sqrt[3]{x} = 0$

$$+\sqrt[3]{x} \qquad \sqrt[3]{x}$$

$$\left(\sqrt[3]{4x^2-4x+1} = \sqrt[3]{x}\right)^3$$

$$4x^2-4x+1 = x$$

$$-x \qquad -x$$

$$4x^2-5x+1 = 0$$

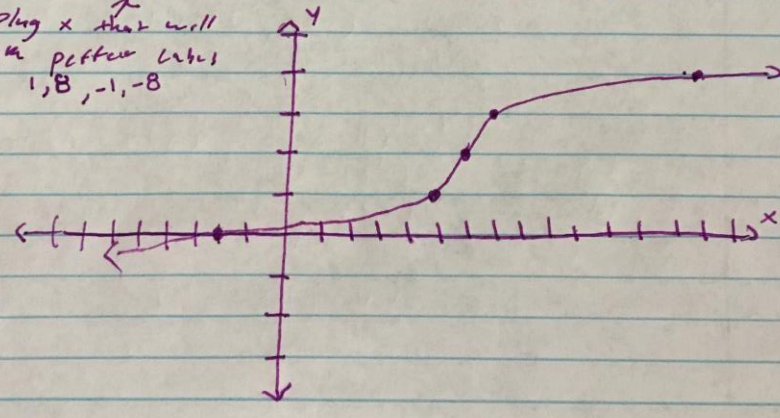
$$(4x-1)(x-1) = 0$$

$x = 1/4 \quad x = 1$ check for extraneous

(18) $F(x) = \sqrt[3]{x-6} + 2$ vertex: $(6,2)$

Plug x that will make perfect cubes 1, 8, -1, -8

x	y
7	3
14	4
5	1
-2	0



(19) $y = 3\sqrt{x-3} + 3$

$\cancel{x} = 3\sqrt{y-3} + 3$

$-3 \qquad\qquad -3$

$\frac{x-3}{3} = \frac{3\sqrt{y-3}}{3}$

$\left(\frac{x-3}{3}\right)^2 = \left(\sqrt{y-3}\right)^2$

$\boxed{\left(\frac{x-3}{3}\right)^2 + 3 = y}$

(20) $\left((3x+28)^{1/2}\right)^2 = (x)^2$

$3x+28 = x^2$

$0 = x^2 - 3x - 28$

$0 = (x-7)(x+4)$

$x = 7 \quad x = \cancel{-4}$ extraneous

(21) $\sqrt[4]{48x^5y^6}$

$\sqrt[4]{3 \cdot 16 \cdot x^4 \cdot x \cdot y^4 \cdot y^2}$

$= 2xy\sqrt[4]{3xy^2}$

(22) $\frac{3}{2}x^{2/3} \cdot y^{1/2} \cdot 4x^{-1/3}y^{3/2}$

$6x^{2/3+(-1/3)}y^{1/2+3/2}$

$\boxed{6x^{1/3}y^2}$

23) $e^{x-7} - 2$ growth Function

$\uparrow$

h. A

domain $-\infty < x < +\infty$ $(-\infty, +\infty)$

Range $-2 < y < +\infty$ $(-2, +\infty)$

24) 30, 32, 34, ... arithmetic: (add)

$a_1 = 30$ $d = 2$

Explicit

$a_n = a_1 + (n-1)d$

$a_n = 30 + (n-1)2$

Recursive

$a_n = a_{n-1} + d$

$a_1 = 30$

geometric (multiply)

1, 3, 9, 27, ... $a_1 = 1$, $r = 3$

Explicit

$a_n = a_1 \cdot r^{n-1}$

$a_n = 1 \cdot 3^{n-1}$

recursive

$a_n = r a_{n-1}$

$a_n = 3a_{n-1}$

$a_1 = 1$

1 2 3 4 5 6 7

30, 32, 34, 36, 38, 40, 42 or plug 7 into $a_n = 30 + (n-1)2$

25) −7, −4, −1, 2, ... $a_1 = -7$ $d = 3$

$a_n = a_1 + (n-d)d$

$a_n = -7 + (n-1)3$

$a_n = -7 + 3n - 3$

a_n $3n - 10$

(26) growth: $y = ab^{x-h} + k$

$k = -1$

$y = a \cdot b^{x-h} - 1$ $\quad$ h - 1st ref point x, h = 3 (3,2)

$y = a \cdot b^{x-3} - 1$ $\quad$ $a + k = 2$

$y = 3 \cdot b^{x-3} - 1$ $\quad$ $a = 3$

plug (4,5) solve for b

$\cancel{y}5 = 3 \cdot b^{4-3} - 1$

$6 = 3b$

$b = 2$

$y = 3 \cdot 2^{x-3} - 1$

division

(27) $\log_5 500 - \log_5 4$

$\log_5 \frac{500}{4} = \log_5 125$

$= \cancel{\log_5} 5^3 = 3$

multiply

(28) $\log_3 27 + \log_3 9$

$= \log_3 243$

$= \cancel{\log_3} 3^5$

$= 5$

(29) $\log_2 128$

$\cancel{\log_2} 2^7$

7

(30) $\log_4(2x-1)+3=5$

$-3 \quad -3$

$\log_4(2x-1)=2$

$\cancel{4}^{\cancel{\log_4}(2x-1)} = 4^2$

$2x-1=16$

$2x=17$ check ✓

$x=17/2$

(31) $3\ln e^{2x+4} = e^{\ln 9}$

$3\cancel{\log_e} e^{2x+4} = \cancel{e}^{\cancel{\log_e}9}$

$\frac{3(2x+4)}{3} = \frac{9}{3}$

$2x+4=3$

$2x=-1$

$x=-1/2$

(32) $\log_4 1/64$

$= \cancel{\log_4 4}^{-3}$

$= -3$

(33) $27^{x+2}=9$

$\underline{3}^{3(x+2)} = \underline{3}^{2}$

$3x+6=2$

$3x=4$

$x=4/3$

(34) Expand $\ln\left(\frac{36x^3}{\sqrt{x}}\right)$ division → subtract

$= \ln 36x^3 - \ln\sqrt{x}$ multiply to add

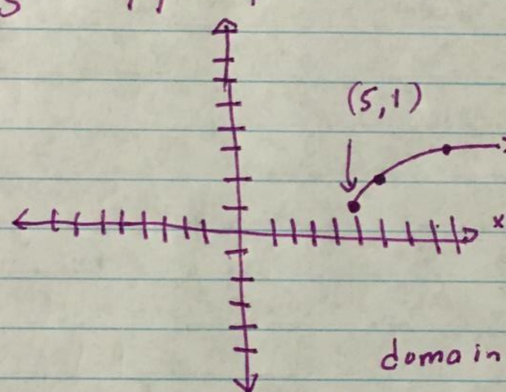
$= \ln 36 + \ln x^3 - \ln\sqrt{x}$ exponents in front

$= \ln 36 + 3\ln x - \ln x^{1/2}$

$= \ln 36 + 3\ln x - \frac{1}{2}\ln x$

(35) $f(x)$ $\sqrt{x-5}+1$

(5, 1)



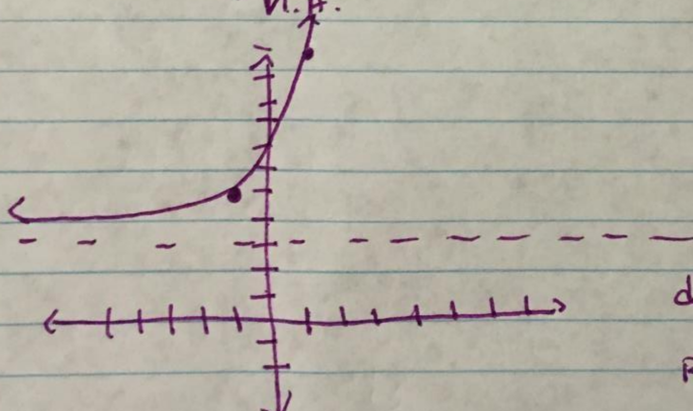
x	y
6	2
9	3

domain: $x \geq 5$ $[5, +\infty)$

$y \geq 1$ $[1, +\infty)$

(36) $2e^{x+1}+3$ growth

↑ H.A.



x	y
-1	5
1	$2e+5 \approx 11$

d: $-\infty < x < +\infty$

$(-\infty, +\infty)$

R: $3 < y < +\infty$

$(3, +\infty)$

(37) $F(x) = \frac{1}{3}\log_3(x-4)+6$

Shift right 4 , up 6
vertical compression 1/3

(38) $17^x = 34$

$\log 17^x = \log 34$

$x = \frac{\log 34}{\log 17}$ or $\frac{\ln 34}{\ln 17}$

(39)

$\log_4 8 = x$

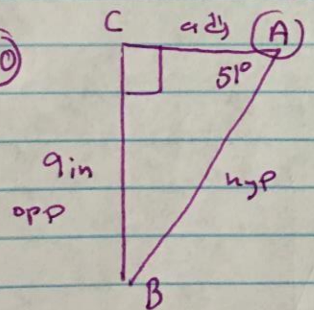
$4^x = 8$

$2^{2x} = 2^3$

$2x = 3$

$x = 3/2$

(40)



$\measuredangle A = 51$ $\measuredangle C = 90°$ $\measuredangle B = 49°$

Looking from A's position

$\tan 51 = \frac{9}{\overline{AC}}$

$\overline{AC}\tan 51 = 9$

$\overline{AC} = \frac{9}{\tan 51}$

$\overline{AC} \approx 7.29$

$\sin 51 = \frac{9}{AB}$

$\overline{AB}\sin 51 = 9$

$\overline{AB} \approx 11.58$

(41) $\frac{11\pi}{8} = \frac{11(\cancel{180}^{45})}{\cancel{8}_2} = \frac{495}{2} = 247.5°$

$220° \times \frac{\pi}{180} = \frac{\cancel{220}^{11}\pi}{\cancel{180}_9} = 11/9\,\pi$

(42) $210° + 360 = 570°$

$210° - 360° = -150°$

$-\frac{2\pi}{5} + 2\pi\left(\frac{5}{5}\right) = \frac{8\pi}{5}$

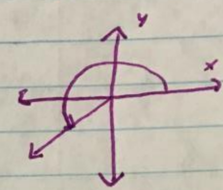
$-\frac{2\pi}{5} - 2\pi\left(\frac{5}{5}\right) = -\frac{12\pi}{5}$

(43) $\cos \frac{37(180)}{4} = \cos 37(45)$

$= \cos 1665$

$= \cos 225°$

$$\begin{array}{r} {}^{3}\,{}^{2} \\ 37 \\ \times 45 \\ \hline 185 \\ 1480 \\ \hline 1665 \end{array}$$



$1665 - 360 = 1305 - 360 = 945 - 360 = 585 - 360 = 225°$

$= -\frac{\sqrt{2}}{2}$

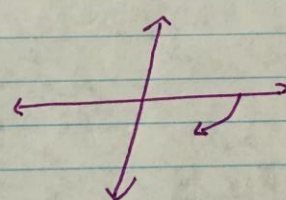
(44) $\tan(-60)$

$= \tan(300°)$

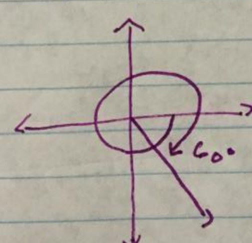
$= -\frac{\sin 60}{\cos 60}$

$= -\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}$

$= -\frac{\sqrt{3}}{2} \cdot \frac{2}{1} = \boxed{-\sqrt{3}}$



(45) $-\frac{7\pi}{3} = \frac{-7(180)}{3} = -7(60) = -420°$



(46) $\dfrac{\cot x}{\csc x} = \cos x$

$= \dfrac{\frac{\cos x}{\sin x}}{\frac{1}{\sin x}}$

$= \dfrac{\cos x}{\cancel{\sin x}} \cdot \dfrac{\cancel{\sin x}}{1}$

$= \cos x$

$\sin^2 x(1+\cot^2 x) = 1$

$= \sin^2 x + \sin^2 x \cot^2 x$

$= \sin^2 x + \cancel{\sin^2 x}\left(\dfrac{\cos^2 x}{\cancel{\sin^2 x}}\right)$

$= \sin^2 x + \cos^2 x$

$= 1$

$\dfrac{\sec x}{\sin x}\left(\dfrac{\cos x}{\cos x}\right) - \dfrac{\sin x}{\cos x}\left(\dfrac{\sin x}{\sin x}\right) = \cot x$

$= \dfrac{1 - \sin^2 x}{\sin x \cos x}$

$= \dfrac{\cos^2 x}{\sin x \cancel{\cos x}}$

$= \dfrac{\cos x}{\sin x}$

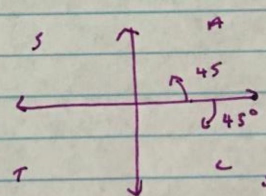
$= \cot x$

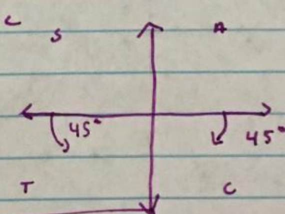
(47) $\dfrac{-4\sqrt{2}}{-8} = \dfrac{-8\cos x}{-8}$

$\cos x = \dfrac{\sqrt{2}}{2}$

$x = \cos^{-1}\dfrac{\sqrt{2}}{2}$

$x = 45^\circ, 315^\circ$





$5 + \sin(x+225) = 5 - \dfrac{\sqrt{2}}{2}$

$-5 \qquad\qquad -5$

$\sin(x+225) = -\dfrac{\sqrt{2}}{2}$

$x + 225 = \sin^{-1}\left(-\dfrac{\sqrt{2}}{2}\right)$

$x + 225 = 225, 315$

$\boxed{x = 0^\circ, 90^\circ}$