

# Pre-Calculus Exam Review

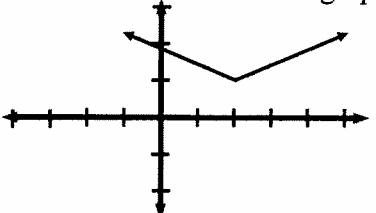
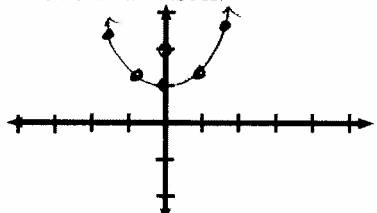
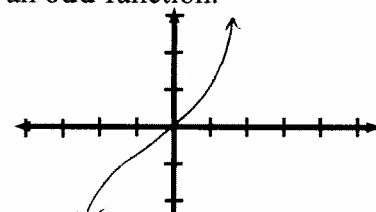
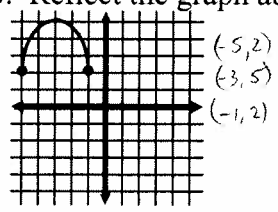
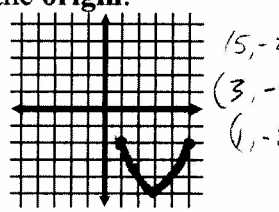
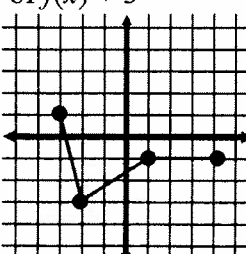
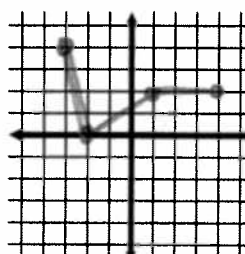
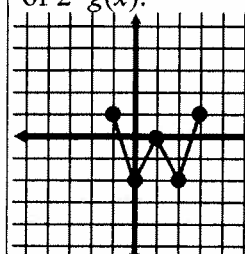
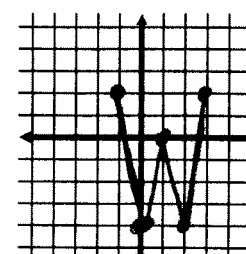
## Semester 1 Exam

### Functions (Chapters 1, 2, and 4)

For questions 1 – 6, let  $f(x) = \frac{1}{x-5}$  and  $g(x) = \frac{1}{x+1}$ .

<p>1. What is <math>f+g</math>?</p> $\left(\frac{1}{x-5}\right) + \left(\frac{1}{x+1}\right)$ $\frac{x+1}{(x-5)(x+1)} + \frac{x-5}{(x+1)(x-5)} \Rightarrow \frac{2x-4}{(x+1)(x-5)} \text{ or } \frac{2(x-2)}{(x+1)(x-5)}$	<p>2. What is <math>f \cdot g</math>?</p> $\left(\frac{1}{x-5}\right) \left(\frac{1}{x+1}\right) = \frac{1}{x^2-4x-5} \text{ or } \frac{1}{(x-5)(x+1)}$
<p>3. What is <math>f \circ g</math>?</p> $f(g(x)) = \frac{1}{\frac{1}{x+1}-5} = \frac{1}{\frac{1-5(x+1)}{x+1}} = \frac{1}{\frac{1-5x-5}{x+1}} = \frac{1}{\frac{-5x-4}{x+1}} = \frac{x+1}{-5x-4} = -\frac{x+1}{5x+4}$ <p><math>D_{f \circ g}</math>: <math>x \neq -1, x \neq -4/5</math></p> <p><math>R_{f \circ g}</math>: <math>y \neq -1/5</math></p>	<p>4. What is <math>g \circ f</math>?</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <math display="block">\frac{x-5}{x-4}</math> </div> <p><math>D_{g \circ f}</math>: <math>x \neq 4, x \neq 5</math></p> <p><math>R_{g \circ f}</math>: <math>y \neq 1</math></p>
<p>5. What is <math>f^{-1}</math>?</p> $y = \frac{1}{x-5}$ $x = \frac{1}{y-5}$ $xy - 5x = 1$ $xy = 1 + 5x$ $y = \frac{5x+1}{x}$ <div style="border: 1px solid black; padding: 5px; display: inline-block;"> <math>f^{-1}(x) = \frac{5x+1}{x}</math> </div> <p><math>x \neq 5</math> <math>x \neq 0</math></p>	<p>6. What is <math>g^{-1}</math>?</p> $y = \frac{1}{x+1}$ $x = \frac{1}{y+1}$ $xy + x = 1$ $xy = 1 - x + 1$ $y = \frac{-x+1}{x}$ <div style="border: 1px solid black; padding: 5px; display: inline-block;"> <math>f^{-1}(x) = \frac{-x+1}{x}, x \neq 0, x \neq -1</math> </div>

### Answer the questions about the graphs of functions.

<p>7. What is the domain for the function in the graph below?</p> <p><math>(-\infty, \infty)</math></p> 	<p>8. Sketch a graph of an <b>even</b> function.</p> 
<p>9. Sketch a graph of an <b>odd</b> function.</p> 	<p>10. Reflect the graph about the <b>origin</b>.</p> <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;">  <p><math>(-5, 2)</math> <math>(3, 5)</math> <math>(-1, 2)</math></p> </div> <div style="text-align: center;">  <p><math>(5, -2)</math> <math>(-3, -5)</math> <math>(1, -2)</math></p> </div> </div>
<p>11. Given the graph of <math>f(x)</math> below, sketch the graph of <math>f(x) + 3</math></p>  	<p>12. Given the graph of <math>g(x)</math> below, sketch the graph of <math>2 \cdot g(x)</math>.</p>  

Find the domain and range for each function below.

13.  $f(x) = \sqrt{3+x}$

$$\begin{aligned} 3+x &\geq 0 \\ x &\geq -3 \end{aligned}$$

Domain:  $[-3, \infty)$

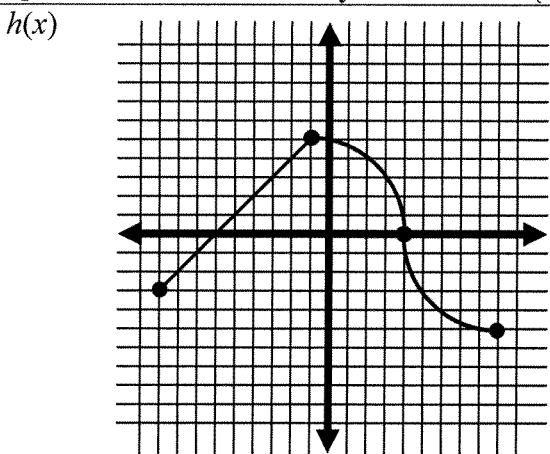
Range:  $[0, \infty)$

14.  $g(x) = \frac{1}{x+8}$       $x+8 \neq 0$   
 $\phantom{g(x) = \frac{1}{x+8}}$       $x \neq -8$

Domain:  $(-\infty, -8) \cup (-8, \infty)$

Range:  $(-\infty, 0) \cup (0, \infty)$

Questions 15 – 18: Analyze function  $h(x)$  in the graph below. (1 point each)



15. What is the value of  $h(6)$ ?

$$h(6) = -4$$

16. For what value of  $x$  does  $h(x) = 3$ ?

$$x = -3, x = 3 \text{ or } \{-3, 3\}$$

17. On what interval(s) is  $h(x)$  decreasing?

$$(-1, 9)$$

18. On what interval(s) is  $h(x) > 0$ ? (above x-axis)

$$(-6, 4)$$

If  $f(x) = \frac{x+2}{x-7}$  and  $g(x) = \frac{3}{x}$ , find an expression for each of the following operations.

19.  $f+g$

$$\begin{aligned} &= \frac{x+2}{x-7} + \frac{3}{x} \\ &= \frac{x(x+2)}{(x-7)(x)} + \frac{3(x-7)}{x(x-7)} \\ f+g &= \frac{x^2+2x+3x-21}{x(x-7)} \\ f+g &= \frac{x^2+5x-21}{x^2-7x} \end{aligned}$$

20.  $\frac{f}{g} = \frac{\frac{x+2}{x-7}}{\frac{3}{x}} = \left(\frac{x+2}{x-7}\right) \left(\frac{x}{3}\right)$

$$\frac{f}{g} = \frac{x^2+2x}{3x-21}$$

If  $f(x) = 2x^2 - 5x + 1$ , find an expression for each of the following.

21.  $f(x) + 3$

$$2x^2 - 5x + 1 + 3$$

$$2x^2 - 5x + 4$$

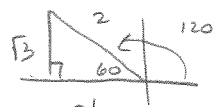
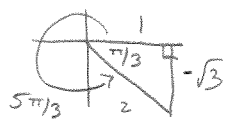

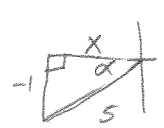
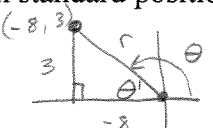


22.  $f(x+3)$

$$\begin{aligned} &2(x+3)^2 - 5(x+3) + 1 \\ &2(x^2+6x+9) - 5x - 15 + 1 \\ &2x^2 + 12x + 18 - 5x - 14 \end{aligned}$$

$$2x^2 + 7x + 4$$

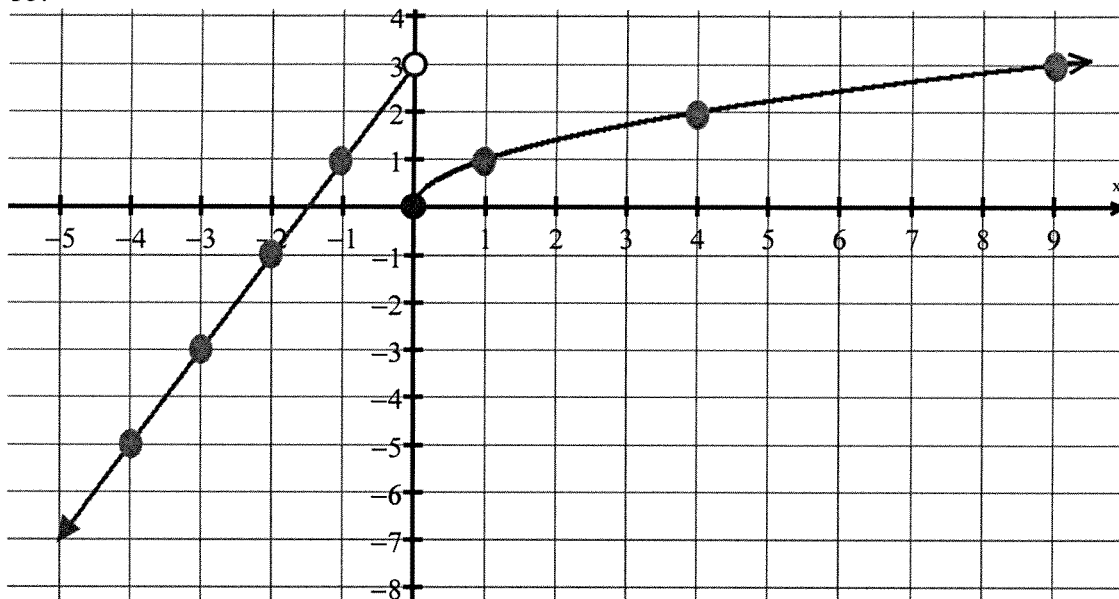
# Trigonometric Functions (Chapter 5)

Evaluate.

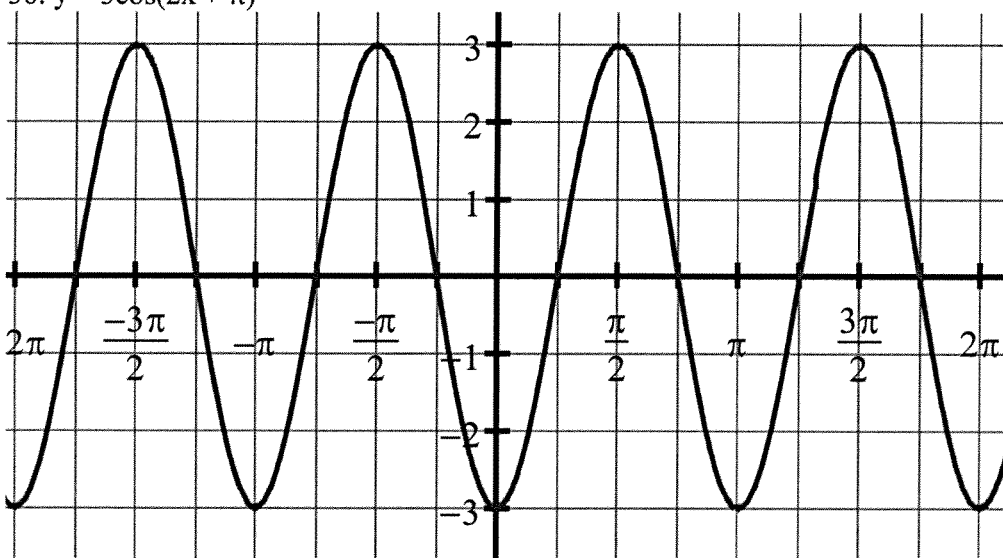
<p>23. Convert <math>\frac{11\pi}{6}</math> to degrees.</p> $\left(\frac{11\pi}{6}\right)\left(\frac{360}{2\pi}\right) = \boxed{330^\circ}$	<p>24. Convert <math>210^\circ</math> to radians.</p> $210\left(\frac{\pi}{180}\right) = \boxed{\frac{7\pi}{6}}$	<p>25. What is the <b>exact</b> value of <math>\tan 120^\circ</math>?</p>  $\boxed{\tan 120^\circ = -\sqrt{3}}$								
<p>26. What is the <b>exact</b> value of <math>\sec \frac{5\pi}{3}</math>?</p>  $\cos\left(\frac{5\pi}{3}\right) = \frac{1}{2}$ $\boxed{\sec\left(\frac{5\pi}{3}\right) = 2}$	<p>27. In which quadrant is <math>\theta</math> if <math>\csc \theta &gt; 0</math> and <math>\tan \theta &lt; 0</math>?</p> <table border="1" data-bbox="633 651 779 798"> <tr> <td>S</td> <td>A</td> </tr> <tr> <td>x</td> <td>o</td> </tr> <tr> <td>T</td> <td>c</td> </tr> <tr> <td></td> <td>o</td> </tr> </table> $\boxed{\text{QUADRANT II}}$	S	A	x	o	T	c		o	<p>28. What is the <b>exact</b> value of <math>\cos \theta</math> if <math>\theta</math> is in quadrant II and <math>\sin \theta = \frac{24}{25}</math>?</p>  $x^2 + y^2 = r^2$ $x^2 + 24^2 = 25^2$ $x^2 + 576 = 625$ $x^2 = 49$ $x = \pm\sqrt{49}$ $\boxed{\cos \theta = -\frac{7}{25}}$ $\boxed{x = -7}$
S	A									
x	o									
T	c									
	o									
<p>29. If <math>\cos \alpha = -\frac{1}{5}</math> and <math>\tan \alpha &gt; 0</math>, what is the exact value of <math>\sin \alpha</math>?</p> <table border="1" data-bbox="97 1113 211 1239"> <tr> <td>S</td> <td>A</td> </tr> <tr> <td>x</td> <td>o</td> </tr> <tr> <td>T</td> <td>c</td> </tr> <tr> <td></td> <td>o</td> </tr> </table>  $x^2 + y^2 = r^2$ $x^2 + (-1)^2 = (5)^2$ $x^2 = 24$ $x = \pm\sqrt{24}$ $\boxed{\sin \alpha = -\frac{2\sqrt{6}}{5}}$	S	A	x	o	T	c		o	<p>30. What is the value of <math>\tan \theta</math> if <math>(-8, 3)</math> is on the terminal side of <math>\theta</math> in standard position?</p>  $\boxed{\tan \theta = -\frac{3}{8}}$	<p>31. Find a negative angle measures that is <b>coterminal</b> with <math>130^\circ</math>.</p> $130 - 360 = \boxed{-230^\circ}$
S	A									
x	o									
T	c									
	o									
<p>32. What is the reference angle of <math>245^\circ</math> in standard position?</p>  $245 - 180 = \boxed{65^\circ}$	<p>33. What is the reference angle of <math>\frac{4\pi}{3}</math> in standard position?</p>  $\frac{4\pi}{3} - \pi = \boxed{\frac{\pi}{3}}$	<p>34. What is the <b>period</b> and <b>phase shift</b> of <math>f(x) = \sin(3x - \pi)</math>?</p> $PD \Rightarrow \frac{2\pi}{b} = \frac{2\pi}{3}$ $PS \Rightarrow 3x - \pi = 0$ $3x = \pi$ $x = \pi/3$ <p>Period: <math>\underline{2\pi/3}</math></p> <p>Phase Shift: <math>\underline{\pi/3}</math> RIGHT</p>								

Pre-Calc Midterm Review Graphing Problems

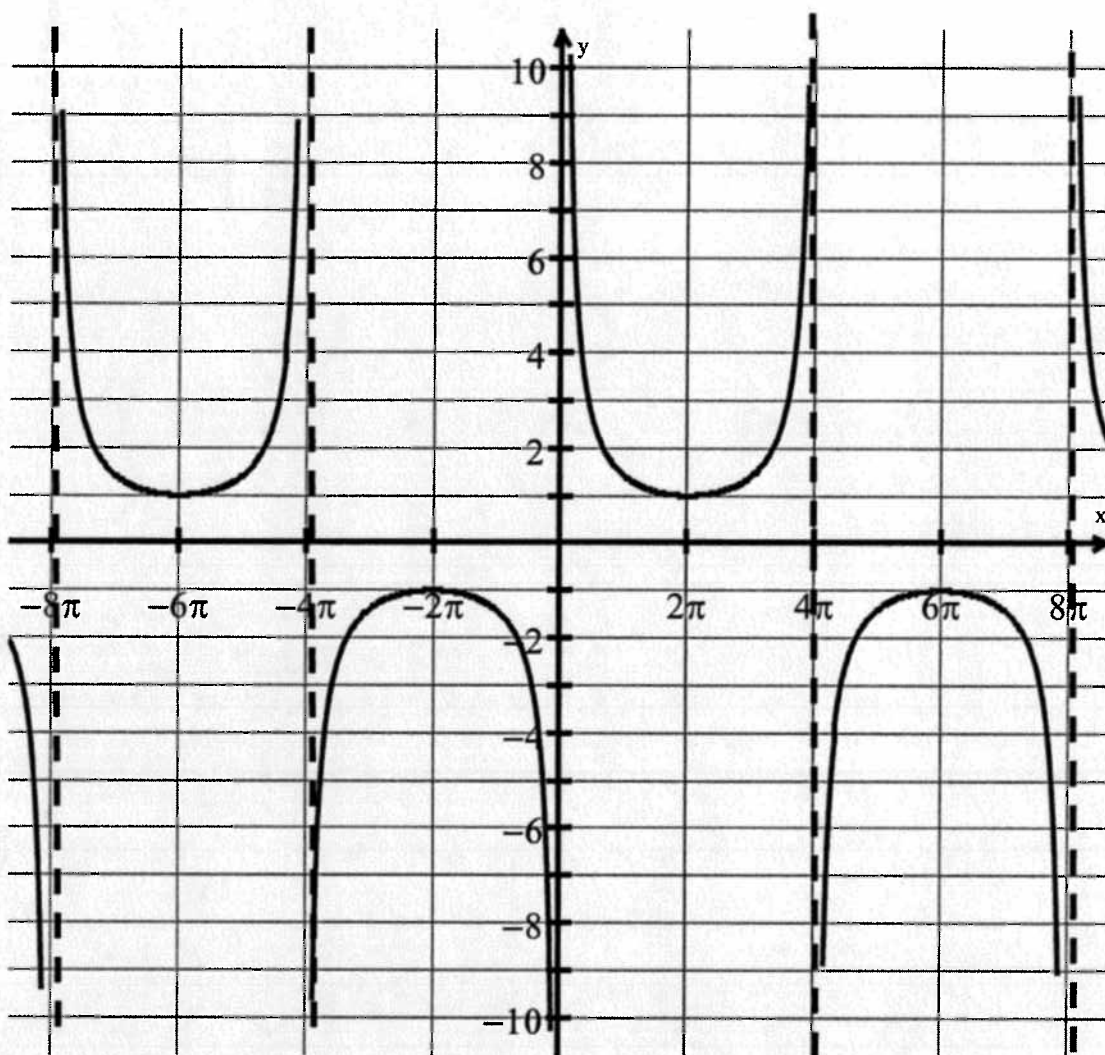
35.



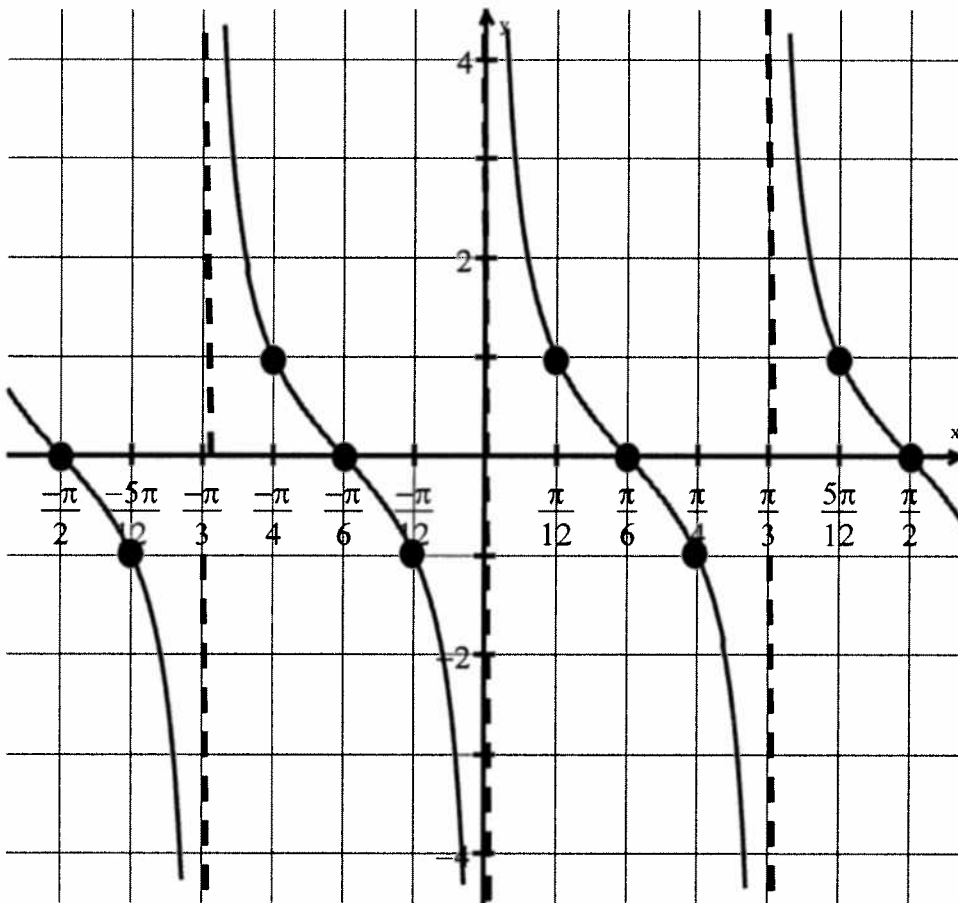
36.  $y = 3\cos(2x + \pi)$



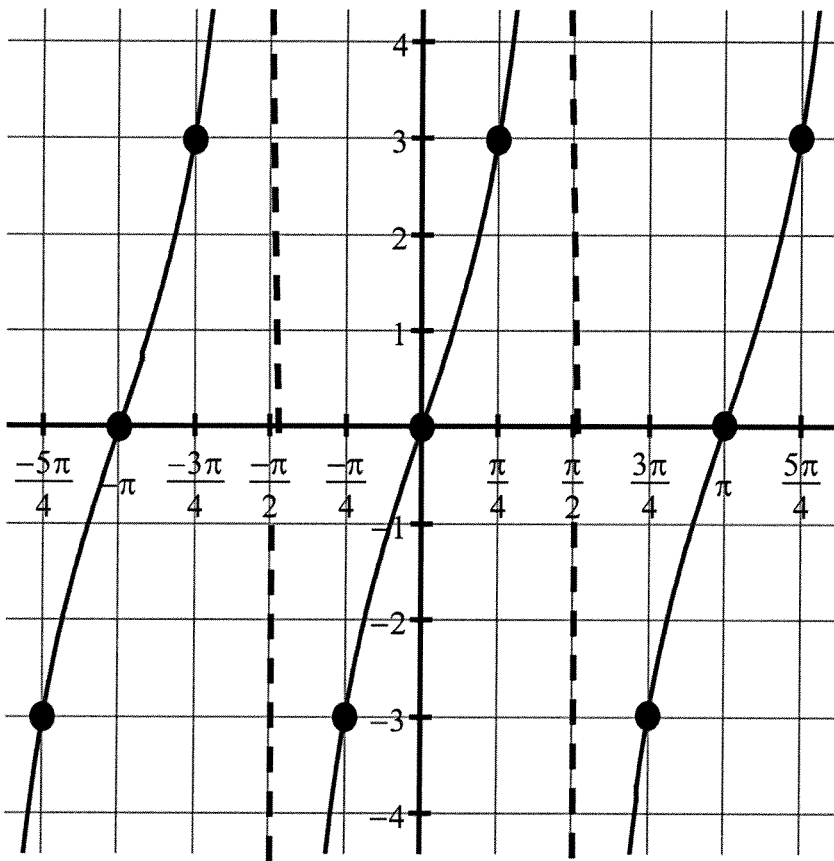
37.



38.  $y = \cot(3x)$



39.



40.  $y = -4\cos(2/5)x$

## Analytic Trigonometry (Chapter 6)

Find values to answer the following questions.

41. If  $\tan \alpha = -\frac{3}{4}$  when  $\frac{\pi}{2} < \alpha < \pi$  and  $\cos \beta = \frac{5}{13}$  when  $\frac{3\pi}{2} < \beta < 2\pi$ , what is the *exact* value of  $\tan(\alpha + \beta)$ ?

42. When  $P(2, -5)$  is on the terminal side of  $\theta$  in standard position, what is the value of  $\cos 2\theta$ ?

43. If  $\cos \alpha = \frac{3}{5}$  and  $\frac{3\pi}{2} < \alpha < 2\pi$ , what is the value of  $\sin \frac{\alpha}{2}$ ?

44. What is the solution of  $\sin \theta = 0.8704$  to the nearest hundredth of a degree? DEGREE MODE!

$$\sin \theta = 0.8704$$

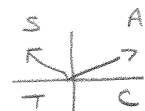
$$\theta = \sin^{-1}(0.8704)$$

$$\theta \approx 60.51^\circ$$

$$\theta' = 180 - \theta$$

$$\theta' = 180 - 60.51$$

$$\theta' \approx 119.49^\circ$$



2 ANSWERS..

1 in QUAD I,

1 in QUAD II

45. What is the solution of  $\sec \theta = 1.8492$  to the nearest hundredth of a radian? RADIAN MODE!

$$\sec \theta = 1.8492$$

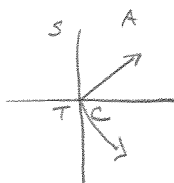
$$\cos \theta = \frac{1}{1.8492}$$

$$\theta = \cos^{-1}\left(\frac{1}{1.8492}\right)$$

$$\theta \approx 1.00$$

$$\theta' = 2\pi - \theta$$

$$\theta' \approx 5.28$$



46. What are the solution(s) of  $2\cos^2 \theta + 1 = -3\cos \theta$  if  $\theta$  is in the interval  $[0, 2\pi)$ ?



Prove the following identities.

$$47. \frac{\tan \theta}{\csc \theta} = \sec \theta - \cos \theta$$

$$48. \frac{\sin \theta}{\tan \theta} + \frac{\cos \theta}{\cot \theta} = \sin \theta + \cos \theta$$

Use reference triangles to find the exact value of each of the following expressions.

$$49. \sec \left[ \sin^{-1} \left( -\frac{5}{6} \right) \right] = \frac{6}{\sqrt{11}} = \frac{6\sqrt{11}}{11}$$



$$x^2 + y^2 = r^2$$

$$x^2 + (-5)^2 = 6^2$$

$$x^2 + 25 = 36$$

$$x^2 = 11$$

$$x = \pm\sqrt{11}$$

$$x = -\sqrt{11}$$

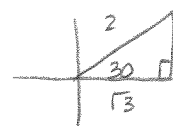
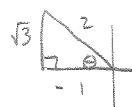
$$50. \cos^{-1} \left[ \sin \frac{2\pi}{3} \right] = \frac{\pi}{6}$$



$\sin \frac{2\pi}{3}$  is positive

$$\sin \frac{2\pi}{3} = \frac{\sqrt{3}}{2}$$

$$\cos^{-1} \left( \frac{\sqrt{3}}{2} \right) = \frac{\pi}{6}$$



What ANGLE has a cosine of  $\sqrt{3}/2$ ?

Solve the equations below on the interval  $0 \leq \theta < 2\pi$ .

$$51. \cos^2 \theta - 1 = 0$$

$$52. 2 \sin \theta + 3 = 4$$

**Conics:** (Chapter 9)

**Classify each conic section.**

53. $5x^2 - 3x - y + 20 = 0$ PARABOLA	54. $16y^2 - 25x^2 + 8y = 50$ HYPERBOLA	55. $12x^2 + 7y^2 - x + y = 8$ ELLIPISE	56. $x^2 + y^2 - 4x = 100$ CIRCLE
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**Find the standard form of the equation the conic given...**

<p>57. a parabola whose vertex is (5, 2) and focus is (3, 2). (h, k) (x, y)</p> <p>① <math>a = V - F</math> <math>a = 5 - 3</math> <math>a = 2</math></p> $(y - k)^2 = -4a(x - h)$ $(y - 2)^2 = -4(2)(x - 5)$ $(y - 2)^2 = -8(x - 5)$	<p>58. an ellipse whose center is at (3, 2), <math>a = 3c</math> and foci at (1, 2) and (5, 2).</p> <p><math>c = 2</math> <math>a = 3(2)</math> <math>a = 6</math></p> $b^2 = a^2 - c^2$ $b^2 = (6)^2 - 2^2$ $b^2 = 32$ $\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$ $\frac{(x - 3)^2}{36} + \frac{(y - 2)^2}{32} = 1$	<p>59. a hyperbola whose vertices are (0, 2) and (6, 2) and whose asymptotes are <math>y = \frac{2}{3}x</math> and <math>y = 4 - \frac{2}{3}x</math>.</p> <p>Since hyperbola opens left/right, Asymptotes are <math>y = \frac{b}{a}x</math> <math>\therefore b = 2, a = 3</math></p> $\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$ $\frac{(x - 3)^2}{9} - \frac{(y - 2)^2}{4} = 1$
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**Classify and graph each conic. Identify and label the essential elements to draw the graph.**

60.  $4x^2 + 25y^2 + 16x - 150y + 141 = 0$

61.  $y^2 - 4x^2 - 2y - 48x + 113 = 0$

60)  $4x^2 + 25y^2 + 16x - 150y + 141 = 0$

$4x^2 + 16x + 25y^2 - 150y = -141$

$4(x^2 + 4x + 4) + 25(y^2 - 6y + 9) = -141 + 16 + 225$

$4(x+2)^2 + 25(y-3)^2 = 100$

$\frac{(x+2)^2}{25} + \frac{(y-3)^2}{4} = 1$

ELLIPSE

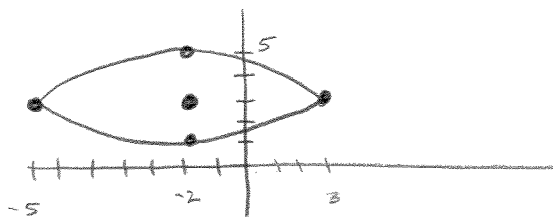
CENTER: (-2, 3)

$a^2 = 25 \Rightarrow a = 5$

$b^2 = 4 \Rightarrow b = 2$

MAJOR AXIS =  $2a \Rightarrow 10$

MINOR AXIS =  $2b \Rightarrow 4$



61)  $y^2 - 4x^2 - 2y - 48x + 113 = 0$

$y^2 - 2y - 4x^2 - 48x = -113$

$(y^2 - 2y + 1) - 4(x^2 + 12x + 36) = -113 + 1 - 144$

$(y-1)^2 - 4(x+6)^2 = -256$

$-\frac{(y-1)^2}{256} + \frac{(x+6)^2}{64} = 1$

$\frac{(x+6)^2}{64} - \frac{(y-1)^2}{256} = 1$

Hyperbola  
 $> <$

Center: (-6, 1)

$a^2 = 64 \Rightarrow a = 8$

$b^2 = 256 \Rightarrow b = 16$

Asymptotes:  $(y-k) = \pm \frac{b}{a}(x-h)$   
 $y-1 = \pm \frac{16}{8}(x+6)$   
 $y-1 = \pm 2(x+6)$

$y-1 = \pm 2(x+6)$

$y-1 = 2(x+6)$

$y-1 = 2x+12$

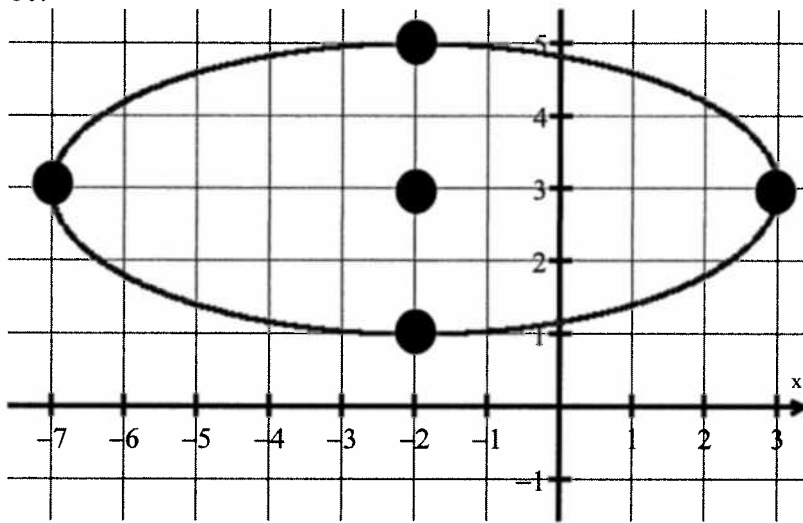
$y = 2x+13$

$y-1 = -2(x+6)$

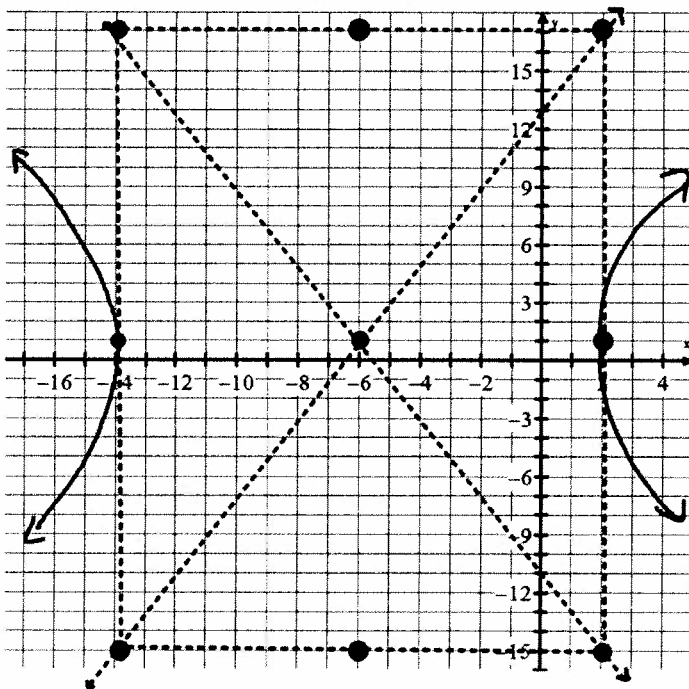
$y-1 = -2x-12$

$y = -2x-11$

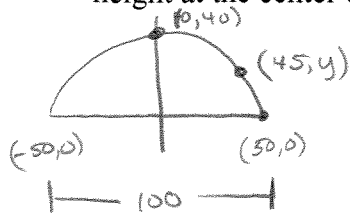
60.



61.



62. A semielliptical arch over a tunnel for a road through a mountain has a major axis of 100 feet and a height at the center of 40 feet. How high is the arch 5 feet from the edge of the tunnel?



$$a = 50$$

$$b = 40$$

$$(h, k) = (0, 0)$$

When  $x = 45$ ,  $y = \underline{\hspace{2cm}}$

$$\frac{x^2}{50^2} + \frac{y^2}{40^2} = 1$$

$$\frac{x^2}{2500} + \frac{y^2}{1600} = 1$$

At  $(45, y)$

$$\frac{45^2}{2500} + \frac{y^2}{1600} = 1$$

$$\frac{y^2}{1600} = 1 - \frac{2025}{2500}$$

$$y^2 = \left(1 - \frac{2025}{2500}\right) 1600$$

$$y = \pm \sqrt{\left(1 - \frac{2025}{2500}\right) (1600)}$$

$$y = \pm \sqrt{\left(\frac{2500 - 2025}{2500}\right) (1600)}$$

$$y = \pm \sqrt{\left(\frac{475}{2500}\right) (1600)}$$

$$y = \pm \sqrt{\left(\frac{19}{100}\right) (1600)}$$

$$y = \pm \sqrt{(19)(16)}$$

$$y = \pm 4\sqrt{19}$$

The arch is about 17.435 ft high 5 ft from the edge of the tunnel.

**Matrices:** (Chapter 10)

$$A = \begin{bmatrix} 5 & -1 \\ -2 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 7 \\ 4 & -6 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & -3 \\ 6 & 0 \\ 2 & 4 \end{bmatrix}$$

Find each product, if possible. NO calculators!

63.  $AB$

$$\begin{bmatrix} 5 & -1 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} 0 & 7 \\ 4 & -6 \end{bmatrix}$$

$$\begin{bmatrix} 5 \cdot 0 + (-1) \cdot 4 & 5 \cdot 7 + (-1) \cdot (-6) \\ -2 \cdot 0 + 3 \cdot 4 & -2 \cdot 7 + 3 \cdot (-6) \end{bmatrix}$$

$$\begin{bmatrix} -4 & 41 \\ 12 & -32 \end{bmatrix}$$

64.  $AC$

$$A_{2 \times 2} C_{3 \times 2}$$

No Solution

65.  $CA$

$$\begin{bmatrix} 1 & -3 \\ 6 & 0 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 5 & -1 \\ -2 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 \cdot 5 + (-3) \cdot (-2) & 1 \cdot (-1) + (-3) \cdot 3 \\ 6 \cdot 5 + 0 \cdot (-2) & 6 \cdot (-1) + 0 \cdot 3 \\ 2 \cdot 5 + 4 \cdot (-2) & 2 \cdot (-1) + 4 \cdot 3 \end{bmatrix}$$

$$\begin{bmatrix} 11 & -10 \\ 30 & -6 \\ 2 & 10 \end{bmatrix}$$

Solve each system using inverse matrices or row operations. Choose two different methods!

$$66. \begin{cases} 2x - 5y = 2 \\ 3x - 7y = 1 \end{cases}$$

$$A = \begin{bmatrix} 2 & -5 \\ 3 & -7 \end{bmatrix}$$

$$A^{-1} = \frac{1}{\det A} \begin{bmatrix} -7 & 5 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} -7 & 5 \\ -3 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -5 \\ 3 & -7 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \implies AX = B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -7 & 5 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} \implies X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -7 \cdot 2 + 5 \cdot 1 \\ -3 \cdot 2 + 2 \cdot 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -9 \\ -4 \end{bmatrix}$$

$$67. \begin{cases} 2x + 6z = -9 \\ 3x - 2y + 11z = -16 \\ 3x - y + 7z = -11 \end{cases}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3/4 \\ 0 \\ -5/4 \end{bmatrix}$$

68. The University of Georgia and Florida State University scored a total of 39 points during the 2003 Sugar Bowl. The points came from a total of 11 scoring plays, which were a combination of touchdowns, extra-point kicks, and field goals, worth 6, 1, and 3 points respectively. The same number of touchdowns and field goals were scored. How many touchdowns, extra-point kicks and field goals were scored during the game?

T = touchdowns  
E = extra points  
F = field goals

①  $6T + 1E + 3F = 39$

②  $T + E + F = 11$

③  $T = F$  SUBSTITUTE

$T + E + F = 11$        $6T + 1E + 3F = 39$

$T + E + T = 11$        $6T + 1E + 3T = 39$

$2T + E = 11$        $9T + E = 39$

$E = 11 - 2T$  SUB ↑

$9T + E = 39$

$9T + (11 - 2T) = 39$

$7T + 11 = 39$

$7T = 28$

$T = 4$

$\therefore F = 4$

$T + E + F = 11$

$4 + E + 4 = 11$

$E = 3$

69. The graph of a quadratic function in the form  $y = ax^2 + bx + c$  contains the points  $(-2, -2)$ ,  $(1, 7)$  and  $(3, -7)$ . What are the values for  $a$ ,  $b$ , and  $c$  in this function?

CALC

STAT - EDIT

L1	L2
-2	-2
1	7
3	-7

STAT - CALC - 5: QUAD REG

$y = ax^2 + bx + c$

$a = -2$

$b = 1$

$c = 8$

$y = -2x^2 + x + 8$

70. What is the average rate of change of  $f(x) = 2x^2 - 5x + 9$  from 1 to 3?

$$\begin{aligned} f(3) &= 2(3)^2 - 5(3) + 9 & f(1) &= 2(1)^2 - 5(1) + 9 \\ f(3) &= 18 - 15 + 9 & f(1) &= 2 - 5 + 9 \\ f(3) &= 12 & f(1) &= 6 \end{aligned}$$

$$\frac{f(3) - f(1)}{3 - 1} = \frac{12 - 6}{2} = \frac{6}{2} = 3$$

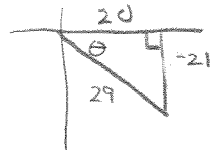
The average rate of change is 3.  
"slope"

Average Rate of Change:

72. For  $\theta$  in the interval  $[0, 2\pi)$ , what are the solutions of the equation below?

$$2\sin(3\theta) + 1 = 0$$

71. If the terminal side of  $\theta$  intersects the unit circle at  $P\left(\frac{20}{29}, -\frac{21}{29}\right)$ , what are the values of the six trigonometric functions?



$$\begin{aligned} \cos &= \frac{x}{r} \\ \sin &= \frac{y}{r} \end{aligned}$$

$$\sin \theta = \frac{-21}{29} \quad \cos \theta = \frac{20}{29} \quad \tan \theta = \frac{-21}{20}$$

$$\csc \theta = \frac{-29}{21} \quad \sec \theta = \frac{29}{20} \quad \cot \theta = \frac{-20}{21}$$

73. Find the *exact value* of the expression below.

$$\tan\left(\sin^{-1}\left(-\frac{4}{5}\right) + \cos^{-1}\frac{5}{13}\right)$$