

2008 SCORING GUIDELINES (Form B)

Question 3

Distance from the river's edge (feet)	0	8	14	22	24
Depth of the water (feet)	0	7	8	2	0

A scientist measures the depth of the Doe River at Picnic Point. The river is 24 feet wide at this location. The measurements are taken in a straight line perpendicular to the edge of the river. The data are shown in the table above. The velocity of the water at Picnic Point, in feet per minute, is modeled by $v(t) = 16 + 2\sin(\sqrt{t+10})$ for $0 \leq t \leq 120$ minutes.

- (a) Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate the area of the cross section of the river at Picnic Point, in square feet. Show the computations that lead to your answer.
- (b) The volumetric flow at a location along the river is the product of the cross-sectional area and the velocity of the water at that location. Use your approximation from part (a) to estimate the average value of the volumetric flow at Picnic Point, in cubic feet per minute, from $t = 0$ to $t = 120$ minutes.
- (c) The scientist proposes the function f , given by $f(x) = 8\sin\left(\frac{\pi x}{24}\right)$, as a model for the depth of the water, in feet, at Picnic Point x feet from the river's edge. Find the area of the cross section of the river at Picnic Point based on this model.
- (d) Recall that the volumetric flow is the product of the cross-sectional area and the velocity of the water at a location. To prevent flooding, water must be diverted if the average value of the volumetric flow at Picnic Point exceeds 2100 cubic feet per minute for a 20-minute period. Using your answer from part (c), find the average value of the volumetric flow during the time interval $40 \leq t \leq 60$ minutes. Does this value indicate that the water must be diverted?

$$(a) \frac{(0+7)}{2} \cdot 8 + \frac{(7+8)}{2} \cdot 6 + \frac{(8+2)}{2} \cdot 8 + \frac{(2+0)}{2} \cdot 2 = 115 \text{ ft}^2$$

$$(b) \frac{1}{120} \int_0^{120} 115v(t) dt = 1807.169 \text{ or } 1807.170 \text{ ft}^3/\text{min}$$

$$(c) \int_0^{24} 8\sin\left(\frac{\pi x}{24}\right) dx = 122.230 \text{ or } 122.231 \text{ ft}^2$$

(d) Let C be the cross-sectional area approximation from part (c). The average volumetric flow is

$$\frac{1}{20} \int_{40}^{60} C \cdot v(t) dt = 2181.912 \text{ or } 2181.913 \text{ ft}^3/\text{min}.$$

Yes, water must be diverted since the average volumetric flow for this 20-minute period exceeds 2100 ft^3/min .

1 : trapezoidal approximation

3 : $\left\{ \begin{array}{l} 1 : \text{limits and average value} \\ \quad \text{constant} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{array} \right.$

2 : $\left\{ \begin{array}{l} 1 : \text{integral} \\ 1 : \text{answer} \end{array} \right.$

3 : $\left\{ \begin{array}{l} 1 : \text{volumetric flow integral} \\ 1 : \text{average volumetric flow} \\ 1 : \text{answer with reason} \end{array} \right.$

**AP[®] CALCULUS AB
2009 SCORING GUIDELINES**

Question 5

x	2	3	5	8	13
$f(x)$	1	4	-2	3	6

Let f be a function that is twice differentiable for all real numbers. The table above gives values of f for selected points in the closed interval $2 \leq x \leq 13$.

- (a) Estimate $f'(4)$. Show the work that leads to your answer.
- (b) Evaluate $\int_2^{13} (3 - 5f'(x)) dx$. Show the work that leads to your answer.
- (c) Use a left Riemann sum with subintervals indicated by the data in the table to approximate $\int_2^{13} f(x) dx$. Show the work that leads to your answer.
- (d) Suppose $f'(5) = 3$ and $f''(x) < 0$ for all x in the closed interval $5 \leq x \leq 8$. Use the line tangent to the graph of f at $x = 5$ to show that $f(7) \leq 4$. Use the secant line for the graph of f on $5 \leq x \leq 8$ to show that $f(7) \geq \frac{4}{3}$.

(a) $f'(4) \approx \frac{f(5) - f(3)}{5 - 3} = -3$

(b) $\int_2^{13} (3 - 5f'(x)) dx = \int_2^{13} 3 dx - 5 \int_2^{13} f'(x) dx$
 $= 3(13 - 2) - 5(f(13) - f(2)) = 8$

(c) $\int_2^{13} f(x) dx \approx f(2)(3 - 2) + f(3)(5 - 3)$
 $+ f(5)(8 - 5) + f(8)(13 - 8) = 18$

- (d) An equation for the tangent line is $y = -2 + 3(x - 5)$.
 Since $f''(x) < 0$ for all x in the interval $5 \leq x \leq 8$, the line tangent to the graph of $y = f(x)$ at $x = 5$ lies above the graph for all x in the interval $5 < x \leq 8$.

Therefore, $f(7) \leq -2 + 3 \cdot 2 = 4$.

An equation for the secant line is $y = -2 + \frac{5}{3}(x - 5)$.

Since $f''(x) < 0$ for all x in the interval $5 \leq x \leq 8$, the secant line connecting $(5, f(5))$ and $(8, f(8))$ lies below the graph of $y = f(x)$ for all x in the interval $5 < x < 8$.

Therefore, $f(7) \geq -2 + \frac{5}{3} \cdot 2 = \frac{4}{3}$.

1 : answer

2 : $\left\{ \begin{array}{l} 1 : \text{uses Fundamental Theorem} \\ \quad \text{of Calculus} \\ 1 : \text{answer} \end{array} \right.$

2 : $\left\{ \begin{array}{l} 1 : \text{left Riemann sum} \\ 1 : \text{answer} \end{array} \right.$

4 : $\left\{ \begin{array}{l} 1 : \text{tangent line} \\ 1 : \text{shows } f(7) \leq 4 \\ 1 : \text{secant line} \\ 1 : \text{shows } f(7) \geq \frac{4}{3} \end{array} \right.$

2010 SCORING GUIDELINES

Question 2

t (hours)	0	2	5	7	8
$E(t)$ (hundreds of entries)	0	4	13	21	23

A zoo sponsored a one-day contest to name a new baby elephant. Zoo visitors deposited entries in a special box between noon ($t = 0$) and 8 P.M. ($t = 8$). The number of entries in the box t hours after noon is modeled by a differentiable function E for $0 \leq t \leq 8$. Values of $E(t)$, in hundreds of entries, at various times t are shown in the table above.

- (a) Use the data in the table to approximate the rate, in hundreds of entries per hour, at which entries were being deposited at time $t = 6$. Show the computations that lead to your answer.
- (b) Use a trapezoidal sum with the four subintervals given by the table to approximate the value of $\frac{1}{8} \int_0^8 E(t) dt$.
Using correct units, explain the meaning of $\frac{1}{8} \int_0^8 E(t) dt$ in terms of the number of entries.
- (c) At 8 P.M., volunteers began to process the entries. They processed the entries at a rate modeled by the function P , where $P(t) = t^3 - 30t^2 + 298t - 976$ hundreds of entries per hour for $8 \leq t \leq 12$. According to the model, how many entries had not yet been processed by midnight ($t = 12$)?
- (d) According to the model from part (c), at what time were the entries being processed most quickly? Justify your answer.

(a) $E'(6) \approx \frac{E(7) - E(5)}{7 - 5} = 4$ hundred entries per hour

(b) $\frac{1}{8} \int_0^8 E(t) dt \approx$
 $\frac{1}{8} \left(2 \cdot \frac{E(0) + E(2)}{2} + 3 \cdot \frac{E(2) + E(5)}{2} + 2 \cdot \frac{E(5) + E(7)}{2} + 1 \cdot \frac{E(7) + E(8)}{2} \right)$
 $= 10.687$ or 10.688

$\frac{1}{8} \int_0^8 E(t) dt$ is the average number of hundreds of entries in the box between noon and 8 P.M.

(c) $23 - \int_8^{12} P(t) dt = 23 - 16 = 7$ hundred entries

(d) $P'(t) = 0$ when $t = 9.183503$ and $t = 10.816497$.

t	$P(t)$
8	0
9.183503	5.088662
10.816497	2.911338
12	8

Entries are being processed most quickly at time $t = 12$.

1 : answer

3 : $\left\{ \begin{array}{l} 1 : \text{trapezoidal sum} \\ 1 : \text{approximation} \\ 1 : \text{meaning} \end{array} \right.$

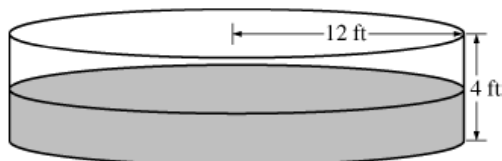
2 : $\left\{ \begin{array}{l} 1 : \text{integral} \\ 1 : \text{answer} \end{array} \right.$

3 : $\left\{ \begin{array}{l} 1 : \text{considers } P'(t) = 0 \\ 1 : \text{identifies candidates} \\ 1 : \text{answer with justification} \end{array} \right.$

AP[®] CALCULUS AB
2010 SCORING GUIDELINES (Form B)

Question 3

t	0	2	4	6	8	10	12
$P(t)$	0	46	53	57	60	62	63



The figure above shows an aboveground swimming pool in the shape of a cylinder with a radius of 12 feet and a height of 4 feet. The pool contains 1000 cubic feet of water at time $t = 0$. During the time interval $0 \leq t \leq 12$ hours, water is pumped into the pool at the rate $P(t)$ cubic feet per hour. The table above gives values of $P(t)$ for selected values of t . During the same time interval, water is leaking from the pool at the rate $R(t)$ cubic feet per hour, where $R(t) = 25e^{-0.05t}$. (Note: The volume V of a cylinder with radius r and height h is given by $V = \pi r^2 h$.)

- Use a midpoint Riemann sum with three subintervals of equal length to approximate the total amount of water that was pumped into the pool during the time interval $0 \leq t \leq 12$ hours. Show the computations that lead to your answer.
- Calculate the total amount of water that leaked out of the pool during the time interval $0 \leq t \leq 12$ hours.
- Use the results from parts (a) and (b) to approximate the volume of water in the pool at time $t = 12$ hours. Round your answer to the nearest cubic foot.
- Find the rate at which the volume of water in the pool is increasing at time $t = 8$ hours. How fast is the water level in the pool rising at $t = 8$ hours? Indicate units of measure in both answers.

(a) $\int_0^{12} P(t) dt \approx 46 \cdot 4 + 57 \cdot 4 + 62 \cdot 4 = 660 \text{ ft}^3$

2 : $\left\{ \begin{array}{l} 1 : \text{midpoint sum} \\ 1 : \text{answer} \end{array} \right.$

(b) $\int_0^{12} R(t) dt = 225.594 \text{ ft}^3$

2 : $\left\{ \begin{array}{l} 1 : \text{integral} \\ 1 : \text{answer} \end{array} \right.$

(c) $1000 + \int_0^{12} P(t) dt - \int_0^{12} R(t) dt = 1434.406$

1 : answer

At time $t = 12$ hours, the volume of water in the pool is approximately 1434 ft^3 .

(d) $V'(t) = P(t) - R(t)$
 $V'(8) = P(8) - R(8) = 60 - 25e^{-0.4} = 43.241$ or $43.242 \text{ ft}^3/\text{hr}$

4 : $\left\{ \begin{array}{l} 1 : V'(8) \\ 1 : \text{equation relating } \frac{dV}{dt} \text{ and } \frac{dh}{dt} \\ 1 : \frac{dh}{dt} \Big|_{t=8} \\ 1 : \text{units of } \text{ft}^3/\text{hr} \text{ and } \text{ft}/\text{hr} \end{array} \right.$

$$V = \pi(12)^2 h$$

$$\frac{dV}{dt} = 144\pi \frac{dh}{dt}$$

$$\frac{dh}{dt} \Big|_{t=8} = \frac{1}{144\pi} \cdot \frac{dV}{dt} \Big|_{t=8} = 0.095 \text{ or } 0.096 \text{ ft/hr}$$

2011 SCORING GUIDELINES

Question 2

t (minutes)	0	2	5	9	10
$H(t)$ (degrees Celsius)	66	60	52	44	43

As a pot of tea cools, the temperature of the tea is modeled by a differentiable function H for $0 \leq t \leq 10$, where time t is measured in minutes and temperature $H(t)$ is measured in degrees Celsius. Values of $H(t)$ at selected values of time t are shown in the table above.

- (a) Use the data in the table to approximate the rate at which the temperature of the tea is changing at time $t = 3.5$. Show the computations that lead to your answer.
- (b) Using correct units, explain the meaning of $\frac{1}{10} \int_0^{10} H(t) dt$ in the context of this problem. Use a trapezoidal sum with the four subintervals indicated by the table to estimate $\frac{1}{10} \int_0^{10} H(t) dt$.
- (c) Evaluate $\int_0^{10} H'(t) dt$. Using correct units, explain the meaning of the expression in the context of this problem.
- (d) At time $t = 0$, biscuits with temperature 100°C were removed from an oven. The temperature of the biscuits at time t is modeled by a differentiable function B for which it is known that $B'(t) = -13.84e^{-0.173t}$. Using the given models, at time $t = 10$, how much cooler are the biscuits than the tea?

(a)
$$H'(3.5) \approx \frac{H(5) - H(2)}{5 - 2}$$

$$= \frac{52 - 60}{3} = -2.666 \text{ or } -2.667 \text{ degrees Celsius per minute}$$

1 : answer

(b) $\frac{1}{10} \int_0^{10} H(t) dt$ is the average temperature of the tea, in degrees Celsius, over the 10 minutes.

$$\frac{1}{10} \int_0^{10} H(t) dt \approx \frac{1}{10} \left(2 \cdot \frac{66 + 60}{2} + 3 \cdot \frac{60 + 52}{2} + 4 \cdot \frac{52 + 44}{2} + 1 \cdot \frac{44 + 43}{2} \right)$$

$$= 52.95$$

3 : $\left\{ \begin{array}{l} 1 : \text{meaning of expression} \\ 1 : \text{trapezoidal sum} \\ 1 : \text{estimate} \end{array} \right.$

(c)
$$\int_0^{10} H'(t) dt = H(10) - H(0) = 43 - 66 = -23$$

The temperature of the tea drops 23 degrees Celsius from time $t = 0$ to time $t = 10$ minutes.

2 : $\left\{ \begin{array}{l} 1 : \text{value of integral} \\ 1 : \text{meaning of expression} \end{array} \right.$

(d)
$$B(10) = 100 + \int_0^{10} B'(t) dt = 34.18275; \quad H(10) - B(10) = 8.817$$

The biscuits are 8.817 degrees Celsius cooler than the tea.

3 : $\left\{ \begin{array}{l} 1 : \text{integrand} \\ 1 : \text{uses } B(0) = 100 \\ 1 : \text{answer} \end{array} \right.$