

Example 2 (graphical)

1. At $t = 4$ seconds, the particle is moving to the right because the velocity is positive.
2. The particle is moving to the left over the interval $5 < t \leq 9$ seconds because the velocity is negative.
3. The acceleration of the particle is negative because the velocity is decreasing, OR the acceleration is the slope of the velocity graph and the slope of the velocity graph at $t = 4$ is negative.
4. Average acceleration over the time interval can be found by dividing the change in velocity by the change in time:

$$\frac{v(4) - v(2)}{4 - 2} = \frac{6 - 9}{4 - 2} = -\frac{3 \text{ ft/sec}}{2 \text{ sec}} = -\frac{3 \text{ ft}}{2 \text{ sec}^2}$$

5. No such time is guaranteed. The Mean Value Theorem does not apply since the function is not differentiable at $t = 3$ due to the sharp turn in the graph. If students have not yet learned the MVT, you can slide a tangent line (toothpick or stick) along the graph to show that no such point exists where the slope of the tangent line would be equal to the slope of a secant line between $t = 2$ and $t = 4$.
6. The particle is farthest to the right at $t = 5$ seconds. Since the velocity is positive during the time interval $0 \leq t < 5$ seconds and negative during the time interval $5 < t \leq 9$ seconds, the particle moves to the right before time $t = 5$ seconds and moves to the left after that time. Therefore, it is farthest to the right at $t = 5$ seconds.

Example 3 (analytic)

1. The particle is moving to the right because $x'(0) = v(0) = 9$ which is positive.
2. At $t = 1$, the velocity of the particle is decreasing because $x''(1) = v'(1) = a(1) = -6$, and if the acceleration is negative then the velocity is decreasing.
3. The particle is moving to the left for all values of t where $v(t) < 0$. We have:

$$v(t) = x'(t) = 3t^2 - 12t + 9 < 0 \text{ for } 1 < t < 3.$$

4.

t	$x(t)$
0	11
1	15
3	11
5	31

The particle moves right 4 spaces, left 4 spaces and then right 20 spaces. Therefore, the particle has traveled a total of 28 units. The common error that students make is to calculate $x(5) - x(0) = 20$, which gives the displacement or net change in position, rather than the total distance traveled. Teachers should reinforce the difference between these two concepts every chance they get.