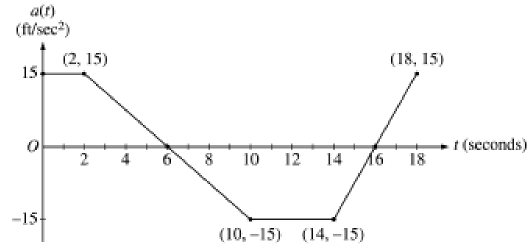
Question 3

A car is traveling on a straight road with velocity 55 ft/sec at time $t = 0$. For $0 \le t \le 18$ seconds, the car's acceleration $a(t)$, in ft/sec^2, is the piecewise linear function defined by the graph above.



(a) Is the velocity of the car increasing at $t = 2$ seconds? Why or why not?

(b) At what time in the interval $0 \le t \le 18$, other than $t = 0$, is the velocity of the car 55 ft/sec? Why?

(c) On the time interval $0 \le t \le 18$, what is the car's absolute maximum velocity, in ft/sec, and at what time does it occur? Justify your answer.

(d) At what times in the interval $0 \le t \le 18$, if any, is the car's velocity equal to zero? Justify your answer.

(a) Since $v'(2) = a(2)$ and $a(2) = 15 > 0$, the velocity is increasing at $t = 2$.	1 : answer and reason
(b) At time $t = 12$ because $v(12) - v(0) = \int_0^{12} a(t)\,dt = 0$.	2 : { 1 : $t = 12$ 1 : reason
(c) The absolute maximum velocity is 115 ft/sec at $t = 6$. The absolute maximum must occur at $t = 6$ or at an endpoint. $v(6) = 55 + \int_0^6 a(t)\,dt$ $= 55 + 2(15) + \frac{1}{2}(4)(15) = 115 > v(0)$ $\int_6^{18} a(t)\,dt < 0$ so $v(18) < v(6)$	4 : { 1 : $t = 6$ 1 : absolute maximum velocity 1 : identifies $t = 6$ and $t = 18$ as candidates or indicates that v increases, decreases, then increases 1 : eliminates $t = 18$
(d) The car's velocity is never equal to 0. The absolute minimum occurs at $t = 16$ where $v(16) = 115 + \int_6^{16} a(t)\,dt = 115 - 105 = 10 > 0$.	2 : { 1 : answer 1 : reason

Question 3

An object moves along the x-axis with initial position $x(0) = 2$. The velocity of the object at time $t \geq 0$ is given by $v(t) = \sin\left(\frac{\pi}{3}t\right)$.

(a) What is the acceleration of the object at time $t = 4$?

(b) Consider the following two statements.

Statement I: For $3 < t < 4.5$, the velocity of the object is decreasing.

Statement II: For $3 < t < 4.5$, the speed of the object is increasing.

Are either or both of these statements correct? For each statement provide a reason why it is correct or not correct.

(c) What is the total distance traveled by the object over the time interval $0 \leq t \leq 4$?

(d) What is the position of the object at time $t = 4$?

(a) $a(4) = v'(4) = \frac{\pi}{3}\cos\left(\frac{4\pi}{3}\right)$

$= -\frac{\pi}{6}$ or -0.523 or -0.524

1 : answer

(b) On $3 < t < 4.5$:

$a(t) = v'(t) = \frac{\pi}{3}\cos\left(\frac{\pi}{3}t\right) < 0$

Statement I is correct since $a(t) < 0$.

Statement II is correct since $v(t) < 0$ and $a(t) < 0$.

3 {
1 : I correct, with reason
1 : II correct
1 : reason for II

(c) Distance $= \int_0^4 |v(t)|\,dt = 2.387$

OR

$x(t) = -\frac{3}{\pi}\cos\left(\frac{\pi}{3}t\right) + \frac{3}{\pi} + 2$

$x(0) = 2$

$x(4) = 2 + \frac{9}{2\pi} = 3.43239$

$v(t) = 0$ when $t = 3$

$x(3) = \frac{6}{\pi} + 2 = 3.90986$

$|x(3) - x(0)| + |x(4) - x(3)| = \frac{15}{2\pi} = 2.387$

3 {
1 : { limits of 0 and 4 on an integral of $v(t)$ or $|v(t)|$
or
uses $x(0)$ and $x(4)$ to compute distance
1 : handles change of direction at student's turning point
1 : answer
0/1 if incorrect turning point or no turning point

(d) $x(4) = x(0) + \int_0^4 v(t)\,dt = 3.432$

OR

$x(t) = -\frac{3}{\pi}\cos\left(\frac{\pi}{3}t\right) + \frac{3}{\pi} + 2$

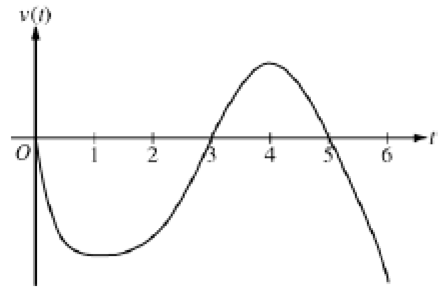
$x(4) = 2 + \frac{9}{2\pi} = 3.432$

2 {
1 : integral
1 : answer

OR

2 {
1 : $x(t) = -\frac{3}{\pi}\cos\left(\frac{\pi}{3}t\right) + C$
1 : answer
0/1 if no constant of integration

Question 4



Graph of v

A particle moves along the x-axis so that its velocity at time t, for $0 \le t \le 6$, is given by a differentiable function v whose graph is shown above. The velocity is 0 at $t = 0$, $t = 3$, and $t = 5$, and the graph has horizontal tangents at $t = 1$ and $t = 4$. The areas of the regions bounded by the t-axis and the graph of v on the intervals $[0, 3]$, $[3, 5]$, and $[5, 6]$ are 8, 3, and 2, respectively. At time $t = 0$, the particle is at $x = -2$.

(a) For $0 \le t \le 6$, find both the time and the position of the particle when the particle is farthest to the left. Justify your answer.

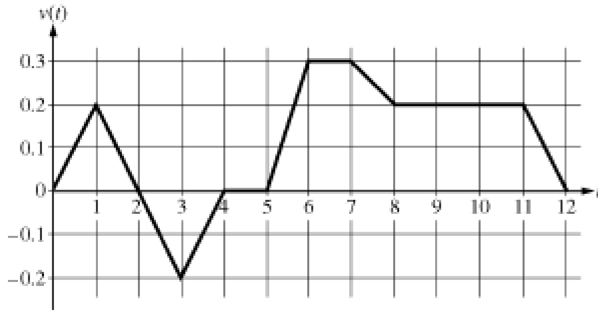
(b) For how many values of t, where $0 \le t \le 6$, is the particle at $x = -8$? Explain your reasoning.

(c) On the interval $2 < t < 3$, is the speed of the particle increasing or decreasing? Give a reason for your answer.

(d) During what time intervals, if any, is the acceleration of the particle negative? Justify your answer.

(a) Since $v(t) < 0$ for $0 < t < 3$ and $5 < t < 6$, and $v(t) > 0$ for $3 < t < 5$, we consider $t = 3$ and $t = 6$. $x(3) = -2 + \int_0^3 v(t)\,dt = -2 - 8 = -10$ $x(6) = -2 + \int_0^6 v(t)\,dt = -2 - 8 + 3 - 2 = -9$ Therefore, the particle is farthest left at time $t = 3$ when its position is $x(3) = -10$.	3 : { 1 : identifies $t = 3$ as a candidate 1 : considers $\int_0^6 v(t)\,dt$ 1 : conclusion
(b) The particle moves continuously and monotonically from $x(0) = -2$ to $x(3) = -10$. Similarly, the particle moves continuously and monotonically from $x(3) = -10$ to $x(5) = -7$ and also from $x(5) = -7$ to $x(6) = -9$. By the Intermediate Value Theorem, there are three values of t for which the particle is at $x(t) = -8$.	3 : { 1 : positions at $t = 3$, $t = 5$, and $t = 6$ 1 : description of motion 1 : conclusion
(c) The speed is decreasing on the interval $2 < t < 3$ since on this interval $v < 0$ and v is increasing.	1 : answer with reason
(d) The acceleration is negative on the intervals $0 < t < 1$ and $4 < t < 6$ since velocity is decreasing on these intervals.	2 : { 1 : answer 1 : justification

Question 1



Caren rides her bicycle along a straight road from home to school, starting at home at time $t = 0$ minutes and arriving at school at time $t = 12$ minutes. During the time interval $0 \le t \le 12$ minutes, her velocity $v(t)$, in miles per minute, is modeled by the piecewise-linear function whose graph is shown above.

(a) Find the acceleration of Caren's bicycle at time $t = 7.5$ minutes. Indicate units of measure.

(b) Using correct units, explain the meaning of $\int_0^{12} |v(t)|\,dt$ in terms of Caren's trip. Find the value of $\int_0^{12} |v(t)|\,dt$.

(c) Shortly after leaving home, Caren realizes she left her calculus homework at home, and she returns to get it. At what time does she turn around to go back home? Give a reason for your answer.

(d) Larry also rides his bicycle along a straight road from home to school in 12 minutes. His velocity is modeled by the function w given by $w(t) = \frac{\pi}{15}\sin\left(\frac{\pi}{12}t\right)$, where $w(t)$ is in miles per minute for $0 \le t \le 12$ minutes. Who lives closer to school: Caren or Larry? Show the work that leads to your answer.

(a) $a(7.5) = v'(7.5) = \frac{v(8) - v(7)}{8 - 7} = -0.1$ miles/minute^2	$2: \begin{cases} 1: \text{answer} \\ 1: \text{units} \end{cases}$
(b) $\int_0^{12} \lvert v(t)\rvert\,dt$ is the total distance, in miles, that Caren rode during the 12 minutes from $t = 0$ to $t = 12$. $\int_0^{12} \lvert v(t)\rvert\,dt = \int_0^2 v(t)\,dt - \int_2^4 v(t)\,dt + \int_4^{12} v(t)\,dt$ $= 0.2 + 0.2 + 1.4 = 1.8$ miles	$2: \begin{cases} 1: \text{meaning of integral} \\ 1: \text{value of integral} \end{cases}$
(c) Caren turns around to go back home at time $t = 2$ minutes. This is the time at which her velocity changes from positive to negative.	$2: \begin{cases} 1: \text{answer} \\ 1: \text{reason} \end{cases}$
(d) $\int_0^{12} w(t)\,dt = 1.6$; Larry lives 1.6 miles from school. $\int_0^{12} v(t)\,dt = 1.4$; Caren lives 1.4 miles from school. Therefore, Caren lives closer to school.	3 : { 2 : Larry's distance from school 1 : integral 1 : value 1 : Caren's distance from school and conclusion