

Pre-Calculus Q1 Cumulative Review

Complete on a Separate Sheet of Paper

Part 1:

For each of the functions below, answer the following questions:

$f(x) = x^2 + 4$	$g(x) = \sqrt{2x - 3} + 1$	$h(x) = \frac{2}{x}$
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1. Find the domain and range for each.

f(x)
 $d: \mathbb{R} (-\infty, \infty)$
 $R: [4, \infty)$

g(x)
 $d: [3/2, \infty)$
 $R: [1, \infty)$

h(x)
 $d: (-\infty, 0) \cup (0, \infty)$
 $r: (-\infty, 0) \cup (0, \infty)$

2. Find the inverse for each function. Are the inverses functions? \Rightarrow pass the horizontal line test?

f(x)
 $x = y^2 + 4$
 $f^{-1}(x) = \sqrt{x - 4}$
NO!

g(x)
 $x = \sqrt{2y - 3} + 1$
 $x - 1 = \sqrt{2y - 3}$
 $x^2 - 2x + 1 = 2y - 3$

h(x)
 $x = \frac{2}{y}$
 $yx = 2$
 $h^{-1}(x) = \frac{2}{x}$
yes!!

3. Find the domain and range for each inverse. $x^2 - 2x + 4 = 2y$

f(x)
 $d: [4, \infty)$
 $r: (-\infty, \infty)$

g(x)
 $d: [1, \infty)$

g^{-1}(x) = \frac{1}{2}x^2 - x + 2
Yes!!

h(x)
 $d \& r: (-\infty, 0) \cup (0, \infty)$

4. Determine whether each function is even, odd, or neither (both algebraically and graphically).

$f(x) = (-x)^2 + 4$
 $= x^2 + 4$
even!!

$g(x) = \sqrt{2(-x) - 3} + 1$
neither!!

$h(x) = \frac{2}{(-x)}$
odd!!

5. Which of the above functions are:

Bounded below? Increasing over their entire domain? A one-to-one function?

f(x)
g(x)

g(x)

g(x)
h(x)

6. Find $g(f(x))$
 $g(x^2 + 4) = \sqrt{2(x^2 + 4) - 3} + 1$
 $= \sqrt{2x^2 + 8 - 3} + 1$
 $= \sqrt{2x^2 + 5} + 1$

7. State the graph transformations from $y = \sqrt{x}$ to $g(x)$.

- ① \uparrow 1
- ② \rightarrow 3
- ③ horiz. shrink baf 2

- OR
- ① \rightarrow 3
 - ② horiz. shrink baf 2
 - ③ \uparrow 1

Part 2:

Use the following functions to evaluate key features of its graph:

$f(x) = \frac{x^2 - 3x - 10}{x^2 - 4}$	$g(x) = x^3 - 5x^2 + 2x + 8$	$h(x) = \frac{x^2 + 2x - 3}{x + 2}$
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$$\begin{array}{r|rr} -2 & 2 & -3 \\ \hline & \downarrow -2 & 0 \\ \hline & 1 & 0 & -3 \end{array}$$

8. Find all asymptotes and removable discontinuities for f(x) and h(x).

$$f(x) = \frac{(x-5)(x+2)}{(x-2)(x+2)}$$

$$h(x) = \frac{(x+3)(x-1)}{(x+2)}$$

V.A.: $x=2$ S.A.: n/a
H.A: $y=1$ R.D. $(-2, 7/4)$

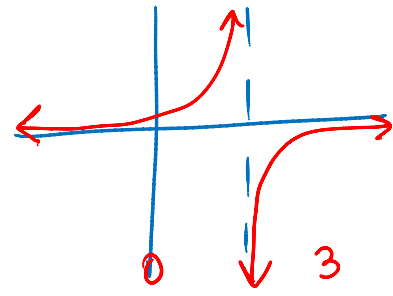
V.A: $x=-2$ S.A: $y=x$
H.A. n/a R.D: n/a

9. Find $\lim_{x \rightarrow -\infty} g(x)$, and $\lim_{x \rightarrow \infty} g(x)$

$$\lim_{x \rightarrow -\infty} g(x) = -\infty \quad \lim_{x \rightarrow \infty} g(x) = \infty$$

10. Find $\lim_{x \rightarrow 2^-} f(x)$, and $\lim_{x \rightarrow 2^+} f(x)$

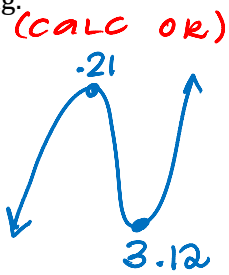
$$\begin{array}{cc} \downarrow & \downarrow \\ \infty & -\infty \end{array}$$



11. State the intervals where g(x) is increasing / decreasing.

$$I: (-\infty, .21) \cup (3.12, \infty)$$

$$D: (.21, 3.12)$$



Part 3:

Answer the following:

12. Write the equation of a line given $f(3) = 1$ and $f(-2) = 6$.

$$y - 1 = -1(x - 3) \quad (3, 1) \quad (-2, 6)$$

OR

$$y - 6 = -1(x + 2)$$

OR

$$y = -x + 4$$

$$M = \frac{5}{-5} = -1$$

$$y = mx + b$$

$$1 = -1(3) + b$$

$$1 = -3 + b$$

$$4 = b$$

13. Write the equation of a parabola whose vertex is at $(-2, 1)$ and contains $(-4, -7)$.

$$y = a(x - h)^2 + k$$

$$-7 = a(-4 + 2)^2 + 1$$

$$-8 = a(4)$$

$$-2 = a$$

$$y = -2(x + 2)^2 + 1$$

14. Solve: $\sqrt{5x+1} = x-1$

$$5x+1 = x^2 - 2x + 1$$

$$0 = x^2 - 7x$$

$$0 = x(x-7)$$

$x = \cancel{0}, 7$
 ↑
 extraneous

15. Write $y = (2x^2 - 8x) + 5$ in VERTEX form.

$$y = 2(x^2 - 4x + 4) + 5 - 8$$

$$y = 2(x-2)^2 - 3$$

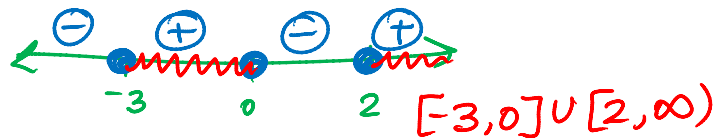
16. Write the equation of a polynomial whose zeros include 2 and $3+2i$.

$$p(x) = (x-2)(x-3-2i)(x-3+2i)$$

17. Solve $x^3 + x^2 - 6x \geq 0$ algebraically and graphically.

$$x(x^2 + x - 6) \geq 0$$

$$x(x+3)(x-2) \geq 0$$



18. Find ALL the zeros for $y = x^4 - x^3 - 3x^2 + 17x - 30$. **CALC OK**

$x = -3, 2$ (zeros on calc)

$$x = -3, 2, 1+2i, 1-2i$$

$$\begin{array}{r|rrrrr} -3 & 1 & -1 & -3 & 17 & -30 \\ & \downarrow & -3 & 12 & -27 & 30 \\ \hline & 1 & -4 & 9 & -10 & 0 \end{array}$$

$$x^2 - 2x + 5 = 0$$

$$x = \frac{2 \pm \sqrt{4 - 4(5)}}{2}$$

$$\begin{array}{r|rrrr} 2 & 1 & -4 & 9 & -10 \\ & \downarrow & 2 & -4 & 10 \\ \hline & 1 & -2 & 5 & 0 \end{array}$$

$$x = \frac{2 \pm \sqrt{-16}}{2}$$

$$x = \frac{2 \pm 4i}{2} \Rightarrow 1 \pm 2i$$

****Also Know: Box Problems, Zeros/Multiplicity, Graphing a "Piece-Wise" function, Imaginary numbers, etc.****