AP® Precalculus Exam

SECTION II: Free Response, Questions

DO NOT OPEN THIS BOOKLET OR BREAK THE SEALS ON PART B UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time
1 hour
Number of Questions
4
Percent of Total Score
37.5%
Writing Instrument
Either pencil or pen with black or dark blue ink
Weight
The questions are weighted equally, but the parts of a question are not necessarily given equal weight.

Part A

Number of Questions
2
Time
30 minutes
Electronic Device
Graphing calculator required
Percent of Section II Score
50%

Part B

Number of Questions
2
Time
30 minutes
Electronic Device
None allowed
Percent of Section II Score
50%

Instructions

The questions for Section II are printed in this booklet. Do not break the seals on Part B until you are told to do so. You may use the pages in this orange booklet for scratch work, but you must write your answers in the separate Section II: Free Response booklet. **No credit will be given for any work written in this orange booklet**. In the Free Response booklet, write your solution to each part of each question in the space provided for that part. Write clearly and legibly. Cross out any errors you make; erased or crossed-out work will not be scored.

- Manage your time carefully. As you begin each part, you may wish to look over both questions for that part before starting to work on them. You are encouraged to use the allotted time to respond to all parts of all questions.
- Show all of your work. Your work will be scored on the correctness and completeness of your responses, including your supporting work and answers. Answers without supporting work may not receive credit in cases where supporting work is requested.
- During Part A, work only on questions 1 and 2. You are expected to use your graphing calculator for tasks such as producing graphs and tables, evaluating functions, solving equations, and performing computations.
- **For Part A, your calculator must be in radian mode**. Avoid rounding intermediate computations on the way to the final result. Unless otherwise specified, any decimal approximations reported in your work should be accurate to three places after the decimal point.
- For Part A, it may be helpful to use your graphing calculator to store information such as computed values for constants, functions you are working with, solutions to equations, and any intermediate values. Computations with the graphing calculator that use the stored information help to maintain as much precision as possible and ensure the desired accuracy in final answers.
- During Part B, questions 3 and 4, no calculator is allowed. Carefully read the instructions provided with the questions. You may continue to work on questions 1 and 2 without the use of a calculator.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

PRECALCULUS

SECTION II, Part A

Time—30 minutes

2 Questions

A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

GO ON TO THE NEXT PAGE.

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x	1	2	3	4	5
$f(x)$	1	3	5	3	1

1. The domain of f consists of the five real numbers 1, 2, 3, 4, and 5. The table defines the function f for these values. The function g is given by $g(x)=2\ln x$.

(A) (i) The function h is defined by $h(x)=(g\circ f)(x)=g(f(x))$. Find the value of $h(4)$ as a decimal approximation, or indicate that it is not defined.

(ii) Find all values of x for which $f(x)=3$, or indicate there are no such values.

(B) (i) Find all values of x, as decimal approximations, for which $g(x)=3$, or indicate there are no such values.

(ii) Determine the end behavior of g as x increases without bound. Express your answer using the mathematical notation of a limit.

(C) (i) Determine if f has an inverse function.

(ii) Give a reason for your answer based on the definition of a function and the table of values of $f(x)$.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

GO ON TO THE NEXT PAGE.

2. An ecologist began studying a certain type of plant species in a wetlands area in 2013. In 2015 ($t = 2$), there were 59 plants. In 2021 ($t = 8$), there were 118 plants.

The number of plants in this species can be modeled by the function P given by $P(t) = ab^t$, where $P(t)$ is the number of plants during year t, and t is the number of years since 2013.

(A) (i) Use the given data to write two equations that can be used to find the values for constants a and b in the expression for $P(t)$.

(ii) Find the values for a and b as decimal approximations.

(B) (i) Use the given data to find the average rate of change of the number of plants, in plants per year, from $t = 2$ to $t = 8$ years. Express your answer as a decimal approximation. Show the computations that lead to your answer.

(ii) Use the average rate of change found in (i) to estimate the number of plants for $t = 10$ years. Show the work that leads to your answer.

(iii) The average rate of change found in (i) can be used to estimate the number of plants during year t for $t > 10$ years. Will these estimates, found using the average rate of change, be less than or greater than the number of plants predicted by the model P during year t for $t > 10$ years? Explain your reasoning.

(C) For which t-value, $t = 6$ years or $t = 20$ years, should the ecologist have more confidence in when using the model P ? Give a reason for your answer in the context of the problem.

Write your responses to this question only on the designated pages in the separate Free Response booklet.
Write your solution to each part in the space provided for that part.

END OF PART A

IF YOU FINISH BEFORE TIME IS CALLED,
YOU MAY CHECK YOUR WORK ON PART A ONLY.

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

PRECALCULUS

SECTION II, Part B

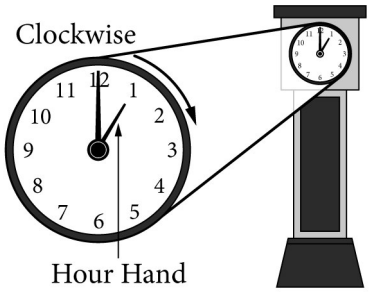
Time—30 minutes

2 Questions

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.

DO NOT BREAK THE SEALS UNTIL YOU ARE TOLD TO DO SO.

NO CALCULATOR ALLOWED

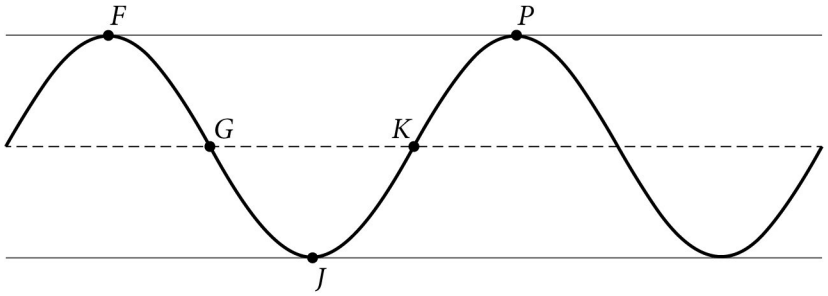


Note: Figure not drawn to scale.

3. The figure shows a clock standing on a level floor with a close-up view of the clock face. The clock face has a 10-centimeter-long moving hour hand. The center of the clock face is 200 centimeters from the floor. At time $t = 0$ hours, the hour hand is pointing directly up to the 12. The next time the hour hand points directly up to the 12 is at time $t = 12$ hours. As the hour hand moves, the distance of the endpoint of the hour hand from the floor periodically decreases and increases.

The sinusoidal function h models the distance, in centimeters, of the endpoint of the hour hand from the floor as a function of time t in hours.

(A) The graph of h and its dashed midline for two full cycles is shown. Five points, F, G, J, K, and P, are labeled on the graph. No scale is indicated, and no axes are presented.
Determine possible coordinates $(t, h(t))$ for the five points: F, G, J, K, and P.



(B) The function h can be written in the form $h(t) = a\sin(b(t+c)) + d$. Find values of constants a, b, c, and d.

NO CALCULATOR ALLOWED

(C) Refer to the graph of h in part (A). The t-coordinate of J is t_1, and the t-coordinate of K is t_2.

(i) On the interval (t_1, t_2), which of the following is true about h ?

a. h is positive and increasing.

b. h is positive and decreasing.

c. h is negative and increasing.

d. h is negative and decreasing.

(ii) Describe how the rate of change of h is changing on the interval (t_1, t_2).

Write your responses to this question only on the designated pages in the separate Free Response booklet.

Write your solution to each part in the space provided for that part.

NO CALCULATOR ALLOWED

4. Directions:

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians.
- Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator.
- Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x+3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3+\ln 5$ should be rewritten in equivalent forms.
- For each part of the question, show the work that leads to your answers.

(A) The functions g and h are given by

$$g(x)=\log_4(2x)$$
$$h(x)=\frac{\left(e^x\right)^5}{e^{1/4}}.$$

(i) Solve $g(x)=3$ for values of x in the domain of g.

(ii) Solve $h(x)=e^{1/2}$ for values of x in the domain of h.

(B) The functions j and k are given by

$$j(x)=\log_{10}(x+1)-5\log_{10}(2-x)+\log_{10}3$$
$$k(x)=\sec x-\cos x.$$

(i) Rewrite $j(x)$ as a single logarithm base 10 without negative exponents in any part of the expression. Your result should be of the form $\log_{10}$ (expression).

(ii) Rewrite $k(x)$ as a product involving $\tan x$ and $\sin x$ and no other trigonometric functions.

(C) The function m is given by

$$m(x)=2\tan^{-1}\left(\sqrt{3}\pi x\right).$$

Find all input values in the domain of m that yield an output value of $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$.

Write your responses to this question only on the designated pages in the separate Free Response booklet.
Write your solution to each part in the space provided for that part.

STOP
END OF EXAM