Parametric

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2012 SCORING GUIDELINES

Question 2

For $t \geq 0$, a particle is moving along a curve so that its position at time t is $(x(t), y(t))$. At time $t = 2$, the particle is at position $(1, 5)$. It is known that $\frac{dx}{dt} = \frac{\sqrt{t+2}}{e^t}$ and $\frac{dy}{dt} = \sin^2 t$.

(a) Is the horizontal movement of the particle to the left or to the right at time $t = 2$? Explain your answer. Find the slope of the path of the particle at time $t = 2$.

(b) Find the x-coordinate of the particle's position at time $t = 4$.

(c) Find the speed of the particle at time $t = 4$. Find the acceleration vector of the particle at time $t = 4$.

(d) Find the distance traveled by the particle from time $t = 2$ to $t = 4$.

(a) $\left.\frac{dx}{dt}\right|_{t=2} = \frac{2}{e^2}$

Because $\left.\frac{dx}{dt}\right|_{t=2} > 0$, the particle is moving to the right at time $t = 2$.

$\left.\frac{dy}{dx}\right|_{t=2} = \frac{\left.dy/dt\right|_{t=2}}{\left.dx/dt\right|_{t=2}} = 3.055$ (or 3.054)

$3 : \begin{cases} 1 : \text{moving to the right with reason} \\ 1 : \text{considers } \frac{dy/dt}{dx/dt} \\ 1 : \text{slope at } t = 2 \end{cases}$

(b) $x(4) = 1 + \int_2^4 \frac{\sqrt{t+2}}{e^t}\, dt = 1.253$ (or 1.252)

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(c) Speed $= \sqrt{(x'(4))^2 + (y'(4))^2} = 0.575$ (or 0.574)

Acceleration $= \langle x''(4), y''(4) \rangle$
$= \langle -0.041, 0.989 \rangle$

$2 : \begin{cases} 1 : \text{speed} \\ 1 : \text{acceleration} \end{cases}$

(d) Distance $= \int_2^4 \sqrt{(x'(t))^2 + (y'(t))^2}\, dt$
$= 0.651$ (or 0.650)

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

Question 1

At time t, a particle moving in the xy-plane is at position $(x(t), y(t))$, where $x(t)$ and $y(t)$ are not explicitly given. For $t \ge 0$, $\frac{dx}{dt} = 4t + 1$ and $\frac{dy}{dt} = \sin(t^2)$. At time $t = 0$, $x(0) = 0$ and $y(0) = -4$.

(a) Find the speed of the particle at time $t = 3$, and find the acceleration vector of the particle at time $t = 3$.

(b) Find the slope of the line tangent to the path of the particle at time $t = 3$.

(c) Find the position of the particle at time $t = 3$.

(d) Find the total distance traveled by the particle over the time interval $0 \le t \le 3$.

(a) Speed $= \sqrt{(x'(3))^2 + (y'(3))^2} = 13.006$ or 13.007

Acceleration $= \langle x''(3), y''(3) \rangle$
$= \langle 4, -5.466 \rangle$ or $\langle 4, -5.467 \rangle$

$2: \begin{cases} 1: \text{speed} \\ 1: \text{acceleration} \end{cases}$

(b) Slope $= \frac{y'(3)}{x'(3)} = 0.031$ or 0.032

$1:$ answer

(c) $x(3) = 0 + \int_0^3 \frac{dx}{dt}\,dt = 21$

$y(3) = -4 + \int_0^3 \frac{dy}{dt}\,dt = -3.226$

At time $t = 3$, the particle is at position $(21, -3.226)$.

$4: \begin{cases} 2: x\text{-coordinate} \\ \quad 1: \text{integral} \\ \quad 1: \text{answer} \\ 2: y\text{-coordinate} \\ \quad 1: \text{integral} \\ \quad 1: \text{answer} \end{cases}$

(d) Distance $= \int_0^3 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt = 21.091$

$2: \begin{cases} 1: \text{integral} \\ 1: \text{answer} \end{cases}$

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Question 3

A particle is moving along a curve so that its position at time t is $(x(t), y(t))$, where $x(t) = t^2 - 4t + 8$ and $y(t)$ is not explicitly given. Both x and y are measured in meters, and t is measured in seconds. It is known that $\frac{dy}{dt} = te^{t-3} - 1$.

(a) Find the speed of the particle at time $t = 3$ seconds.

(b) Find the total distance traveled by the particle for $0 \le t \le 4$ seconds.

(c) Find the time t, $0 \le t \le 4$, when the line tangent to the path of the particle is horizontal. Is the direction of motion of the particle toward the left or toward the right at that time? Give a reason for your answer.

(d) There is a point with x-coordinate 5 through which the particle passes twice. Find each of the following.
  (i) The two values of t when that occurs
  (ii) The slopes of the lines tangent to the particle's path at that point
  (iii) The y-coordinate of that point, given $y(2) = 3 + \frac{1}{e}$

(a) Speed $= \sqrt{(x'(3))^2 + (y'(3))^2} = 2.828$ meters per second

1 : answer

(b) $x'(t) = 2t - 4$

Distance $= \int_0^4 \sqrt{(2t-4)^2 + \left(te^{t-3} - 1\right)^2}\, dt = 11.587$ or 11.588 meters

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(c) $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = 0$ when $te^{t-3} - 1 = 0$ and $2t - 4 \ne 0$

This occurs at $t = 2.20794$.

Since $x'(2.20794) > 0$, the particle is moving toward the right at time $t = 2.207$ or 2.208.

$3 : \begin{cases} 1 : \text{considers } \frac{dy}{dx} = 0 \\ 1 : t = 2.207 \text{ or } 2.208 \\ 1 : \text{direction of motion with reason} \end{cases}$

(d) $x(t) = 5$ at $t = 1$ and $t = 3$

At time $t = 1$, the slope is $\left.\frac{dy}{dx}\right|_{t=1} = \left.\frac{dy/dt}{dx/dt}\right|_{t=1} = 0.432$.

At time $t = 3$, the slope is $\left.\frac{dy}{dx}\right|_{t=3} = \left.\frac{dy/dt}{dx/dt}\right|_{t=3} = 1$.

$y(1) = y(3) = 3 + \frac{1}{e} + \int_2^3 \frac{dy}{dt}\, dt = 4$

$3 : \begin{cases} 1 : t = 1 \text{ and } t = 3 \\ 1 : \text{slopes} \\ 1 : y\text{-coordinate} \end{cases}$

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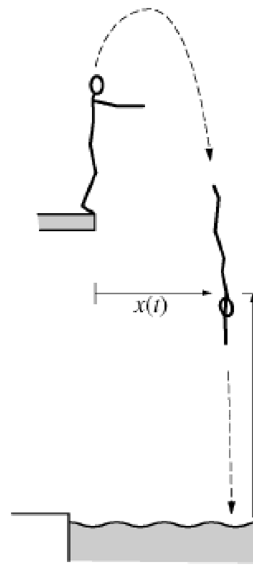
Question 3

A diver leaps from the edge of a diving platform into a pool below. The figure above shows the initial position of the diver and her position at a later time. At time t seconds after she leaps, the horizontal distance from the front edge of the platform to the diver's shoulders is given by $x(t)$, and the vertical distance from the water surface to her shoulders is given by $y(t)$, where $x(t)$ and $y(t)$ are measured in meters. Suppose that the diver's shoulders are 11.4 meters above the water when she makes her leap and that

$$\frac{dx}{dt} = 0.8 \text{ and } \frac{dy}{dt} = 3.6 - 9.8t,$$

for $0 \le t \le A$, where A is the time that the diver's shoulders enter the water.



Note: Figure not drawn to scale.

(a) Find the maximum vertical distance from the water surface to the diver's shoulders.

(b) Find A, the time that the diver's shoulders enter the water.

(c) Find the total distance traveled by the diver's shoulders from the time she leaps from the platform until the time her shoulders enter the water.

(d) Find the angle θ, $0 < \theta < \frac{\pi}{2}$, between the path of the diver and the water at the instant the diver's shoulders enter the water.

(a) $\frac{dy}{dt} = 0$ only when $t = 0.36735$. Let $b = 0.36735$.

The maximum vertical distance from the water surface to the diver's shoulders is

$$y(b) = 11.4 + \int_0^b \frac{dy}{dt}\, dt = 12.061 \text{ meters.}$$

Alternatively, $y(t) = 11.4 + 3.6t - 4.9t^2$, so $y(b) = 12.061$ meters.

3 : { 1 : considers $\frac{dy}{dt} = 0$; 1 : integral or $y(t)$; 1 : answer }

(b) $y(A) = 11.4 + \int_0^A \frac{dy}{dt}\, dt = 11.4 + 3.6A - 4.9A^2 = 0$ when $A = 1.936$ seconds.

2 : { 1 : equation; 1 : answer }

(c) $\int_0^A \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = 12.946$ meters

2 : { 1 : integral; 1 : answer }

(d) At time A, $\frac{dy}{dx} = \left.\frac{dy/dt}{dx/dt}\right|_{t=A} = -19.21913$.

The angle between the path of the diver and the water is $\tan^{-1}(19.21913) = 1.518$ or 1.519.

2 : { 1 : $\frac{dy}{dx}$ at time A; 1 : answer }

Question 3

An object moving along a curve in the xy-plane is at position $(x(t), y(t))$ at time t, where

$$\frac{dx}{dt} = \sin^{-1}\left(1 - 2e^{-t}\right) \text{ and } \frac{dy}{dt} = \frac{4t}{1 + t^3}$$

for $t \geq 0$. At time $t = 2$, the object is at the point $(6, -3)$. (Note: $\sin^{-1} x = \arcsin x$)

(a) Find the acceleration vector and the speed of the object at time $t = 2$.

(b) The curve has a vertical tangent line at one point. At what time t is the object at this point?

(c) Let $m(t)$ denote the slope of the line tangent to the curve at the point $(x(t), y(t))$. Write an expression for $m(t)$ in terms of t and use it to evaluate $\lim_{t \to \infty} m(t)$.

(d) The graph of the curve has a horizontal asymptote $y = c$. Write, but do not evaluate, an expression involving an improper integral that represents this value c.

(a) $a(2) = \langle 0.395 \text{ or } 0.396, -0.741 \text{ or } -0.740 \rangle$ Speed $= \sqrt{x'(2)^2 + y'(2)^2} = 1.207$ or 1.208	$2: \begin{cases} 1: \text{acceleration} \\ 1: \text{speed} \end{cases}$
(b) $\sin^{-1}\left(1 - 2e^{-t}\right) = 0$ $1 - 2e^{-t} = 0$ $t = \ln 2 = 0.693$ and $\frac{dy}{dt} \neq 0$ when $t = \ln 2$	$2: \begin{cases} 1: x'(t) = 0 \\ 1: \text{answer} \end{cases}$
(c) $m(t) = \frac{4t}{1 + t^3} \cdot \frac{1}{\sin^{-1}\left(1 - 2e^{-t}\right)}$ $\lim_{t \to \infty} m(t) = \lim_{t \to \infty} \left(\frac{4t}{1 + t^3} \cdot \frac{1}{\sin^{-1}\left(1 - 2e^{-t}\right)} \right)$ $= 0\left(\frac{1}{\sin^{-1}(1)} \right) = 0$	$2: \begin{cases} 1: m(t) \\ 1: \text{limit value} \end{cases}$
(d) Since $\lim_{t \to \infty} x(t) = \infty$, $c = \lim_{t \to \infty} y(t) = -3 + \int_2^\infty \frac{4t}{1 + t^3}\, dt$	$3: \begin{cases} 1\text{: integrand} \\ 1\text{: limits} \\ 1\text{: initial value consistent} \\ \quad \text{with lower limit} \end{cases}$

Question 3

An object moving along a curve in the xy-plane has position $(x(t), y(t))$ at time $t \geq 0$ with $\frac{dx}{dt} = 3 + \cos\left(t^2\right)$. The derivative $\frac{dy}{dt}$ is not explicitly given. At time $t = 2$, the object is at position $(1, 8)$.

(a) Find the x-coordinate of the position of the object at time $t = 4$.

(b) At time $t = 2$, the value of $\frac{dy}{dt}$ is -7. Write an equation for the line tangent to the curve at the point $(x(2), y(2))$.

(c) Find the speed of the object at time $t = 2$.

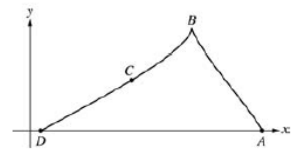
(d) For $t \geq 3$, the line tangent to the curve at $(x(t), y(t))$ has a slope of $2t + 1$. Find the acceleration vector of the object at time $t = 4$.

(a) $x(4) = x(2) + \int_2^4 \left(3 + \cos\left(t^2\right)\right) dt$ $= 1 + \int_2^4 \left(3 + \cos\left(t^2\right)\right) dt = 7.132$ or 7.133	3 : { 1 : $\int_2^4 \left(3 + \cos\left(t^2\right)\right) dt$ 1 : handles initial condition 1 : answer
(b) $\left.\frac{dy}{dx}\right\|_{t=2} = \left.\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right\|_{t=2} = \frac{-7}{3 + \cos 4} = -2.983$	2 : { 1 : finds $\left.\frac{dy}{dx}\right\|_{t=2}$ 1 : equation
(c) The speed of the object at time $t = 2$ is $\sqrt{(x'(2))^2 + (y'(2))^2} = 7.382$ or 7.383.	1 : answer
(d) $x''(4) = 2.303$ $y'(t) = \frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} = (2t + 1)\left(3 + \cos\left(t^2\right)\right)$ $y''(4) = 24.813$ or 24.814 The acceleration vector at $t = 4$ is $\langle 2.303, 24.813 \rangle$ or $\langle 2.303, 24.814 \rangle$.	3 : { 1 : $x''(4)$ 1 : $\frac{dy}{dt}$ 1 : answer

Question 2

A particle starts at point A on the positive x-axis at time $t = 0$ and travels along the curve from A to B to C to D, as shown above. The coordinates of the particle's position $\left(x(t),\, y(t)\right)$ are differentiable functions of t, where

$x'(t) = \dfrac{dx}{dt} = -9\cos\left(\dfrac{\pi t}{6}\right)\sin\left(\dfrac{\pi\sqrt{t+1}}{2}\right)$ and $y'(t) = \dfrac{dy}{dt}$ is not explicitly given.



At time $t = 9$, the particle reaches its final position at point D on the positive x-axis.

(a) At point C, is $\dfrac{dy}{dt}$ positive? At point C, is $\dfrac{dx}{dt}$ positive? Give a reason for each answer.

(b) The slope of the curve is undefined at point B. At what time t is the particle at point B?

(c) The line tangent to the curve at the point $\left(x(8),\, y(8)\right)$ has equation $y = \dfrac{5}{9}x - 2$. Find the velocity vector and the speed of the particle at this point.

(d) How far apart are points A and D, the initial and final positions, respectively, of the particle?

(a) At point C, $\dfrac{dy}{dt}$ is not positive because $y(t)$ is decreasing along the arc BD as t increases.

At point C, $\dfrac{dx}{dt}$ is not positive because $x(t)$ is decreasing along the arc BD as t increases.

$2: \begin{cases} 1: \dfrac{dy}{dt} \text{ not positive with reason} \\ 1: \dfrac{dx}{dt} \text{ not positive with reason} \end{cases}$

(b) $\dfrac{dx}{dt} = 0$; $\cos\left(\dfrac{\pi t}{6}\right) = 0$ or $\sin\left(\dfrac{\pi\sqrt{t+1}}{2}\right) = 0$

$\dfrac{\pi t}{6} = \dfrac{\pi}{2}$ or $\dfrac{\pi\sqrt{t+1}}{2} = \pi$; $t = 3$ for both.

Particle is at point B at $t = 3$.

$2: \begin{cases} 1: \text{sets } \dfrac{dx}{dt} = 0 \\ 1: t = 3 \end{cases}$

(c) $x'(8) = -9\cos\left(\dfrac{4\pi}{3}\right)\sin\left(\dfrac{3\pi}{2}\right) = -\dfrac{9}{2}$

$\dfrac{y'(8)}{x'(8)} = \dfrac{dy}{dx} = \dfrac{5}{9}$

$y'(8) = \dfrac{5}{9}x'(8) = -\dfrac{5}{2}$

The velocity vector is $< -4.5, -2.5 >$.

Speed $= \sqrt{4.5^2 + 2.5^2} = 5.147$ or 5.148

$3: \begin{cases} 1: x'(8) \\ 1: y'(8) \\ 1: \text{speed} \end{cases}$

(d) $x(9) - x(0) = \displaystyle\int_0^9 x'(t)\,dt$

$= -39.255$

The initial and final positions are 39.255 apart.

$2: \begin{cases} 1: \text{integral} \\ 1: \text{answer} \end{cases}$

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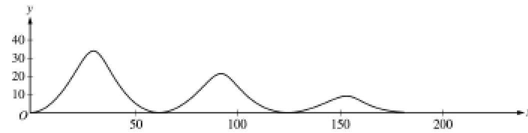
Question 3



The figure above shows the path traveled by a roller coaster car over the time interval $0 \le t \le 18$ seconds. The position of the car at time t seconds can be modeled parametrically by $x(t) = 10t + 4\sin t$, $y(t) = (20 - t)(1 - \cos t)$, where x and y are measured in meters. The derivatives of these functions are given by

$x'(t) = 10 + 4\cos t$, $y'(t) = (20 - t)\sin t + \cos t - 1$.

(a) Find the slope of the path at time $t = 2$. Show the computations that lead to your answer.

(b) Find the acceleration vector of the car at the time when the car's horizontal position is $x = 140$.

(c) Find the time t at which the car is at its maximum height, and find the speed, in m/sec, of the car at this time.

(d) For $0 < t < 18$, there are two times at which the car is at ground level ($y = 0$). Find these two times and write an expression that gives the average speed, in m/sec, of the car between these two times. Do not evaluate the expression.

(a) Slope $= \left.\dfrac{dy}{dx}\right|_{t=2} = \dfrac{y'(2)}{x'(2)} = \dfrac{18\sin 2 + \cos 2 - 1}{10 + 4\cos 2}$

$= 1.793$ or 1.794

1 : answer using $\dfrac{dy}{dx} = \dfrac{dy}{dt} \Big/ \dfrac{dx}{dt}$

(b) $x(t) = 10t + 4\sin t = 140$; $t_0 = 13.647083$

$x''(t_0) = -3.529$, $y''(t_0) = 1.225$ or 1.226

Acceleration vector is $\langle -3.529, 1.225 \rangle$

or $\langle -3.529, 1.226 \rangle$

2 $\begin{cases} 1: \text{identifies acceleration vector as derivative of velocity vector} \\ 1: \text{computes acceleration vector when } x = 140 \end{cases}$

(c) $y'(t) = (20 - t)\sin t + \cos t - 1 = 0$

$t_1 = 3.023$ or 3.024 at maximum height

Speed $= \sqrt{\left(x'(t_1)\right)^2 + \left(y'(t_1)\right)^2} = \left|x'(t_1)\right|$

$= 6.027$ or 6.028

3 $\begin{cases} 1: \text{sets } y'(t) = 0 \\ 1: \text{selects first } t > 0 \\ 1: \text{speed} \end{cases}$

(d) $y(t) = 0$ when $t = 2\pi$ and $t = 4\pi$

Average speed $= \dfrac{1}{2\pi}\displaystyle\int_{2\pi}^{4\pi} \sqrt{\left(x'(t)\right)^2 + \left(y'(t)\right)^2}\, dt$

$= \dfrac{1}{2\pi}\displaystyle\int_{2\pi}^{4\pi} \sqrt{(10 + 4\cos t)^2 + ((20 - t)\sin t + \cos t - 1)^2}\, dt$

3 $\begin{cases} 1: t = 2\pi, t = 4\pi \\ 1: \text{limits and constant} \\ 1: \text{integrand} \end{cases}$

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Question 1

An object moving along a curve in the xy-plane has position $(x(t), y(t))$ at time t with

$$\frac{dx}{dt} = \cos(t^3) \text{ and } \frac{dy}{dt} = 3\sin(t^2)$$

for $0 \le t \le 3$. At time $t = 2$, the object is at position (4,5).

(a) Write an equation for the line tangent to the curve at (4,5).

(b) Find the speed of the object at time $t = 2$.

(c) Find the total distance traveled by the object over the time interval $0 \le t \le 1$.

(d) Find the position of the object at time $t = 3$.

(a) $\frac{dy}{dx} = \frac{3\sin(t^2)}{\cos(t^3)}$ $\left.\frac{dy}{dx}\right\rvert_{t=2} = \frac{3\sin(2^2)}{\cos(2^3)} = 15.604$ $y - 5 = 15.604(x - 4)$	1 : tangent line
(b) Speed $= \sqrt{\cos^2(8) + 9\sin^2(4)} = 2.275$	1 : answer
(c) Distance $= \int_0^1 \sqrt{\cos^2(t^3) + 9\sin^2(t^2)}\, dt$ $= 1.458$	3 : { 2 : distance integral $<-1>$ each integrand error $<-1>$ error in limits 1 : answer
(d) $x(3) = 4 + \int_2^3 \cos(t^3)\, dt = 3.953$ or 3.954 $y(3) = 5 + \int_2^3 3\sin(t^2)\, dt = 4.906$	4 : { 1 : definite integral for x 1 : answer for $x(3)$ 1 : definite integral for y 1 : answer for $y(3)$

AP Calculus BC–4 2000

A moving particle has position $(x(t), y(t))$ at time t. The position of the particle at time $t = 1$ is (2,6) and the velocity vector at any time $t > 0$ is given by $\left(1 - \frac{1}{t^2}, 2 + \frac{1}{t^2}\right)$.

(a) Find the acceleration vector at time $t = 3$.

(b) Find the position of the particle at time $t = 3$.

(c) For what time $t > 0$ does the line tangent to the path of the particle at $(x(t), y(t))$ have a slope of 8?

(d) The particle approaches a line as $t \to \infty$. Find the slope of this line. Show the work that leads to your conclusion.

(a) acceleration vector $= (x''(t), y''(t)) = \left(\frac{2}{t^3}, -\frac{2}{t^3}\right)$ $(x''(3), y''(3)) = \left(\frac{2}{27}, -\frac{2}{27}\right)$	2 { 1 : components of acceleration vector as a function of t 1 : acceleration vector at $t = 3$
(b) $(x(t), y(t)) = \left(t + \frac{1}{t} + C_1,\ 2t - \frac{1}{t} + C_2\right)$ $(2,6) = (x(1), y(1)) = (2 + C_1, 1 + C_2)$ $C_1 = 0,\ C_2 = 5$ $(x(3), y(3)) = \left(3 + \frac{1}{3}, 6 - \frac{1}{3} + 5\right) = \left(\frac{10}{3}, \frac{32}{3}\right)$	3 { 1 : antidifferentiation 1 : uses initial condition at $t = 1$ 1 : position at $t = 3$ Note: max 1/3 [1–0–0] if no constants of integration
(c) $\frac{dy}{dx} = \frac{2 + \frac{1}{t^2}}{1 - \frac{1}{t^2}} = 8$ $2 + \frac{1}{t^2} = 8\left(1 - \frac{1}{t^2}\right); \quad t^2 = \frac{9}{6}$ $t = \sqrt{\frac{3}{2}}$	2 { 1 : $\frac{dy}{dx} = 8$ as equation in t 1 : solution for t
(d) $\lim_{t\to\infty} \frac{dy}{dx} = \lim_{t\to\infty} \frac{2 + \frac{1}{t^2}}{1 - \frac{1}{t^2}} = 2$ – or – Since $x(t) \to \infty$ as $t \to \infty$, the slope of the line is $\lim_{t\to\infty} \frac{y(t)}{x(t)} = \lim_{t\to\infty} \frac{2t - \frac{1}{t} + 5}{t + \frac{1}{t}} = 2$	2 { 1 : considers limit of $\frac{dy}{dx}$ or $\frac{y(t)}{x(t)}$ 1 : answer Note: 0/2 if no consideration of limit