AP Calculus AB Summer Assignment

This assignment is designed to make the transition to AP Calculus AB a smooth one. You will be practicing skills you have acquired in earlier math classes. It is imperative that you are able to do most of these problems with ease, **without the use of a calculator.The entire assignment is due on the first day of class.**There will be a test on this material during the first week of school in the Fall.

Directions: In order to receive credit all work must be completed **in pencil**. Remember that we care about process, so show your work carefully on lined paper. **This should include: problem numbers, calculations done neatly, sketches drawn carefully, and labeled answers (circled, underlined, or boxed). Graphs should be done on graph paper.** Organize your work into columns and work down, not across the paper. **No Calculators! No Calculator, unless directed to do so! When appropriate, answers should be in simplified rational or fractional form. Last thing: ABSOLUTELY NO CALCULATORS, unless directed to do so!**

Let $f(x) = x^2$, $g(x) = 2x + 5$, and $h(x) = x^2 - 1$. Evaluate:

1. $f\left[g(x-1)\right]$

2. $g\left[h\left(x^3\right)\right]$

3. $g\left(f\left(a+h\right)\right)$

Find $\dfrac{f(x+h) - f(x)}{h}$ for each given function $f(x)$. Let h represent some constant.

4. $f(x) = 9x + 3$

5. $f(x) = x^2 - 10$

6. $f(x) = x^2 - 3x + 2$

Find the x and y intercepts for each equation:

7. $y = x\sqrt{16 - x^2}$

8. $y = x^2\left(x-1\right)^2$

9. $y = \dfrac{x - \dfrac{1}{2}}{x+2}$

The domain is the set of all the possible values for x (everything you can input), and the range is the set of all possible values for y (every possible output.)

Find the domain and range of each function. Write your answer in interval notation.

10. $f(x) = x^2 - 5$

11. $f(x) = -\sqrt{x+3}$

12. $f(x) = 3\sin x$

13. $f(x) = \dfrac{x}{x^2 - 9}$

14. $f(x) = \dfrac{x-1}{x^2 - x - 6}$

15. $f(x) = \dfrac{3x^2}{x^2 + 1}$

16. $f(x) = \dfrac{2}{x-1}$

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17. Write the equation of a line with a slope of 3 and a y-intercept of 5 in slope-intercept form.

18. Write the equation of a line passing through the point $(0,5)$ with a slope of $\dfrac{2}{3}$.

19. Write the equation of a line passing through the points $(-3,6)$ and $(1,2)$.

20. Write the equation of a line passing through the points $(-4,2)$ with a slope of zero.

21. Write the equation of a line passing through the points $(-4,2)$ with an undefined slope.

22. Write the equation of a line passing through the points $(2,8)$ and parallel to the line $y = \dfrac{5}{6}x - 1.$

23. Write the equation of a line passing through the point $(2,-3)$ normal to $y = \dfrac{5}{6}x - 1.$

Find the value without a calculator.

24. $\tan\left(\arccos\dfrac{2}{3}\right)$ 25. $\sec\left(\sin^{-1}\dfrac{12}{13}\right)$ 26. $\sin\left(\arctan\dfrac{12}{5}\right)$ 27. $\sin\left(\sin^{-1}\dfrac{7}{8}\right)$

The vertical asymptote occurs where the function is undefined (where the denominator is equal to zero).

Determine the vertical asymptote(s) for the function.

28. $f(x)=\dfrac{1}{x^2}$ 29. $f(x)=\dfrac{x^2}{x^2-4}$ 30. $f(x)=\dfrac{2+x}{x^2(1-x)}$

Determining the horizontal asymptote.
Case 1: The degree of the numerator is less than the degree of the denominator. The function has a horizontal asymptote at $y=0$.
Case 2: The degree of the numerator is equal to the degree of the denominator. The function has a horizontal asymptote equal to the ration of the leading coefficients.
Case 3: The degree of the numerator is greater than the degree of the denominator. The function has no horizontal asymptote.

Determine the horizontal asymptote.

31. $f(x)=\dfrac{x^2-4x+10}{x^3+x-5}$ 32. $f(x)=\dfrac{5x^3-2x^2+8}{4x-3x^3+5}$ 33. $f(x)=\dfrac{4x^5}{x^2-7}$

Solve:

34. $x^2-6x+5=0$ 35. $3x^3-4x=1$ 36. $2e^{-t}-2te^{-t}=0$

37. $\sin^2 x-\sin x=0$, for $0\le x\le\dfrac{3\pi}{2}$

Simplify:

38. $\dfrac{x^2-a^2}{x^4-a^4}$ 39. $\sqrt{x}(x-1)^2$ 40. $\dfrac{\dfrac{x+h}{x+h+2}-\dfrac{x}{x^2}}{h}$ 41. $\dfrac{\dfrac{1}{(x+h)^2}-\dfrac{1}{x^2}}{h}$

42. $\left(\dfrac{1}{x}+\dfrac{1}{y}\right)(x+y)^{-1}$ 43. $\left(x^{-1}+y^{-1}\right)^{-1}$ 44. $\dfrac{x^2-2x-8}{x^3+x^2-2x}$

Simplify completely. Leave no variables in the denominator.

45. $\dfrac{(2a^2)^3}{b}$ 46. $\dfrac{a\left(\dfrac{2}{b}\right)}{\dfrac{3}{a}}$ 47. $\dfrac{ab-a}{b^2-b}$ 48. $\dfrac{a^{-1}}{\left(b^{-1}\right)\sqrt{a}}$ 49. $\left(\dfrac{a^{\frac{2}{3}}}{b^{\frac{1}{2}}}\right)^2\left(\dfrac{b^{\frac{3}{2}}}{a^{\frac{1}{2}}}\right)$

Solve for the indicated variable:

50. Solve for **z** in terms of **x** and **y**: $3xy^2z+y^3+2x^2yz+2xy^2=0$

51. Solve for **z** in terms of **y**: $z-z\sin y=1$

52. Solve for **y** in terms of **x**: $\dfrac{1}{2}e^{2y}=x^3+2$

53. If $f(x) = \dfrac{1}{3}x^3$, then find k so that $f(k) - f(-3) = 0$.

54. If $f(x) = 3x^2 - 6x + 12$, then find the minimum value of f.

55. If $f(x) = \dfrac{2}{9}\left(x^3 + 1\right)^{\frac{3}{2}}$, then find the value of $\dfrac{1}{2-0}\left[f(2) - f(0)\right]$.

56. If $f(x) = 3\ln x + C$, and $f(e) = 2$, then find the value of C.

57. If $f(x) = \dfrac{x^2}{2}\ln x - \dfrac{3}{4}x^2 + C$, and $f(1) = 6$, then find the value of C.

58. If $f(x) = \sec^2\left(2x\right)$, then find $f\left(\dfrac{\pi}{6}\right)$.

59. If $f(x) = e^x - \dfrac{x^2}{2}$, then find the value of $f(4) - f(0)$.

60. If $f(x) = \dfrac{3}{2}\sqrt{1 + \tan x} \cdot \sec^2 x$, then find $f(0)$.

61. If $f(x) = \dfrac{3}{5}\left(2x\right)^{\frac{5}{3}}$, then find $f(4)$.

62. If $f(x) = -x - \dfrac{x^2}{2} + \dfrac{x^3}{3}$, then find $f(3)$.

63. If $f(x) = -\dfrac{x}{2} - x^2 + \dfrac{x^3}{3}$, then find $f(4) - f(2)$.

64. If $f(x) = \begin{cases} 2x^2, & \text{when } x < 0 \\ 3x, & \text{when } 0 \le x \le 1 \\ |6 - x|, & \text{when } x > 1 \end{cases}$, then find $f(-3) + f(1)$.

Multiply:

65. $2^3 \cdot 2^m$

66. $\left(a^k + 3\right)\left(a^k - 2\right)$

67. $x^{\frac{5}{8}}\left(x^{\frac{3}{8}} - 10x^{\frac{11}{8}}\right)$

Without using a calculator, evaluate the following: (Imperative that you can do this with ease w/o a calculator quickly!)

68. $\cos 210°$ 　　69. $\sin\dfrac{5\pi}{4}$ 　　70. $\tan^{-1}\left(-1\right)$ 　　71. $\sin^{-1}\left(-1\right)$ 　　72. $\cos\dfrac{9\pi}{4}$

73. $\sin^{-1}\left(\dfrac{\sqrt{3}}{2}\right)$ 　　74. $\tan\dfrac{7\pi}{6}$ 　　75. $\cos^{-1}\left(-1\right)$ 　　76. $\sec\left(\dfrac{\pi}{6}\right)$

77. Divide $x^5 - x^4 + x^3 + 2x^2 - x + 4$ by $x^3 + 1$.
78. The equation $12x^3 - 23x^2 - 3x + 2 = 0$ has a solution $x = 2$. Find all other solutions.
79. Find the point of intersection of the lines: $3x - y - 7 = 0$ and $x + 5y + 3 = 0$.
Write as a single equation in **x** and **y**: 80. $\begin{cases} x = t+1 \\ y = t^2 - t \end{cases}$ 81. $\begin{cases} x = \sqrt[3]{t} - 1 \\ y = t^2 - t \end{cases}$ 82. $\begin{cases} x = \sin t \\ y = \cos t \end{cases}$

Graph the following. You should recall how they are graphed from Pre-Cal. Use a calculator only if you absolutely need to:

83. $y = \ln x$ 84. $y = \dfrac{|x|}{x}$ 85. $y = e^x$ 86. $y = e^{-x}$ 87. $y = 2^x$ 88. $y = \sqrt{x}$ 89. $y = \begin{cases} \sqrt{x+1}, & 0 \le x \le 3 \\ 5 - x, & 3 < x \le 5 \end{cases}$

If all of the above was not enough, you MUST be able to crunch fractions in a timely manner! Do the following as efficiently as possible <u>without the use of a calculator</u>!

Given: $f(x) = \dfrac{1}{2}x^2 + 2x - 3$ and $g(x) = \dfrac{2}{3}x^3 - \dfrac{1}{2}x^2 + \dfrac{1}{5}x + 7$, Find: 90. $f\left(\dfrac{1}{5}\right)$ 91. $g(3)$ 92. $g\left(\dfrac{1}{2}\right) - f\left(\dfrac{1}{2}\right)$

Using the Graphing Calculator: These are the only problems that you should use a calculator for!!!!!!!!!!

93. Solve $6e^{2x} = 18x^2$.
94. How many solutions exist for the equation $\dfrac{\cos^2 x}{x} - \dfrac{1}{5} = 0$ on the interval $0 < x < 10$?
95. If $f(x) = 2xe^{2x}$, then find the absolute minimum of **f** and the range of **f**.
96. If $f(x) = 80 - 10\cos\left(\dfrac{\pi x}{12}\right)$, then (a) graph f over the interval $0 \le x \le 24$, and (b) find x-values such that $f(x) \ge 78$.
97. For $0 \le t \le 10$, how many values of t satisfy the equation $\cos t = -2e^{-2t}$? Do not find specific values.
98. Solve: $1 - 3e^{-0.2\sqrt{x}} = 0$.
99. Find the range of $f(x) = xe^{1-x}$.
100. Find the domain of $y = \ln\left(\dfrac{x}{x-1}\right)$.

I will post an answer key on <u>qayumi.weebly.com</u> so that you can check your answers. Make sure that you show work for any problem that requires work to be shown.