

### Calculus BC HW # 3

For each of the following questions enter a T (true) or F (false), as appropriate, on the line at the beginning of the statement.

\_\_\_\_\_ (1) The function  $\sin(x)$  is defined for all real values of  $x$ .

\_\_\_\_\_ (2) For all real values  $x$ , it follows that  $\sin^2(x) = \cos^2(x) - 1$ .

\_\_\_\_\_ (3) For  $-\frac{\pi}{2} < x < \frac{\pi}{2}$ , it follows that  $\tan(x) \cos(x) = \sin(x)$ .

\_\_\_\_\_ (4) For  $0 < x < \frac{\pi}{2}$ , it follows that  $\tan(x) \cot(x) = 1$ .

\_\_\_\_\_ (5) For  $-\frac{\pi}{2} < x < \frac{\pi}{2}$ , it follows that  $\cos(x) \sec(x) = 1$ .

\_\_\_\_\_ (6) For  $-\frac{\pi}{2} < x < \frac{\pi}{2}$ , it follows that  $\tan^2(x) - \sec^2(x) = 1$ .

\_\_\_\_\_ (7) For  $0 < x < \pi$ , it follows that  $\csc(x) \sin(x) = 1$ .

\_\_\_\_\_ (8) For all  $x \in \mathbb{R}$ , it follows that  $0 \leq \sin(x) \leq 1$ .

\_\_\_\_\_ (9) The function  $\sin^{-1}(x)$  is defined for all real values of  $x$ .

\_\_\_\_\_ (10) For  $-\frac{\pi}{2} < x < \frac{\pi}{2}$ , it follows that  $1 + \tan(x) > 1$ .

\_\_\_\_\_ (11) For  $-1 \leq x \leq 1$ , it follows that  $\sin(\sin^{-1}(x)) = x$ .

\_\_\_\_\_ (12) The function  $\tan^{-1}(x)$  is defined for all real values of  $x$ .

\_\_\_\_\_ (13) For  $-\frac{\pi}{2} < x < \frac{\pi}{2}$ , it follows that  $1 + \tan^2(x) = \sec^2(x)$ .

\_\_\_\_\_ (14) For all real values of  $x$ , it follows that  $\sin(x + \frac{\pi}{2}) = \cos(x)$ .

\_\_\_\_\_ (15) For all real values of  $x$ , it follows that  $\sin(x) + \cos(x) = \cos(2x)$ .

\_\_\_\_\_ (16) For all real values of  $x$ , it follows that  $2 \sin(x) \cos(x) = \sin(2x)$ .

\_\_\_\_\_ (17) For all real values of  $x$ , it follows that  $1 - 2 \sin^2(x) = \cos(2x)$ .

\_\_\_\_\_ (18) For all real values of  $x$ , it follows that  $2 \cos^2(x) - 1 = \cos(2x)$ .

\_\_\_\_\_ (19) For all real values of  $\theta$ , it follows that  $\cos\left(\frac{\theta}{2}\right) = \frac{\cos(\theta)+1}{2}$ .

For each of the following questions enter a T (true) or F (false), as appropriate, on the line at the beginning of the statement.

\_\_\_\_\_ (7) Even though the function  $e^x$  is defined for all real numbers  $x$ , its inverse function  $\ln x$ , is defined for only positive real numbers.

\_\_\_\_\_ (8) The function  $e^x$ ,  $x \in \mathbb{R}$ , never assumes a negative value.

\_\_\_\_\_ (9) The function  $\log_2 x$ ,  $x \in \mathbb{R}$ , never assumes the value 0.

\_\_\_\_\_ (10) Even though the function  $\ln e^x$  is defined for all real numbers  $x$ , the function  $e^{\ln x}$  is not defined for all real numbers  $x$ .

\_\_\_\_\_ (11) The graphs of the functions  $e^x$ ,  $-\infty < x < \infty$ , and  $\ln x$ ,  $0 < x < \infty$ , never intersect each other.

\_\_\_\_\_ (12)  $(4)^\pi = 3^{\pi \log_3 4}$ .

\_\_\_\_\_ (13) Let  $x$  denote a positive real number. If  $a$  and  $b$  denote real numbers, then  $(x^a)(x^b) = x^{ab}$ .

\_\_\_\_\_ (14) Let  $x$  denote a positive real number. If  $a$  and  $b$  denote real numbers, then  $(x^a)^b = x^{ab}$ .

\_\_\_\_\_ (15) Let  $x$  denote a positive real number. If  $a$  denotes a real number, then  $x^{-a} = (\frac{1}{x})^a$ .

\_\_\_\_\_ (16) Let  $x$  and  $y$  denote positive real numbers with  $x < y$ . If  $a$  denotes a real number, then  $x^a < y^a$ .

\_\_\_\_\_ (17) Let  $x$  denote a positive real number. If  $a$  and  $b$  denote real numbers, then  $x^a + x^b = x^{a+b}$ .