1.

Express the number $\frac{1}{2} + \frac{1}{3+\frac{3}{4}}$ in simplest form (as an irreducible quotient of two integers).

2.

The expression

$$\frac{x+1-\frac{1}{x+1}}{\frac{1}{x+1}},$$

such that $x \neq -1$, can be written in the form $ax^2 + bx$. Express a and b as integers.

3.

The value of $\sin\left(\frac{\pi}{4} + \frac{\pi}{6}\right)$ can be written in the form $\frac{\sqrt{2}(\sqrt{3}+1)}{\alpha}$. Express α as an integer.

4.

The value of $\cos\left(\frac{\pi}{4} + \frac{\pi}{6}\right)$ can be written in the form $\frac{\sqrt{2}(\sqrt{3}-1)}{\alpha}$. Express α as an integer.

5.

For some real number θ, $0 \leq \theta \leq 2\pi$, it is known that $\sin(\theta) = \frac{\sqrt{3}}{2}$ and $\tan(\theta) = -\sqrt{3}$. The value of θ can be written in the form $\alpha\pi$. Express α as an irreducible quotient of two integers.

6.

For some real number θ, it is known that $\sin(\theta) = \frac{1}{4}$ and $\cos(\theta) > 0$. The value of $\sin(2\theta)$ can be written in the form $\frac{\sqrt{15}}{\alpha}$. Express α as an integer.

7.

For some real number x, $\cos(x) = \frac{1}{3}$ and $\sin\left(\frac{x}{2}\right) > 0$. The value of $\sin\left(\frac{x}{2}\right)$ can be written as $\frac{1}{\sqrt{\alpha}}$. Express α as an integer.

8.

For some real number x, $\sin(x) = \frac{4}{5}$. Express the value of $\sin(-x)$ as an irreducible quotient of two integers.

9.

The value of $\sin(\theta)$ is $\frac{1}{5}$. The value of $\cos^2(\theta)$ can be written in the form $6(\alpha^2)$. Express the value of α^2 in simplest form (as an irreducible quotient of two integers).

10.

Express as an integer the value of $3^{\log_2 4}$.

11.

Express as an integer the value of $\frac{1}{2}\log_{\frac{1}{4}} 80 - \frac{1}{2}\log_{\frac{1}{4}} 5$.

12.

Express as an integer the value of α that satisfies $(2^{\log_4 2})(4^{\log_2 4}) = 2^{\alpha/2}$.

For each of the following questions enter a T (true) or F (false), as appropriate, on the line at the beginning of the statement.

_____(1) For positive real numbers a and b,

$$\log_{10} a + \log_{10} b = \log_{10}(ab).$$

_____(2) For positive real numbers a and b,

$$\log_3 a - \log_3 b = \log_3 \frac{a}{b}.$$

_____(3) For positive real numbers a and b,

$$(\ln a)(\ln b) = \ln(ab).$$

_____(4) If $\ln a < 0$ for some real number a, then $a < 0$.

_____(5) For positive real numbers a and b,

$$\frac{\log_{10} b}{a} = \log_{10}(b^{1/a}).$$

_____(6) $(3)^\pi = e^{\pi \ln 3}$.