1.1 Solving Equations

Essential Question: How do you solve an equation in one variable?

⊘ Explore   Solving Equations by Guess-and-Check or by Working Backward

An **equation** is a mathematical sentence that uses the equal sign $=$ to show two expressions are equivalent. The expressions can be numbers, variables, constants, or combinations thereof.

There are many ways to solve an equation. One way is by using a method called *guess-and-check*. A guess-and-check method involves guessing a value for the variable in an equation and checking to see if it is the solution by substituting the value in the equation. If the resulting equation is a true statement, then the value you guessed is the solution of the equation. If the equation is not a true statement, then you adjust the value of your guess and try again, continuing until you find the solution.

Another way to solve an equation is by *working backward*. In this method, you begin at the end and work backward toward the beginning.

Solve the equation $x - 6 = 4$ using both methods.

Use the *guess-and-check* method to find the solution of the equation $x - 6 = 4$.

(A) Guess 11 for x.

$$x - 6 = 4$$

$$\boxed{} - 6 \overset{?}{=} 4$$

$$\boxed{} \overset{?}{=} 4$$

Is 11 the solution

of $x - 6 = 4$? _____

(B) The value 11 is too high.

Guess 10 for x.

$$x - 6 = 4$$

$$\boxed{} - 6 \overset{?}{=} 4$$

$$\boxed{} \overset{?}{=} 4$$

Is 10 the solution

of $x - 6 = 4$? _____

(C) Use the *working backward* method to find the solution of the equation $x - 6 = 4$.

$4 + 6 = \boxed{}$ Is this the value of x before taking away 6?

$\boxed{} - 6 \overset{?}{=} 4$ _____

Reflect

1. **Discussion** Which method of solving do you think is more efficient? Explain your answer.

_____

_____

🎸 Explain 1 — Solving One-Variable Two-Step Equations

A **solution of an equation** is a value for the variable that makes the equation true. To determine the solution of an equation, you will use the Properties of Equality.

Properties of Equality		
Words	**Numbers**	**Algebra**
Addition Property of Equality You can add the same number to both sides of an equation, and the statement will still be true.	$3 = 3$ $3 + 2 = 3 + 2$ $5 = 5$	$a = b$ $a + c = b + c$
Subtraction Property of Equality You can subtract the same number from both sides of an equation, and the statement will still be true.	$7 = 7$ $7 - 5 = 7 - 5$ $2 = 2$	$a = b$ $a - c = b - c$
Multiplication Property of Equality You can multiply both sides of an equation by the same number, and the statement will still be true.	$3 = 3$ $3 \cdot 4 = 3 \cdot 4$ $12 = 12$	$a = b$ $a \cdot c = b \cdot c$
Division Property of Equality You can divide both sides of an equation by the same nonzero number, and the statement will still be true.	$15 = 15$ $\dfrac{15}{3} = \dfrac{15}{3}$ $5 = 5$	$a = b$ $\dfrac{a}{c} = \dfrac{b}{c}$, where $c \neq 0$

Example 1   Solve the equation by using Properties of Equality.

(A) $3x - 2 = 6$

Use the Addition Property of Equality.	$3x - 2 + 2 = 6 + 2$
Combine like terms.	$3x = 8$
Now use the Division Property of Equality.	$\dfrac{3x}{3} = \dfrac{8}{3}$
Simplify.	$x = \dfrac{8}{3}$

(B) $\frac{1}{2}z + 4 = 10$

Use the Subtraction Property of Equality.	$\frac{1}{2}z + 4 - \boxed{} = 10 - \boxed{}$
Combine like terms.	$\frac{1}{2}z = \boxed{}$
Now use the Multiplication Property of Equality to multiply each side by 2.	$2 \cdot \frac{1}{2}z = 2 \cdot \boxed{}$
Simplify.	$z = \boxed{}$

2. **Discussion** What is the goal when solving a one-variable equation?

_____

Your Turn

Solve the equation by using Properties of Equality.

3. $5x - 10 = 20$

4. $\frac{1}{3}x + 9 = 21$

🎯 **Explain 2** **Solving Equations to Define a Unit**

One useful application of algebra is to use an equation to determine what a unit of measure represents. For instance, if a person uses the unit of time "score" in a speech and there is enough information given, you can solve an equation to find the quantity that a "score" represents.

Example 2 **Solve an equation to determine the unknown quantity.**

Ⓐ In 1963, Dr. Martin Luther King, Jr., began his famous "I have a dream" speech with the words "Five score years ago, a great American, in whose symbolic shadow we stand, signed the Emancipation Proclamation." The proclamation was signed by President Abraham Lincoln in 1863. But how long is a score? We can use algebra to find the answer.

Let s represent the quantity (in years) represented by a score.

$s =$ number of years in a score

Calculate the quantity in years after President Lincoln signed the Emancipation Proclamation.

$$1963 - 1863 = 100$$

Dr. Martin Luther King, Jr. used "five score" to describe this length of time. Write the equation that shows this relationship.

Use the Division Property of Equality to solve the equation.

$$5s = 100$$
$$\frac{5s}{5} = \frac{100}{5}$$
$$s = 20$$

A score equals 20 years.

(B) An airplane descends in altitude from 20,000 feet to 10,000 feet. A gauge at Radar Traffic Control reads that the airplane's altitude drops 1.8939 miles. How many feet are in a mile?

Let m represent the quantity (in feet) represented by a mile.

m = number of feet in a mile

Calculate the quantity in feet of the descent. $20{,}000 - \boxed{} = \boxed{}$

A gauge described this quantity as 1.8939 miles. Write the equation that shows this relationship.

$$1.8939m = \boxed{}$$

Use the Division Property of Equality to solve the equation. $$\dfrac{1.8939m}{\boxed{}} = \dfrac{\boxed{}}{\boxed{}}$$

Round to the nearest foot. $m \approx \boxed{}$

There are 5280 feet in a mile.

Your Turn

Solve an equation to determine the unknown quantity.

5. An ostrich that is 108 inches tall is 20 inches taller than 4 times the height of a kiwi. What is the height of a kiwi in inches?

6. An emu that measures 60 inches in height is 70 inches less than 5 times the height of a kakapo. What is the height of a kakapo in inches?

💬 Elaborate

7. How do you know which operation to perform first when solving an equation?

_____

8. How can you create an equivalent equation by using the Properties of Equality?

_____

9. When a problem involves more than one unit for a characteristic (such as length), how can you tell which unit is more appropriate to report the answer in?

_____

_____

10. Essential Question Check-In Describe each step in a solution process for solving an equation in one variable.

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Use the *guess-and-check* method to find the solution of the equation. Show your work.

1. $2x + 5 = 19$

Use the *working backward* method to find the solution of the equation. Show your work.

2. $4y - 1 = 7$

Solve each equation using the Properties of Equality. Check your solutions.

3. $4a + 3 = 11$ **4.** $8 = 3r - 1$

5. $42 = -2d + 6$ **6.** $3x + 0.3 = 3.3$

7. $15y + 31 = 61$ **8.** $9 - c = -13$

9. $\dfrac{x}{6} + 4 = 15$

10. $\dfrac{1}{3}y + \dfrac{1}{4} = \dfrac{5}{12}$

11. $\dfrac{2}{7}m - \dfrac{1}{7} = \dfrac{3}{14}$

12. $15 = \dfrac{a}{3} - 2$

13. $4 - \dfrac{m}{2} = 10$

14. $\dfrac{x}{8} - \dfrac{1}{2} = 6$

Justify each step.

15. $2x - 5 = -20$

$2x = -15$ _____

$x = -\dfrac{15}{2}$ _____

16. $\dfrac{x}{3} - 7 = 11$

$\dfrac{x}{3} = 18$ _____

$x = 6$ _____

17. $\dfrac{9x}{4} = -9$

$9x = -36$ _____

$x = -4$ _____

18. In 2003, the population of Zimbabwe was about 12.6 million people, which is 1 million more than 4 times the population in 1950. Write and solve an equation to find the approximate population p of Zimbabwe in 1950.

19. Julio is paid 1.4 times his normal hourly rate for each hour he works over 30 hours in a week. Last week he worked 35 hours and earned $436.60. Write and solve an equation to find Julio's normal hourly rate, r. Explain how you know that your answer is reasonable.

20. The average weight of the top 5 fish caught at a fishing tournament was 12.3 pounds. Some of the weights of the fish are shown in the table.

Top 5 Fish	
Caught by	**Weight (lb)**
Wayne S.	
Carla P.	12.8
Deb N.	12.6
Vincente R.	11.8
Armin G.	9.7

What was the weight of the heaviest fish?

21. Paul bought a student discount card for the bus. The card allows him to buy daily bus passes for $1.50. After one month, Paul bought 15 passes and spent a total of $29.50. How much did he spend on the student discount card?

22. Jennifer is saving money to buy a bike. The bike costs $245. She has $125 saved, and each week she adds $15 to her savings. How long will it take her to save enough money to buy the bike?

23. **Astronomy** The radius of Earth is 6378.1 km, which is 2981.1 km greater than the radius of Mars. Find the radius of Mars.
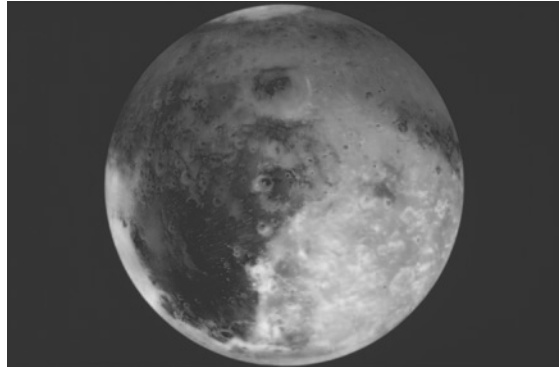
24. Maggie's brother is 3 years younger than twice her age. The sum of their ages is 24. How old is Maggie?

25. **Analyze Relationships** One angle of a triangle measures 120°. The other two angles are congruent. Write and solve an equation to find the measure of the congruent angles.

26. **Explain the Error** Find the error in the solution, and then solve correctly.

$$9x + 18 + 3x = 1$$
$$9x + 18 = -2$$
$$9x = -20$$
$$x = -\frac{20}{9}$$

27. **Check for Reasonableness** Marietta was given a raise of $0.75 per hour, which gave her a new wage of $12.25 per hour. Write and solve an equation to determine Marietta's hourly wage before her raise. Show that your answer is reasonable.

Lesson Performance Task

The formula $p = 8n - 30$ gives the profit p when a number of items n are each sold at $8 and expenses totaling $30 are subtracted.

a. If the profit is $170.00, how many items were bought?

b. If the same number of items were bought but the expenses changed to $40, would the profit increase or decrease, and by how much? Explain.