Section II: Free-Response

The following are examples of the kinds of free-response questions found on the exam.

PART A

A graphing calculator is required for these questions.

x	1	2	3	4	5
$f(x)$	-10	-5	4	17	34

1. Let f be an increasing function defined for $x \geq 0$. The table gives values of $f(x)$ at selected values of x. The function g is given by $g(x) = \dfrac{x^3 - 14x - 27}{x + 2}$.

   (A) (i) The function h is defined by $h(x) = (g \circ f)(x) = g(f(x))$. Find the value of $h(5)$ as a decimal approximation, or indicate that it is not defined.

   (ii) Find the value of $f^{-1}(4)$, or indicate that it is not defined.

   (B) (i) Find all values of x, as decimal approximations, for which $g(x) = 3$, or indicate there are no such values.

   (ii) Determine the end behavior of g as x decreases without bound. Express your answer using the mathematical notation of a limit.

   (C) (i) Use the table of values of $f(x)$ to determine if f is best modeled by a linear, quadratic, exponential, or logarithmic function.

   (ii) Give a reason for your answer based on the relationship between the change in the output values of f and the change in the input values of f.

2. Students who completed a class participated in a year-long study to see how much content from the class they retained over the following year. At the end of the class, students completed an initial test to determine the group's content knowledge. At that time ($t = 0$), the group of students achieved a score of 75 out of 100 points. For the next 12 months, the group was evaluated at the end of each month to track their retention of the content. After 3 months ($t = 3$), the group's score was 70.84 points.

The group's score can be modeled by the function R given by $R(t) = a + b\ln(t + 1)$, where $R(t)$ is the score, in points, for month t, and t is the number of months since the initial test.

(A) (i) Use the given data to write two equations that can be used to find the values for constants a and b in the expression for $R(t)$.

(ii) Find the values for a and b.

(B) (i) Use the given data to find the average rate of change of the scores, in points per month, from $t = 0$ to $t = 3$ months. Express your answer as a decimal approximation. Show the computations that lead to your answer.

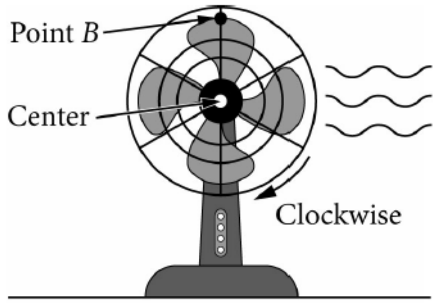
(ii) Interpret the meaning of your answer from (i) in the context of the problem.

(iii) Consider the average rates of change of R from $t = 3$ to $t = p$ months, where $p > 3$. Are these average rates of change less than or greater than the average rate of change from $t = 0$ to $t = 3$ months found in (i)? Explain your reasoning.

(C) The leaders of the study decide to use model R to make predictions about the group's score beyond 12 months (1 year). For a given year, model R is an appropriate model if the group's predicted score at the end of the year is at least 1 point lower than the group's predicted score at the end of the previous year. Based on this information, for how many years is model R an appropriate model? Give a reason for your answer. (Note: The end of a year occurs every 12 months from the initial evaluation—$t = 12, t = 24, \ldots$)

PART B

No calculator is allowed for these questions.

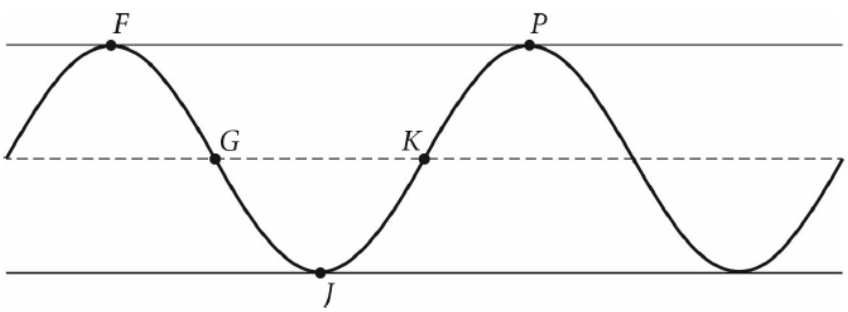


Note: Figure not drawn to scale.

3. The blades of an electric fan rotate in a clockwise direction and complete 5 rotations every second. Point B is on the tip of one of the fan blades and is located directly above the center of the fan at time $t = 0$ seconds, as indicated in the figure. Point B is 6 inches from the center of the fan. The center of the fan is 20 inches above a level table on which the fan sits. As the fan blades rotate at a constant speed, the distance between B and the surface of the table periodically decreases and increases.

   The sinusoidal function h models the distance between B and the surface of the table, in inches, as a function of time t in seconds.

   (A) The graph of h and its dashed midline for two full cycles is shown. Five points, F, G, J, K, and P are labeled on the graph. No scale is indicated, and no axes are presented.
   Determine possible coordinates $(t, h(t))$ for the five points: F, G, J, K, and P.



   (B) The function h can be written in the form $h(t) = a\sin(b(t + c)) + d$. Find values of constants a, b, c, and d.

   (C) Refer to the graph of h in part (A). The t-coordinate of K is t_1, and the t-coordinate of P is t_2.

   (i) On the interval (t_1, t_2), which of the following is true about h?

   a. h is positive and increasing.
   b. h is positive and decreasing.
   c. h is negative and increasing.
   d. h is negative and decreasing.

   (ii) Describe how the rate of change of h is changing on the interval (t_1, t_2).

4. Directions:

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians.
- Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator.
- Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x + 3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms.
- For each part of the question, show the work that leads to your answers.

(A) The functions g and h are given by

$$g(x) = 3\ln x - \frac{1}{2}\ln x$$
$$h(x) = \frac{\sin^2 x - 1}{\cos x}.$$

(i) Rewrite $g(x)$ as a single natural logarithm without negative exponents in any part of the expression. Your result should be of the form $\ln(\text{expression})$.

(ii) Rewrite $h(x)$ as an expression in which $\cos x$ appears once and no other trigonometric functions are involved.

(B) The functions j and k are given by

$$j(x) = 2(\sin x)(\cos x) - \cos x$$
$$k(x) = 8e^{(3x)} - e.$$

(i) Solve $j(x) = 0$ for values of x in the interval $\left[0, \frac{\pi}{2}\right]$.

(ii) Solve $k(x) = 3e$ for values of x in the domain of k.

(C) The function m is given by

$$m(x) = \cos(2x) + 4.$$

Find all input values in the domain of m that yield an output value of $\frac{9}{2}$.