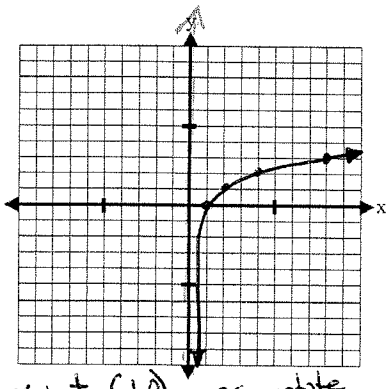
Algebra 2
Semester 2 Final Exam Study Guide
Unit 4 – Exponential and Logarithmic Functions

Name:_____ (104)
Period:_____

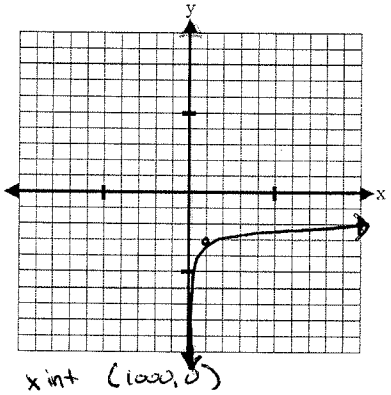
Identify the following for each function: x-intercept, y-intercept, and asymptote. Then graph the function.

1. $f(x) = \log_2 x$



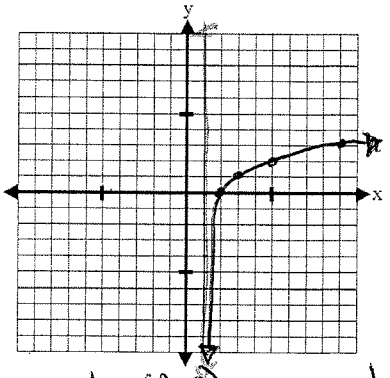
x-int (1,0) asymptote
y-int none x=0

2. $f(x) = \log(x) - 3$



x int (1000,0)
y int none
asym x=0

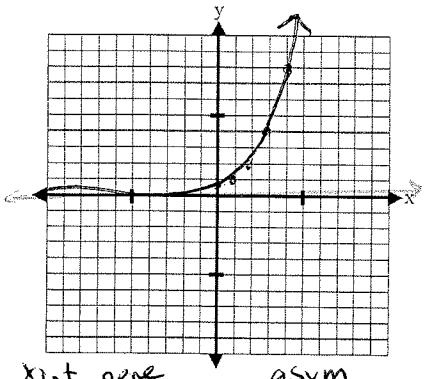
3. $f(x) = \log_2(x - 1)$



x int (2,0)
y int none
asymptote x=1

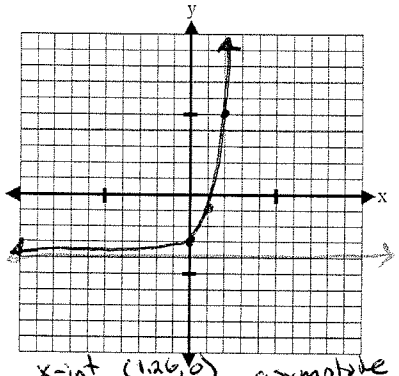
Identify the following for each function: x-intercept, y-intercept, and asymptote. Then graph the function.

4. $f(x) = 2^{x-1}$



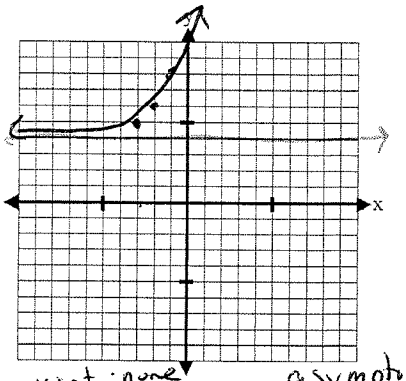
x int none
y int (0,0.5)
asym y=0

5. $f(x) = 3^x - 4$



x-int (1.26,0)
y-int (0,-3)
asymptote y=-4

6. $f(x) = 2^{x+3} + 4$



x int none
y int (0,12)
asymptote y=4

For numbers 7 through 9, determine whether the data in the table can be modeled by a linear function or exponential function. Then write the equation that represents the function.

7.

x	0	1	2	3	4	5
F(x)	3.5	7	14	28	56	112

+1 +1 +1 +1
×2 ×2 ×2 ×2

exponential
$y = 3.5(2)^x$

8.

x	1	2	3	4	5
F(x)	5	10	15	20	25

+1 +1 +1 +1
+5 +5 +5 +5

linear
$y = 5x$

9.

x	0	1	2	3	4	5
F(x)	1280	320	80	20	5	$\frac{5}{4}$

+1 +1 +1 +1 +1
×¼ ×¼ ×¼ ×¼ ×¼

$y = 1280\left(\frac{1}{4}\right)^x$

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For 10 through 12, write the transformations of the function compared to the parent function $f(x) = 2^x$

10. $g(x) = -2^x + 4$

reflected over the x-axis; up 4

11. $h(x) = 3 \cdot 2^{x-1}$

vertical stretch; right 1

12. $r(x) = 2^{x+4} - 9$

left 4; down 9

For 13 through 15, write the transformations of the function compared to the parent function $f(x) = \log x$

13. $g(x) = -\log(x + 12)$

reflection over x-axis; 12 left

14. $h(x) = 5\log(x) - 2$

vertical stretch; down 2

15. $r(x) = \log(x - 4) + 1$

right 4; up 1

Convert each log equation to an exponential equation.

16. $\log_2 x = 4$

$2^4 = x$

$16 = x$

17. $\log 3 = a$

$10^a = 3$

18. $\log_5(x - 1) = 3$

$5^3 = x - 1$

Convert each exponential expression to a logarithm.

19. $2^x = 5$

$\log_2 5 = x$

20. $e^{2x} = 3$

$\log_e 3 = 2x$ (or) $\ln 3 = 2x$

21. $10^y = 12$

$\log_{10} 12 = y$

Condense each expression to a single log.

22. $\log a - \log b$

$\log \frac{a}{b}$

23. $3\log x + 4\log y$

$\log x^3 y^4$

24. $2\log x + \log y - 3\log z$

$\log \frac{x^2 y}{z^3}$

Expand each log.

25. $\log(2x)$

$\log 2 + \log x$

26. $\log\left(\frac{2y}{5}\right)$

$\log 2 + \log y - \log 5$

27. $\log\left(\frac{3y^4}{x^5}\right)$

$\log 3 + 4\log y - 5\log x$

Solve the following equations:

28. $6^{2x} - 5 = 45$

$6^{2x} = 50$

$\log_6 50 = 2x$

$x \approx 1.09$

29. $2^{3x-1} = 32$

$\log_2 32 = 3x - 1$

$5 = 3x - 1$

$2 = x$

(or)

$2^{3x-1} = 2^5$

$3x - 1 = 5$

$3x = 6$

$x = 2$

30. $\log(2x - 1) = \log(8x + 14)$

$2x - 1 = 8x + 14$

$-15 = 6x$

$-2.5 = x$

31. $\log_2(3x + 1) = 3$

$2^3 = 3x + 1$

$8 = 3x + 1$

$7 = 3x$

$x = \frac{7}{3}$

Algebra 2
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Unit 5 – Radical and Rational Functions

Name:_____
Period: _____

1. Simplify $\frac{x^2-7x-30}{x^2-5x-24}$ $\frac{(x-10)(x+3)}{(x-8)(x+3)}$

$\frac{x-10}{x-8}$; $x \neq 8, -3$

2. Simplify $\frac{x^2+8x+12}{x^2+3x-18}$ $\frac{(x+6)(x+2)}{(x+6)(x-3)}$

$\frac{x+2}{x-3}$; $x \neq -6, 3$

3. Simplify $\frac{2x^2+10x-48}{8x+64}$ $\frac{2(x^2+5x-24)}{8(x+8)}$

$\frac{2(x+8)(x-3)}{8(x+8)}$ $= \frac{x-3}{4}$; $x \neq -8$

4. Simplify $\frac{x^2+3x-28}{x^2-49}$ $\frac{(x+7)(x-4)}{(x-7)(x+7)}$

$\frac{x-4}{x-7}$; $x \neq 7, -7$

5. Simplify each expression

a) $\frac{3}{(x+7)} + \frac{4}{(x-8)}$ $\frac{3(x-8)}{(x+8)(x+7)} + \frac{4(x+7)}{(x-8)(x+7)}$ $3(x-8) + 4(x+7)$

$\frac{3x-24+4x+28}{(x-8)(x+7)}$

$\frac{7x+4}{(x-8)(x+7)}$; $x \neq 8, -7$

b) $\frac{7x}{(x+1)} + \frac{8}{(x-7)}$ $\frac{7x(x-7)}{(x-7)(x+1)} + \frac{8(x+1)}{(x-7)(x+1)}$

$\frac{7x(x-7)+8(x+1)}{(x+1)(x-7)}$

$\frac{7x^2-49x+8x+8}{(x+1)(x-7)}$

$\frac{7x^2-41x+8}{(x+1)(x-7)}$; $x \neq 7, -1$

c) $\frac{6}{(x-1)} - \frac{5x}{4}$ $\frac{4 \cdot 6}{4(x-1)} - \frac{5x(x-1)}{4(x-1)}$

$\frac{24-5x^2+5x}{4(x-1)}$

$\frac{-5x^2+5x+24}{4(x-1)}$; $x \neq 1$

6. Solve the equation. $\frac{4}{(x-2)} + \frac{1}{(4x)} = \frac{6}{x}$ $4x(x-2)$

$\frac{16x}{4x(x-2)} + \frac{x-2}{4x(x-2)} = \frac{24(x-2)}{4x(x-2)}$

$16x + x - 2 = 24x - 48$

$46 = 7x$

$x = \frac{46}{7}$

Check

$\frac{4}{(46/7)-2} + \frac{1}{4(46/7)} = \frac{6}{46/7}$

$\frac{7}{8} + \frac{7}{184} = \frac{21}{23}$

$\frac{21}{23} = \frac{21}{23}$

7. Solve the equation. $\frac{(x+5)}{(x+8)} = \frac{1}{(x+1)} + \frac{6}{(x+1)}$ $(x+1)(x+8)$

$\frac{x^2+6x+5}{(x+1)(x+8)} = \frac{(x^2+9x+8)+6x+48}{(x+1)(x+8)}$

$x^2+6x+5 = x^2+15x+56$

$-51 = 9x$

$x = -17/3$

Check!

8. Solve the equation. $\frac{(x-3)}{(x-1)} - \frac{2}{(x+1)} = \frac{(x-5)}{(x^2-1)}$ $(x+1)(x-1)$

$\frac{(x^2-2x-3)-2x+2}{(x+1)(x-1)} = \frac{x-5}{(x+1)(x-1)}$

$\frac{x^2-4x-1}{(x+1)(x-1)} = \frac{x-5}{(x+1)(x-1)}$

$x^2-4x-1 = x-5$

$x^2-5x+4=0$

$(x-4)(x-1)=0$

$x = 4, 1$ (1 extraneous)

Check!

9. Solve the equation. $\frac{3}{(x-4)} + \frac{2(x+1)}{(x+1)} = \frac{4x-11}{x^2-3x-4}$ $(x-4)(x+1)$

$\frac{(3x+3)+(2x-8)}{(x-4)(x+1)} = \frac{4x-11}{(x-4)(x+1)}$

$5x-5 = 4x-11$

$x = -1$ ← extraneous

NO SOLUTION

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10 Identify any horizontal or vertical asymptotes or holes in the graph.

a) $f(x)=\frac{x-2}{x-4}$

Vertical Asymptote: $x=4$

Horizontal Asymptote: $y=1$

Hole(s): none

b) $f(x)=\frac{2x+4}{x^2-4}$ $\frac{2(x+2)}{(x+2)(x-2)}$

Vertical Asymptote: $x=2$

Horizontal Asymptote: $y=0$

Hole(s): $x=-2$

11. Describe the function $g(x)$ as a transformation of $f(x)=\frac{1}{x}$.

a) $g(x)=\frac{5}{x}-7$

down 7
vertical stretch by 5

b) $3+\frac{1}{x-2}$

up 3
right 2

c) $-\frac{4}{x+1}-6$

reflection over x-axis
left 1
down 6

12. Solve $\sqrt[3]{3x+4}-1=26$

$(\sqrt[3]{3x+4})^3=(27)^3$
$3x+4=19,683$
$3x=19679$
$+1 \quad +1$
$x=6559.67$
CHECK!

13. Solve $\sqrt[3]{2x-7}-2=3$

$(\sqrt[3]{2x-7})^3=(5)^3$ $+2$
$2x-7=125$
$2x=132$
$x=66$
check!

14. State the vertical asymptote(s), hole(s), and horizontal asymptote for the following function: $\frac{3x^2-12x}{x^2-2x-3}$

$\frac{3x(x-4)}{(x-3)(x+1)}$ V.A: $x=3,-1$ hole: none H.A: $y=3$

15. Find the solution for the equation: $x=\sqrt{110-x}$. Be sure to clearly state any solution(s) and/or extraneous solution(s).

$x^2=(\sqrt{110-x})^2$
$x^2=110-x$
$x^2+x-110=0$
$(x+11)(x-10)=0$
$x=-11, 10$

Check:
$10=\sqrt{110-10}$
$10=\sqrt{100}$
$10=10$ ✓

$x=10$
11 is extraneous

Check:
$-11=\sqrt{110-(-11)}$
$-11=\sqrt{121}$
$-11\neq 11$

16. Solve: $4-\sqrt[3]{x+1}=5$

$-4 \quad -4$
$\frac{-\sqrt[3]{x+1}}{-1}=\frac{1}{-1}$
$(\sqrt[3]{x+1})^3=(-1)^3$
$x+1=-1$
$x=-2$

Check:
$4-\sqrt[3]{-2+1}=5$
$4-\sqrt[3]{-1}=5$
$4-(-1)=5$
$5=5$ ✓

17. What are the asymptotes for $\frac{x^2+16x-63}{x^2-2x-35}$?

$\frac{-(x^2-16x+63)}{(x-7)(x+5)} \rightarrow \frac{-(x-7)(x-9)}{(x-7)(x+5)}$

V.A: $x=-5$ hole: $x=7$
H.A: $y=-1$

18. Simplify $\frac{x^2-2x-15}{x^2-6x+5}$

$\frac{(x-5)(x+3)}{(x-5)(x-1)}$ $\frac{x+3}{x-1}$; $x\neq 1,5$

19. Find the product of $\frac{8x-56}{8x+48}\cdot\frac{x^2+9x+18}{8x^2+24x}$

$\frac{8(x-7)}{8(x+6)}\cdot\frac{(x+6)(x+3)}{8x(x+3)}$

$\frac{x-7}{8x}$; $x\neq -6,0,-3$

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Unit 6 - Trigonometry

Name:_____
Period: _____

1. Convert the following from radians to degrees.
   a. $\frac{3\pi}{4}$ radians $\quad \frac{3\pi}{4} \times \frac{180^\circ}{\pi} = 135^\circ$
   b. 6π radians $\quad \frac{6\pi}{1} \times \frac{180^\circ}{\pi} = 1080^\circ$
   c. $\frac{11\pi}{6}$ radians $\quad \frac{11\pi}{6} \times \frac{180^\circ}{\pi} = 330^\circ$

2. Convert the following from degrees to radians.
   a. 60^0 $\quad \frac{60}{1} \times \frac{\pi}{180} = \frac{\pi}{3}$
   b. 360^0 $\quad \frac{360}{1} \times \frac{\pi}{180} = 2\pi$
   c. 720^0 $\quad \frac{720}{1} \times \frac{\pi}{180} = 4\pi$

3. Circle all of the angle measures that are co-terminal with 380^0
   a. 680^0
   (b. 20^0) $\quad 380 - 360$
   c. $\frac{\pi}{9}$
   d. -360^0
   (e. -700^0) $\quad 380 - 360 - 360 - 360$
   f. 6π

4. Find the values of all six trigonometric functions of θ when $\theta = 315^0$.

   $\sin\theta = \frac{-\sqrt{2}}{2}$ $\quad$ $\csc\theta = -\sqrt{2}$ $\quad \frac{1}{\sin\theta} = \frac{1}{-\sqrt{2}/2} = \frac{-2}{\sqrt{2}} \frac{\sqrt{2}}{\sqrt{2}} = \frac{-2\sqrt{2}}{2} = -\sqrt{2}$

   $\cos\theta = \frac{\sqrt{2}}{2}$ $\quad$ $\sec\theta = \sqrt{2}$ $\quad \frac{1}{\cos\theta} = \frac{1}{\sqrt{2}/2} = \frac{2}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{2\sqrt{2}}{2} = \sqrt{2}$

   $\tan\theta = -1$ $\quad \frac{\sin\theta}{\cos\theta} = \frac{-\sqrt{2}/2}{\sqrt{2}/2}$ $\quad$ $\cot\theta = -1$ $\quad \frac{\cos\theta}{\sin\theta} = \frac{\sqrt{2}/2}{-\sqrt{2}/2}$

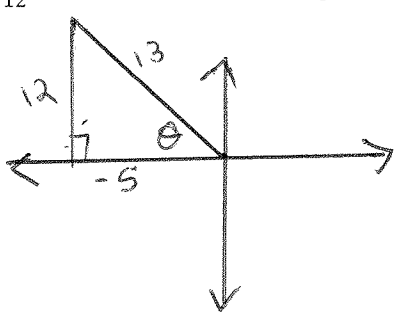
5. Find the values of all six trigonometric functions of θ when $\theta = \frac{3\pi}{2}$.

   $\sin\theta = -1$ $\quad$ $\csc\theta = -1$ $\quad \frac{1}{\sin\theta} = \frac{1}{-1}$

   $\cos\theta = 0$ $\quad$ $\sec\theta =$ undefined $\quad \frac{1}{\cos\theta} = \frac{1}{0}$

   $\tan\theta =$ undefined $\quad \frac{-1}{0}$ $\quad$ $\cot\theta = 0$ $\quad \frac{\cos\theta}{\sin\theta} = \frac{0}{-1}$

6. If $\cot\theta = \frac{-5}{12}$ and θ is in the second quadrant, use the trigonometric identities to find the value of $\sin\theta$.



$5^2 + 12^2 = c^2$
$13 = c$

$\sin\theta = \frac{12}{13}$

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7. Simplify each expression in terms of $\sin\theta$ and $\cos\theta$.

   a. $\tan\theta \cdot \cos\theta$

   $\frac{\sin\theta}{\cos\theta} \cdot \frac{\cos\theta}{1} = \sin\theta$

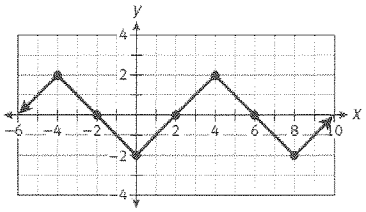
   b. $\cot\theta \cdot \sin\theta$

   $\frac{\cos\theta}{\sin\theta} \cdot \frac{\sin\theta}{1} = \cos\theta$

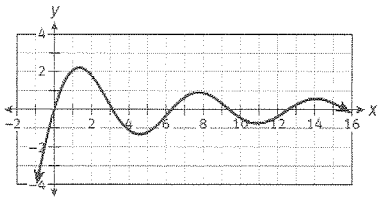
   c. $\frac{1}{\tan\theta}$ $\frac{1}{\frac{\sin\theta}{\cos\theta}} = \frac{\cos\theta}{\sin\theta} = \cot\theta$

8. Circle all of the graphs that represent a periodic function.

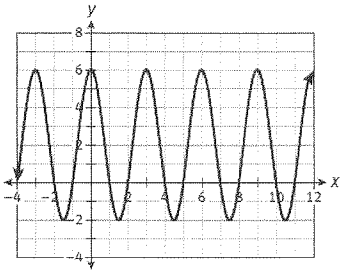
   a.



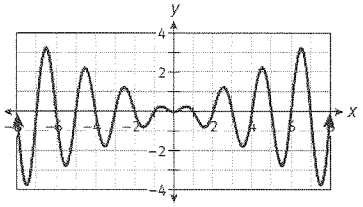
   b.



   c.

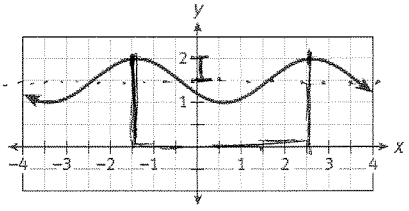


   d.



9. State the period and amplitude of each function.

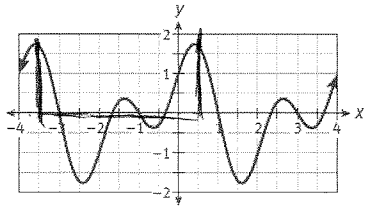
   a.



amplitude = 1

period = 4

   b.



period = 4

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Unit 7 - Statistics

Name: Key
Period: _____

1. Ben is on the baseball team at his school and asks everyone if they have a part-time job. Why might his data be a poor estimate of the actual percentage of students at his school with part-time jobs?

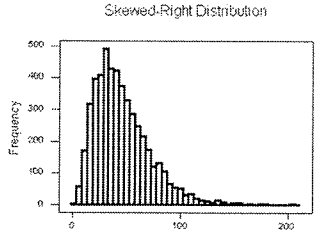
Not all students play baseball and athletes are less likely to have a part-time job.

2. Name a few ways to reduce the margin of error on a survey.

Increase the size of a random sample.

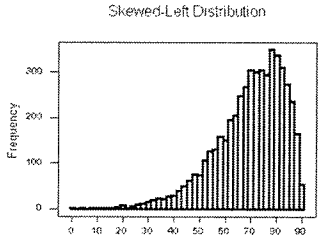
3. How does the mean compare to the median in each of the following scenarios?

a.



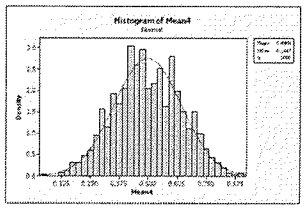
mean > median

b.



mean < median

c.

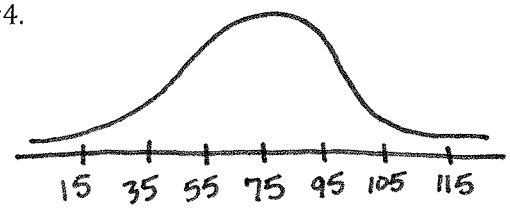


mean = median

4. The mean and standard deviation of a normally distributed data set are 75 and 20, respectively. What is the z-score of a data point whose value is 100?

$$Z = \frac{100 - 75}{20} = 1.25$$

5. Set up and label the normal curve for Question #4.



6. Jamie wants to evaluate the negative effects of computer gaming on student grades. She has decided to use the ninth-grade class as the population for her study. After some consideration, she decides to interview ninth-graders who shop in the local used-game store as a sample of the ninth-grade population. Is Jamie's sample free from bias? Why or why not?

No. All students do not have an equal chance to be selected because not all ninth-graders go to that store.

7. A new nutritional supplement claims to increase long-distance runner endurance by 25%. A study to evaluate the claim is planned using a representative sample of 50 male and 50 female long-distance runners. A study is designed to give the supplement to 100 runners before they race in Florida. The study will compare the results to the same runners when they raced in Colorado before taking the supplement. Is this a reasonable study method? Why or why not?

No. Running in Colorado and Florida are different (thin air vs. humidity).