1. The sequence **converges** to a value of $-\frac{2}{5}$

$$\lim_{n\to\infty}\frac{6n^4-n^2+9}{2+8n-15n^4}=-\frac{2}{5}$$

2. To do this a quick rewrite on the sequence terms is needed then we'll see that the sequence **converges** to a value of $\ln\left(\frac{9}{19}\right)$.

$$\lim_{n\to\infty}\left[\ln(8+5n)-\ln(1+19n)\right]=\lim_{n\to\infty}\ln\left(\frac{8+5n}{1+19n}\right)=\ln\left(\frac{5}{19}\right)$$

3. First write out the first few terms of the series and we'll see that the sequence **diverges.**

$$\left\{\cos(n\pi)\right\}_{n=0}^{\infty}=\{1,-1,1,-1,1,-1,\cdots\}=\left\{(-1)^n\right\}_{n=0}^{\infty} \qquad \lim_{n\to\infty}(-1)^n - \text{ Does Not Exist}$$

8. $\lim\limits_{n\to\infty}d_n=\lim\limits_{n\to\infty}\frac{3+8n}{4-7n}=-\frac{8}{7}$

(a) From the limit above we can see that the sequence **converges** to a value of $-\frac{8}{7}$.

(b) From the limit above and the **Divergence Test** we can see that the series will **diverge**.

Remember that all we need for a sequence to converge is for the limit of the terms to exist and be a finite value (which we have here). Also, in order for a series to converge the limit of the terms MUST be zero and because we don't have that here the series can't converge.

1. We can use the integral test or a quick rewrite shows that $p=\frac{11}{12}\leq 1$ and so the series will **diverge.**

$$\sum_{n=2}^{\infty}\frac{8}{n^{\frac{2}{3}}\,n^{\frac{1}{4}}}=\sum_{n=2}^{\infty}\frac{8}{n^{\frac{11}{12}}}$$

so by the **Integral Test** the series will **converge.** You were able to L'Hospital's Rule that first term right?

2. (2 pts) In this case the terms are clearly positive and the some quick Calc I shows,

$$f(x)=xe^{-\frac{1}{6}x} \qquad f'(x)=\left(1-\tfrac{1}{6}x\right)e^{-\frac{1}{6}x}$$

This is negative for $x>6$ and so the function, and hence the series terms, are eventually decreasing. The Integral Test can therefore be used. I'll leave it to you to verify the integration by parts work.

$$\int_1^{\infty}xe^{-\frac{1}{6}x}\,dx=\lim_{t\to\infty}\int_1^t xe^{-\frac{1}{6}x}\,dx=\lim_{t\to\infty}\left(-(6x+36)e^{-\frac{1}{6}x}\right)\Bigg|_1^t=\lim_{t\to\infty}\left[-\frac{6t+36}{e^{\frac{1}{6}t}}+42e^{-\frac{1}{6}}\right]=42e^{-\frac{1}{6}}$$

The integral converges.

3. In this case the terms are clearly positive for $n \geq 3$ and increasing *n* will ***only*** increase the denominator we can see that these terms are increasing and so the integral test can be used.

$$\int_3^{\infty} \frac{1}{(6x+1)^{\frac{5}{3}}}\,dx = \lim_{t\to\infty}\int_3^{t} \frac{1}{(6x+1)^{\frac{5}{3}}}\,dx = \lim_{t\to\infty}\left(-\frac{1}{4(6x+1)^{\frac{2}{3}}}\right)\Bigg|_3^t$$

$$= \lim_{t\to\infty}\left(-\frac{1}{4(6t+1)^{\frac{2}{3}}}+\frac{1}{4\left(19^{\frac{2}{3}}\right)}\right) = \frac{1}{4\left(19^{\frac{2}{3}}\right)}$$

The integral converges and so by the **Integral Test** the series will **converge.**

4. First, the terms are positive. This looks like it will converge so we'll need a larger function we can prove converges.

$$\frac{\sqrt[3]{n^6-4}}{n^7+n} \leq \frac{\sqrt[3]{n^6}}{n^7+n} \qquad \text{b/c } n^6 > n^6-4 \qquad (\text{we've made numer. larger})$$

$$\leq \frac{\left(n^6\right)^{\frac{1}{3}}}{n^7} = \frac{1}{n^5} \qquad \text{b/c } n^7 < n^7+n \qquad (\text{we've made denom. smaller})$$

Now, $\sum_{n=1}^{\infty}\frac{1}{n^5}$ converges and so by the **Comparison Test** the original series will **converge.**

5. (2 pts) First, the terms are positive. This looks like it will diverge so we'll need a smaller function we can prove diverges.

$$\frac{4\cos^2(n)+7n^2}{n^3\mathbf{e}^{-n}} \geq \frac{0+7n^2}{n^3\mathbf{e}^{-n}} \qquad \text{b/c } 0 \leq 4\cos^2(n) \leq 1 \qquad (\text{we've made numer. smaller})$$

$$\geq \frac{n^2}{n^3(1)} = \frac{1}{n} \qquad \text{b/c } \mathbf{e}^{-n} \leq 1 \text{ for } n \geq 0 \qquad (\text{we've made denom. larger})$$

Now, $\sum_{n=1}^{\infty}\frac{1}{n}$ diverges and so by the **Comparison Test** the original series will **diverge.**

6. (2 pts) First, the terms are positive. We'll use the limit comparison test with $\sum_{n=1}^{\infty}\frac{1}{n}$ as the second series and note that this diverges.

$$c = \lim_{n\to\infty}\frac{n^4-n^2-3}{n^5+n^3+7}\frac{n}{1}\lim_{n\to\infty}\frac{n^5\left(1-\frac{1}{n^2}-\frac{3}{n^4}\right)}{n^5\left(1+\frac{1}{n^2}+\frac{7}{n^5}\right)} = 1$$

So, $0 < c = 1 < \infty$ and so by the **Limit Comparison Test** both series will have the same convergence. The second series diverges and so the original series will **diverge.**

7. First note that $\cos(n\pi) = (-1)^n$ (recall #3 from the previous homework set…) and so this really is an alternating series with,

$$b_n = \frac{1}{n^2+4} > 0 \qquad \lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{1}{n^2+4} = 0$$

Also, increasing *n* only increases the denominator and so the b_n are decreasing and so by the **Alternating Series Test** the series will **converge.**

8. (2 pts) In this case we have,

$$b_n = \frac{n}{2+n^2} > 0 \qquad \lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{n}{2+n^2} = 0$$

We'll need to do a little Calc I for the decreasing part here.

$$f(x) = \frac{x}{2+x^2} \qquad f'(x) = \frac{2-x^2}{\left(2+x^2\right)^2}$$

From this we can see that the function , and hence the series terms, will increase in the range $0 \le n < \sqrt{2}$ and decrease in the range $n > \sqrt{2}$ and so the series terms will be decreasing eventually. Therefore, by the **Alternating Series Test** the series will **converge**.

9. In this case we have,

$$b_n = \frac{2n+1}{6n+5} > 0 \qquad \lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{2n+1}{6n+5} = \frac{1}{3} \ne 0$$

So, the Alternating series test will not work for this series. The Divergence Test can then be used.

$$\lim_{n\to\infty} \frac{(-1)^{n+8}(2n+1)}{6n+5} = \lim_{n\to\infty}\left[(-1)^{n+8}\frac{2n+1}{6n+5}\right]$$

The second term is clearly going towards $\frac{1}{3}$ while the first time is oscillating between 1 and -1. Therefore as $n\to\infty$ is getting closer and closer to just oscillating between $\frac{1}{3}$ and $-\frac{1}{3}$ and so the limit will not exist. So, by the **Divergence Test** this series will **diverge**.

10. (2 pts) By the Ratio Test the series will **converge.**

$$L = \lim_{n\to\infty}\left|\frac{1+8(n+1)}{(2(n+1)+2)!}\,\frac{(2n+2)!}{1+8n}\right| = \lim_{n\to\infty}\left|\frac{9+8n}{(2n+4)!}\,\frac{(2n+2)!}{1+8n}\right|$$

$$= \lim_{n\to\infty}\left|\frac{9+8n}{(2n+4)(2n+3)(2n+2)!}\,\frac{(2n+2)!}{1+8n}\right| = \lim_{n\to\infty}\left|\frac{9+8n}{(2n+4)(2n+3)(1+8n)}\right| = 0$$

11. By the Ratio Test the series will **diverge.**

$$L = \lim_{n\to\infty} \left| \frac{3^{3+2n}\left((n+1)^2 - (n+1)\right)}{6^{n-1}} \frac{6^{n-2}}{3^{1+2n}\left(n^2 - n\right)} \right| = \lim_{n\to\infty} \left| \frac{3^2\left((n+1)^2 - (n+1)\right)}{6\left(n^2 - n\right)} \right| = \frac{3}{2} > 1$$

12. By the Root Test the series will **diverge**.

$$L = \lim_{n\to\infty} \left| \frac{\mathbf{e}^{2n}}{(-3)^{n+1}} \right|^{\frac{1}{n}} = \lim_{n\to\infty} \frac{\mathbf{e}^2}{3^{1+\frac{1}{n}}} = \frac{\mathbf{e}^2}{3} > 1$$

13. By the Root Test the series will **diverge**.

$$L = \lim_{n\to\infty} \left| \left(\frac{2-6n}{4-7n} \right)^{3-n} \right|^{\frac{1}{n}} = \lim_{n\to\infty} \left| \frac{2-6n}{4-7n} \right|^{\frac{3}{n}-1} = \left| \frac{-6}{-7} \right|^{-1} = \frac{7}{6} > 1$$