Name _____ Date _____

Don't Take This Out of Context
Analyzing Polynomial Functions

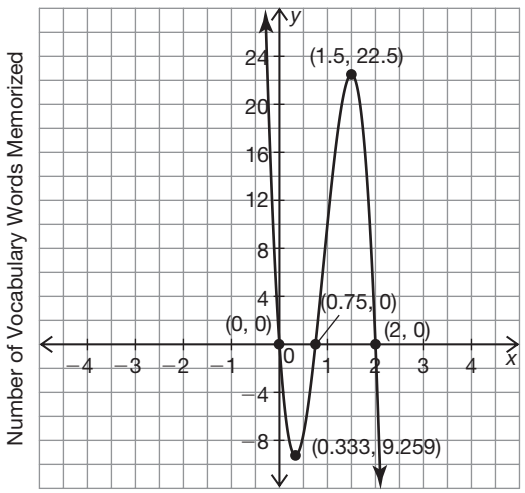
Vocabulary

Write a definition for the term in your own words.

1. average rate of change

Problem Set

The graph shows the number of vocabulary words a student is able to memorize based on the amount of time spent studying. Use the graph to answer the questions.



1. How many vocabulary words does the student know at the start of the study? Where is this information located on the graph?

   The student knows 0 vocabulary words at the start of the study. This event is represented by the *y*-intercept at the origin.

2. Describe the relative minimum in terms of the problem situation.

3. What is the minimum amount of time the student studies before they begin to remember the vocabulary? Where is this information located on the graph?

4. How long did the student need to study in order to remember 22 vocabulary words? Where is this information located on the graph?

5. The graph has an x-intercept at (2, 0). Describe the activity of the student at this point in terms of the problem situation.

6. Does the graph accurately describe the problem situation? Explain your reasoning.

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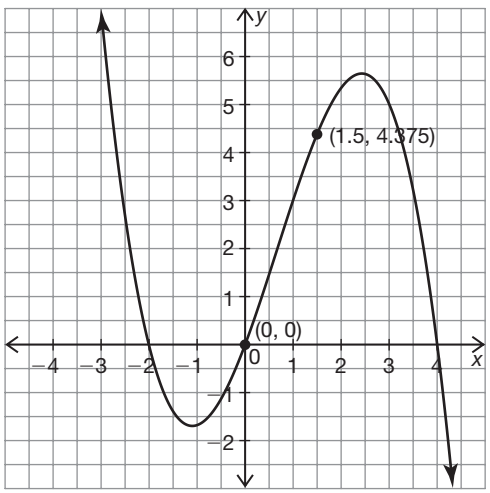
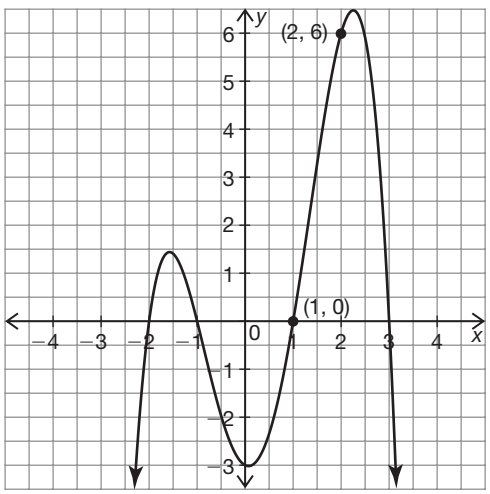
Determine the average rate of change for the given interval for each polynomial function.

7. (0, 1.5)

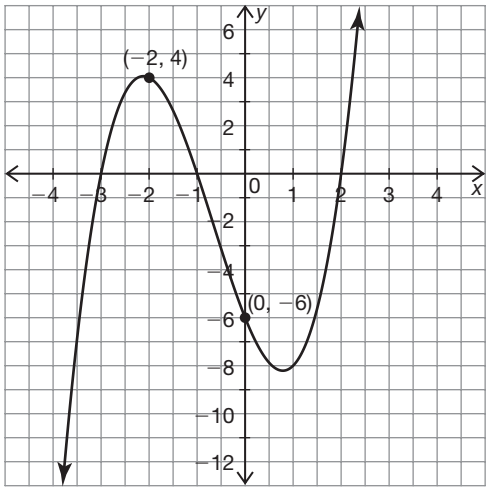


$$\frac{f(b) - f(a)}{b - a} = \frac{f(1.5) - f(0)}{1.5 - 0}$$
$$= \frac{4.375 - 0}{1.5 - 0}$$
$$= \frac{4.375}{1.5}$$
$$\approx 2.92$$

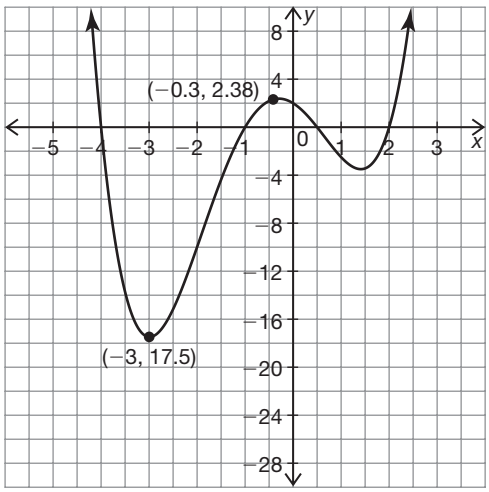
8. (1, 2)



9. (−2, 0)



10. (−3, −0.3)



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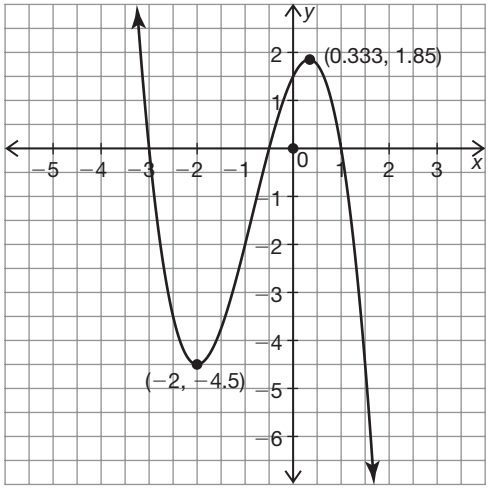
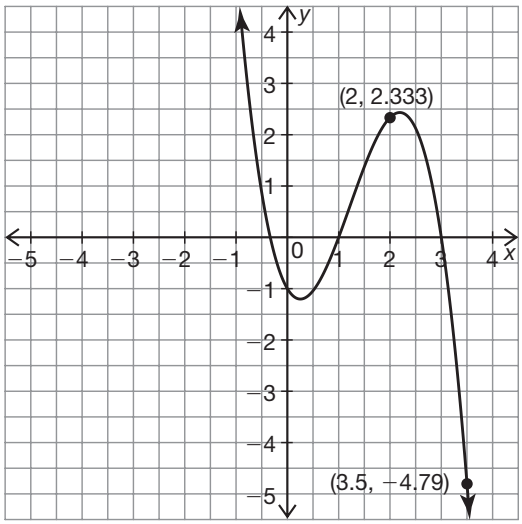
11. (−2, 0.333)

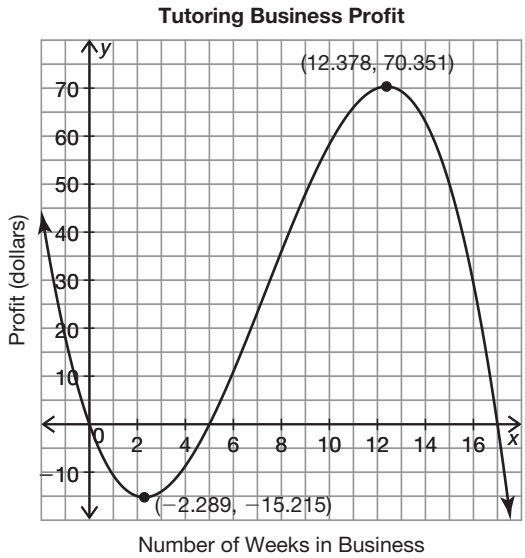


12. (2, 3.5)



Solve each equation using the information found in the graph.

13. The graph models the profit a group of students earns running a tutoring business. After how many weeks did it take the group to earn a profit? Where is this information located on the graph?



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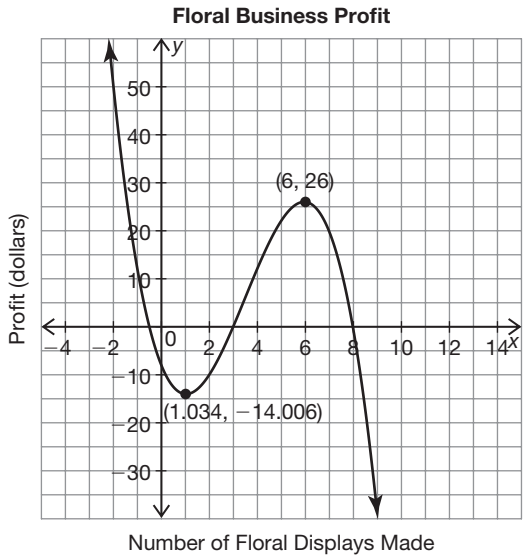
The group earns a profit after 5 weeks in business. The information is located at the *x*-intercept of (5, 0).

14. The graph models the amount of money a company makes producing floral displays. What is the maximum number of floral displays that the company can create and make a profit? Where is this information located on the graph?



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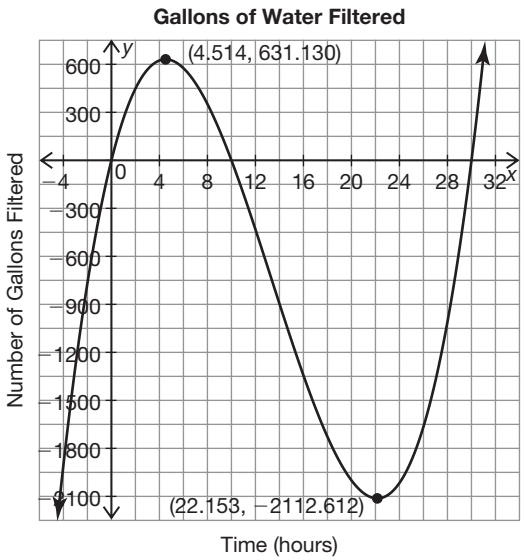
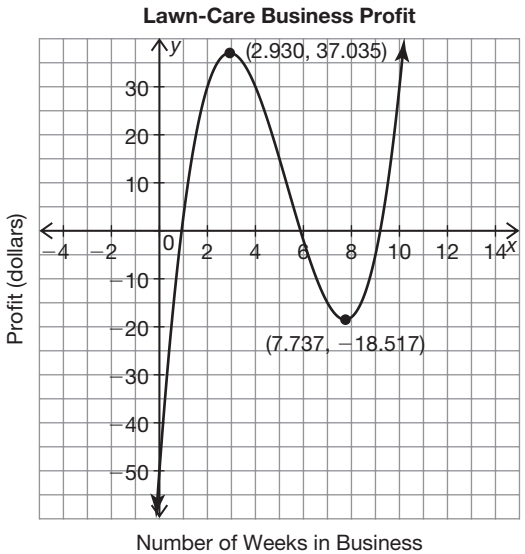
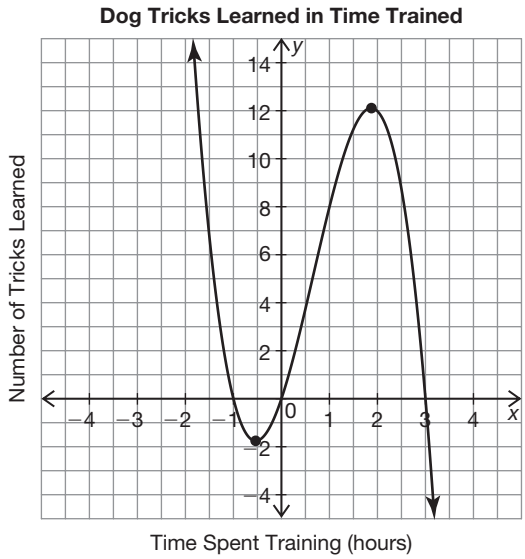
15. The graph models the number of gallons of water that are filtered at a filtration plant hourly. How many gallons of water has the plant filtered after running for about 4.5 hours? Where is this information located on the graph?



16. The graph models the amount of profit Emilio earns from his own lawn-care business. How much did Emilio initially invest to start his business? Where is this information located on the graph?



17. The graph models the number of tricks that a dog can perform based on the number of hours it is trained. Estimate how long it takes the dog to learn 8 tricks. Where is this information located on the graph?



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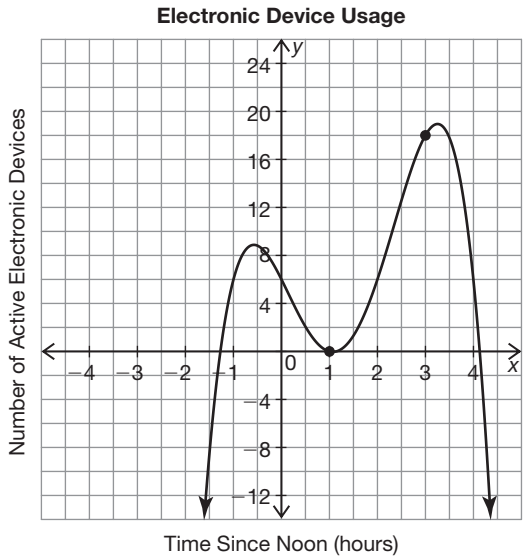
18. The graph models the number of electronic devices that are being use in a home during the hours of noon and 4:00 pm. Estimate the time when the greatest number of electronic devices are being used. Where is this information located on the graph?



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The Great Polynomial Divide

Polynomial Division

Vocabulary

Write an example for each term. Write the dividend as the product of the divisor and the quotient plus the remainder.

1. Polynomial long division

2. Synthetic division

Problem Set

Write the zero that corresponds to each factor.

1. $x + 5$
   $x = -5$

2. $x - 12$

3. $2x + 1$

4. $10x - 9$

5. $x - 13$

6. $3x + 4$

Write the factor that corresponds to each zero.

7. $x = 2$

$x - 2$

8. $x = -7$

9. $x = -75$

10. $x = \frac{2}{3}$

11. $x = -\frac{3}{8}$

12. $x = \frac{5}{4}$

Determine if the given factor is a factor of each polynomial. Explain your reasoning.

13. Is $x - 1$ a factor of $x^4 - 3x^3 + 6x^2 - 12x + 8$?

$$
\begin{array}{r}
x^3 - 2x^2 + 4x - 8 \\
x - 1\overline{)\,x^4 - 3x^3 + 6x^2 - 12x + 8} \\
\underline{x^4 - x^3} \\
-2x^3 + 6x^2 \\
\underline{-2x^3 + 2x^2} \\
4x^2 - 12x \\
\underline{4x^2 - 4x} \\
-8x + 8 \\
\underline{-8x + 8} \\
0
\end{array}
$$

Yes, $x - 1$ is a factor of $x^4 - 3x^3 + 6x^2 - 12x + 8$ because it divides into the polynomial without a remainder.

14. Is $x - 1$ a factor of $x^4 + 6x^3 - 12x^2 - 38x - 21$?

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15. Is $3x + 2$ a factor of
$3x^5 + 20x^4 + 9x^3 - 92x^2 - 60x$?

16. Is $x - 3$ a factor of
$x^3 + 12x^2 + 17x - 30$?

17. Is $x + 4$ a factor of $2x^3 + 7x^2 - 10x - 24$?

18. Is $x + 2$ a factor of $x^4 - 2x^3 - x^2 - 4x - 6$?

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Determine each quotient using polynomial long division. Write the dividend as the product of the divisor and the quotient plus the remainder.

19. $x - 4\overline{)2x^3 - 7x^2 - 19x + 60}$

$$
\begin{array}{r}
2x^2 + x - 15 \\
x - 4\overline{)2x^3 - 7x^2 - 19x + 60} \\
\underline{2x^3 - 8x^2} \\
x^2 - 19x \\
\underline{x^2 - 4x} \\
-15x + 60 \\
\underline{-15x + 60} \\
0
\end{array}
$$

$2x^3 - 7x^2 - 19x + 60 = (x - 4)(2x^2 + x - 15)$

20. $x - 2\overline{)2x^3 - x^2 - 13x - 6}$

21. $x + 3\overline{)x^3 + 8x^2 + 7x + 5}$

22. $x + 2\overline{)3x^3 + 5x^2 - 2x}$

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23. $x + 1\overline{)4x^4 + 9x^3 - 82x^2 - 57x + 18}$

24. $x - 3\overline{)x^4 + 5x^3 - 33x^2 + 27x}$

Determine each quotient using synthetic division. Write the dividend as the product of the divisor and the quotient plus the remainder.

25. $(x^4 + 8x^3 - 3x^2 - 24x) \div (x - 3)$

3	1	8	−3	−24	0
		3	33	90	108
	1	11	30	66	108

$x^4 + 8x^3 - 3x^2 - 24x = (x - 3)\left((x^3 + 11x^2 + 30x + 66) + \frac{108}{x - 3}\right)$

26. $(x^4 - 3x^3 + 6x^2 - 12x + 8) \div (x - 1)$

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27. $(2x^3 + 21x^2 + 22x - 45) \div (2x + 5)$

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28. $(x^3 + x^2 - 16x - 16) \div (x + 2)$

29. $(x^4 - 6x^3 - 19x^2 + 24x) \div (x + 3)$

30. $(x^4 + 5x^3 - 33x^2 + 27x) \div (x - 9)$

4

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The Factors of Life

The Factor Theorem and Remainder Theorem

Vocabulary

Remainder Theorem	Factor Theorem

Choose the term from the box that best completes each statement.

1. The _____ states that a linear polynomial $(x - r)$ is a factor of a polynomial $p(x)$ if and only if $p(r) = 0$ and $\frac{p(x)}{(x - r)}$ has a remainder of zero.
2. The _____ states that when any polynomial equation or function is divided by a linear factor $(x - r)$, the remainder is the value of the equation or function when $x = r$.

Problem Set

Determine each function value using the Remainder Theorem. Explain your reasoning.

1. Determine $p(3)$ if $p(x) = 2x^3 - 6x^2 - 36x - 36$.

$$\begin{array}{r} 2x^2 + 0x^2 - 36 \\ x - 3\overline{)2x^3 - 6x^2 - 36x - 36} \\ \underline{2x^3 - 6x^2} \\ -36x - 36 \\ \underline{-36x + 108} \\ -144 \end{array}$$

When $p(x)$ is divided by $x - 3$, the remainder is -144. So, by the Remainder Theorem $p(3) = -144$.

2. Determine $p(-2)$ if $p(x) = x^4 - 10x^3 + 8x^2 + 106x - 105$.

3. Determine $p(-3)$ if $p(x) = 2x^4 + 5x^3 + 8x^2 + 15x + 6$.

4. Determine $p(1)$ if $p(x) = x^4 + 3x^3 - 6x^2 - 8x$.

5. Determine $p(10)$ if $p(x) = 6x^3 + 11x^2 - 3x - 2$.

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6. Determine $p\left(\frac{1}{3}\right)$ if $p(x) = x^4 - x^3 + 7x^2 - 9x - 18$.

Use the Factor Theorem to determine whether the given expression is a factor of each polynomial. Explain your reasoning.

7. Is $x - 2$ a factor of $f(x) = x^3 + 8x^2 - 31x + 22$?

If $x - 2$ is a factor of $f(x)$, then by the Factor Theorem $f(2) = 0$.

$f(2) = (2)^3 + 8(2)^2 - 31(2) + 22$

$f(2) = 8 + 32 - 62 + 22$

$f(2) = 0$

When $f(x)$ is evaluated at 2, the result is 0. According to the Factor Theorem $x - 2$ is a factor of $f(x)$.

8. Is $x - 3$ a factor of $f(x) = 4x^4 - x^3 - 52x^2 - 35x + 12$?

9. Is $x - 12$ a factor of $f(x) = x^4 - 12x^3 + x^2 - 12x$?

10. Is $x - 8$ a factor of $f(x) = x^3 - 7x^2 - 14x + 48$?

11. Is $x - 5$ a factor of $f(x) = x^3 + 5x^2 - x - 5$?

12. Is $3x + 4$ a factor of $f(x) = 3x^3 + 13x^2 + 18x + 8$?

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Use the Factor Theorem to determine whether $g(x)$ is the factored form of $f(x)$. Explain your reasoning.

13. Is $g(x) = (x + 8)(x - 1)(x + 2)$ the factored form of $f(x) = x^3 - 7x^2 - 10x + 16$?

$f(-8) = (-8)^3 - 7(-8)^2 - 10(-8) + 16$
$f(-8) = 512 - 448 + 80 + 16$
$f(-8) = 160$

$f(-2) = (-2)^3 - 7(-2)^2 - 10(-2) + 16$
$f(-2) = -8 - 28 + 20 + 16$
$f(-2) = 0$

$f(1) = (1)^3 - 7(1)^2 - 10(1) + 16$
$f(1) = 1 - 7 - 10 + 16$
$f(1) = 0$

No, the function $g(x)$ is not the factored form of $f(x)$. Since $f(-8) = 160$, $x + 8$ is not a factor of $f(x)$ by the Factor Theorem.

14. Is $g(x) = (x - 3)(x + 5)(x + 2)(x - 1)$ the factored form of $f(x) = x^4 + 3x^3 - 15x^2 - 19x + 30$?

15. Is $g(x) = (x - 2)(x + 9)(x + 1)$ the factored form of $f(x) = x^3 + 8x^2 - 11x - 18$?

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16. Is $g(x) = x(x - 4)(x - i\sqrt{7})(x + i\sqrt{7})$ the factored form of $f(x) = x^4 + 4x^3 + 7x^2 + 28x$?

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17. Is $g(x) = (x + 1)(x + 2)(4x + 7)$ the factored form of $f(x) = 4x^3 - 11x^2 - x + 14$?

18. Is $g(x) = (x - 1)(x + 1)(x - 3i)(x + 3i)$ the factored form of $f(x) = x^4 + 8x^2 - 9$?

Use the Factor Theorem to determine the unknown coefficient so that the given linear expression is a factor of the function.

19. Determine a if $x + 3$ is a factor of $f(x) = x^3 + 9x^2 + ax + 15$.

If $x + 3$ is a factor, then by the Factor Theorem $f(-3) = 0$.

$f(-3) = (-3)^3 + 9(-3)^2 + a(-3) + 15$

$= -27 + 81 - 3a + 15$

$= 69 - 3a$

By the Transitive Property, $69 - 3a = 0$.

$69 - 3a = 0$

$69 = 3a$

$a = 23$

20. Determine a if $x - 4$ is a factor of $f(x) = x^3 + ax^2 - 20x - 48$.

21. Determine a if $x - 1$ is a factor of $f(x) = ax^3 - 10x^2 - 13x + 20$.

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22. Determine a if $x - 7$ is a factor of $f(x) = x^4 - 4x^3 + ax^2 - 8x - 42$.

23. Determine a if $x + 2$ is a factor of $f(x) = x^3 - x^2 + ax - 36$.

24. Determine a if $x - 8$ is a factor of $f(x) = x^4 + ax^3 - 5x^2 - 21x - 24$.

4

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Break It Down

Factoring Higher Order Polynomials

Problem Set

Factor each expression completely.

1. $x^2 + 12x - 13$

$x^2 + 12x - 13 = (x + 13)(x - 1)$

2. $x^2 + 6x + 8$

3. $x^2 - 12x - 28$

4. $x^2 + 30x + 81$

5. $x^2 - 5x - 14$

6. $x^2 - 16x - 36$

Factor each expression by factoring out the greatest common factor.

7. $2x^5 - 8x^4 + 10x^3$

$2x^5 - 8x^4 + 10x^3 = 2x^3(x^2 - 4x + 10)$

8. $-9x^4 + 45x^3 - 9x^2$

9. $105x^3 - 147x$

10. $-\frac{3}{5}x^4 + \frac{3}{5}x^3 - \frac{27}{5}x^2$

11. $\frac{1}{3}x^4 - \frac{8}{3}x^3 + \frac{1}{3}x^2 - \frac{11}{3}x$

12. $8x^4 - 16x^3 + 56x^2 - 24x$

Factor each expression completely using the chunking method.

13. $4x^2 + 8x + 3$

$4x^2 + 8x + 3 = (2x)^2 + 4(2x) + 3$

Let $z = 2x$

$= z^2 + 4z + 3$

$= (z + 1)(z + 3)$

$= (2x + 1)(2x + 3)$

14. $25x^2 - 35x + 12$

15. $121x^2 - 44x - 12$

16. $49x^2 + 63x + 18$

17. $9x^2 + 30x - 11$

18. $169x^2 - 130x + 24$

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Factor each expression completely using the factor by grouping method.

19. $x^3 - 2x^2 + 3x - 6$

$x^3 - 2x^2 + 3x - 6 = x^2(x - 2) + 3(x - 2)$

$= (x^2 + 3)(x - 2)$

$= (x + i\sqrt{3})(x - i\sqrt{3})(x - 2)$

20. $x^3 + x^2 - 4x - 4$

21. $x^3 - 6x^2 - 9x + 54$

22. $x^4 - 3x^3 - x^2 - 3x$

23. $-x^3 + 5x^2 + 16x - 80$

24. $x^3 - 3x^2 - 4x + 12$

Factor each quartic expression completely using the quadratic form method.

25. $x^4 - 13x^2 + 36$

$x^4 - 13x^2 + 36 = (x^2 - 4)(x^2 - 9)$

$= (x - 2)(x + 2)(x - 3)(x + 3)$

26. $x^4 - 50x^2 + 49$

27. $x^4 - 29x^2 + 100$

28. $x^4 - 25x^2 + 144$

29. $x^4 - 164x^2 + 6{,}400$

30. $x^4 - 61x^2 + 900$

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Factor each binomial using the sum or difference of perfect cubes formula.

31. $x^3 + 27$

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$x^3 + 27 = (x)^3 + (3)^3$

$= (x + 3)(x^2 - 3x + 9)$

32. $x^3 - 8y^3$

33. $8x^3 - 125$

34. $x^3 + 64y^3$

35. $343x^3 - 1$

36. $216x^3 + 125y^3$

Factor each binomial completely over the set of real numbers using the difference of squares method.

37. $x^2 - 100$

$a^2 - b^2 = (a + b)(a - b)$.

$x^2 - 100 = (x + 10)(x - 10)$

38. $x^4 - 36$

39. $49x^2 - 4y^2$

40. $x^{10} - 81$

41. $9x^4 - 121y^2$

42. $4x^{14} - 9y^8$

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Factor each perfect square trinomial.

43. $4x^2 + 12x + 9$

$a^2 + 2ab + b^2 = (a + b)^2$

$4x^2 + 12x + 9 = (2x)^2 + 2(2x)(3) + (3)^2$

$= (2x + 3)^2$

44. $x^2 - 12xy + 36y^2$

45. $16x^2 + 104x + 169$

46. $25x^2 + 80x + 64$

47. $9x^4 + 42x^2y + 49y^2$

48. $64x^2 + 16xy^2 + y^4$

4

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Getting to the Root of It All
Rational Root Theorem

Vocabulary

Write a definition for the term in your own words.

1. Rational Root Theorem

Problem Set

Determine the possible rational roots of each polynomial using the Rational Root Theorem.

1. $x^3 - 4x^2 + 6x - 8 = 0$

   $p = \pm 1, \pm 2, \pm 4, \pm 8$

   $q = \pm 1$

   $\frac{p}{q} = \pm 1, \pm 2, \pm 4, \pm 8$

2. $2x^4 - 4x^2 + 15 = 0$

3. $-2x^3 + 5x + 18 = 0$

4. $x^3 + 12x^2 - 21x + 32 = 0$

5. $5x^4 - 7x^3 + 5x - 30 = 0$

6. $12x^4 - 15x^3 + 24x^2 + 11x - 2 = 0$

Use the Rational Root Theorem to determine the possible rational roots for each polynomial equation. Then, solve completely. Use the graph, if given, to identify possible zeros.

7. $x^3 + 3x^2 - 18x - 40 = 0$

- Possible rational roots:

  $p = \pm 1, \pm 2, \pm 4, \pm 5, \pm 8, \pm 10, \pm 20, \pm 40$

  $q = \pm 1$

  $\frac{p}{q} = \pm 1, \pm 2, \pm 4, \pm 5, \pm 8, \pm 10, \pm 20, \pm 40$

- Solve completely:

$$\begin{array}{r|cccc} 4 & 1 & 3 & -18 & -40 \\ & \downarrow & 4 & 28 & 40 \\ \hline & 1 & 7 & 10 & 0 \end{array}$$

$x^3 + 3x^2 - 18x - 40 = (x - 4)(x^2 + 7x + 10)$

$= (x - 4)(x + 2)(x + 5)$

$x = 4, -2, -5$

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8. $6x^3 + 35x^2 - 52x - 21 = 0$

- Possible rational roots:

- Solve completely:

9. $x^4 + 4x^3 - 21x^2 - 36x + 108 = 0$

- Possible rational roots:

- Solve completely:

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10. $x^2 - 8x + 17 = 0$

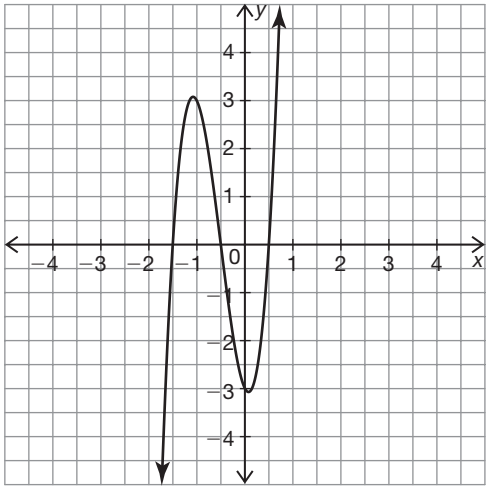
- Possible rational roots:

- Solve completely:

11. $8x^3 + 12x^2 - 2x - 3 = 0$

- Possible rational roots:

- Solve completely:



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12. $x^4 - 4x^3 + 5x^2 - 4x + 4 = 0$

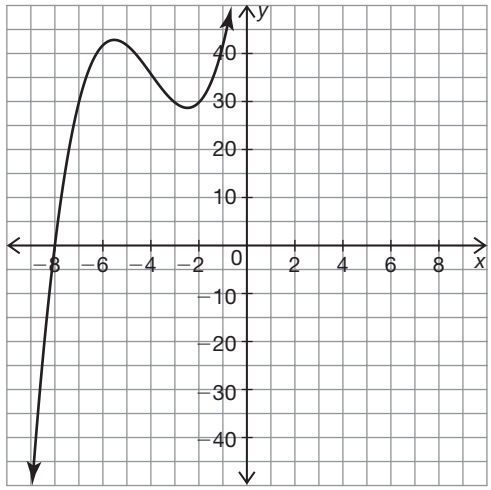
- Possible rational roots:

- Solve completely:

13. $x^3 + 12x^2 + 41x + 72 = 0$

- Possible rational roots:

- Solve completely:



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14. $x^3 - 1.5x^2 - 1.5x + 1 = 0$

- Possible rational roots:

- Solve completely:

4

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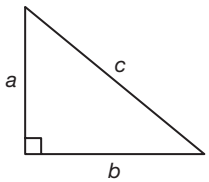
Identity Theft
Exploring Polynomial Identities

Vocabulary

1. Given positive integers r and s, where $r > s$, write the terms in Euclid's Formula that correspond to each side length in a right triangle, a, b, and c.



Problem Set

Use polynomial identities and number properties to perform each calculation.

1. 109^2

Answers will vary.

$109^2 = (100 + 9)^2$

$= 100^2 + 2(100)(9) + 9^2$

$= 10{,}000 + 1800 + 81$

$= 11{,}881$

2. 54^3

3. 38^3

4. 99^2

5. 127^2

6. 75^3

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Determine whether each set of numbers is a Pythagorean triple. Explain your reasoning.

7. 5, 12, 13

$5^2 + 12^2 \stackrel{?}{=} 13^2$

$25 + 144 \stackrel{?}{=} 169$

$169 = 169$

These numbers are a Pythagorean triple because $5^2 + 12^2 = 13^2$.

8. 60, 61, 11

9. 8, 15, 16

10. 4, 8, 12

11. 10, 24, 26

12. 1, 2, $\sqrt{5}$

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Generate a Pythagorean triple using each pair of given numbers and Euclid's Formula.

13. 3 and 8

$(8^2 + 3^2)^2 = (8^2 - 3^2)^2 + (2(8)(3))^2$

$(64 + 9)^2 = (64 - 9)^2 + (6(8))^2$

$73^2 = 55^2 + 48^2$

$5329 = 5329$

The Pythagorean triple is 48, 55, 73.

14. 4 and 12

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15. 11 and 9

16. 7 and 13

17. 50 and 60

18. 25 and 100

Verify each algebraic statement by transforming one side of the equation to show that it is equivalent to the other side of the equation.

19. $g^6 - h^6 = (g^2 - h^2)(g^2 - gh + h^2)(g^2 + gh + h^2)$

Method 1:

$$g^6 - h^6 \stackrel{?}{=} (g^2 - h^2)(g^2 - gh + h^2)(g^2 + gh + h^2)$$
$$\stackrel{?}{=} (g^4 - g^3h + g^2h^2 - g^2h^2 + gh^3 - h^4)(g^2 + gh + h^2)$$
$$\stackrel{?}{=} g^6 + g^5h + g^4h^2 - g^5h - g^4h^2 - g^3h^3 + g^4h^2 + g^3h^3 + g^2h^4 - g^4h^2 - g^3h^3 - g^2h^4 + g^3h^3 + g^2h^4 - gh^5 - g^2h^4 - gh^5 - h^6$$
$$= g^6 - h^6$$

Method 2:

$$g^6 - h^6 \stackrel{?}{=} (g^2 - h^2)(g^2 - gh + h^2)(g^2 + gh + h^2)$$
$$(g^3 + h^3)(g^3 - h^3) \stackrel{?}{=}$$
$$(g + h)(g^2 - gh + h^2)(g - h)(g^2 + gh + h^2) \stackrel{?}{=}$$
$$(g^2 - h^2)(g^2 - gh + h^2)(g^2 + gh + h^2) =$$

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20. $(m^2 + n^2)^3 = (m^2 + n^2)(m^4 + 2m^2n^2 + n^4)$

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21. $p^8 - q^8 = (p - q)(p + q)(p^2 + q^2)(p^4 + q^4)$

22. $r^4 - s^4 = (r^2 + s^2)(r + s)(r - s)$

23. $a^{15} + b^{15} = (a^5 + b^5)(a^{10} - a^5b^5 + b^{10})$

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24. $(v^6 + w^6)^2 = (v^6 - w^6)^2 + (2v^3w^3)^2$

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The Curious Case of Pascal's Triangle

Pascal's Triangle and the Binomial Theorem

Vocabulary

Write a definition for the term in your own words.

1. Binomial Theorem

Problem Set

Use Pascal's Triangle to expand each binomial.

1. $(a + b)^4 =$

   $(a + b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

2. $(a + b)^7 =$

3. $(a + b)^8 =$

4. $(a + b)^9 =$

5. $(a + b)^{10} =$

6. $(a + b)^{13} =$

Perform each calculation and simplify.

7. $7! =$

$7! = (7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1)$

$= 5040$

8. $12! =$

9. $3!4! =$

10. $5!8! =$

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11. $\frac{6!}{4!} =$

12. $\frac{17!}{14!3!} =$

Perform each calculation and simplify.

13. $\binom{5}{3} =$

$$\binom{5}{3} = \frac{5!}{3!(5-3)}$$
$$= \frac{5 \cdot 4 \cdot \cancel{3 \cdot 2 \cdot 1}}{(\cancel{3 \cdot 2 \cdot 1})(2 \cdot 1)}$$
$$= \frac{20}{2}$$
$$= 10$$

14. ${}_6C_3 =$

15. $\binom{10}{4} =$

16. ${}_{11}C_5 =$

17. $\binom{16}{2} =$

18. ${}_{13}C_4 =$

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Use the Binomial Theorem and substitution to expand each binomial.

19. $(x - 2y)^6 =$

$(a + b)^6 = \binom{6}{0}a^6b^0 + \binom{6}{1}a^5b^1 + \binom{6}{2}a^4b^2 + \binom{6}{3}a^3b^3 + \binom{6}{4}a^2b^4 + \binom{6}{5}a^1b^5 + \binom{6}{6}a^0b^6$

$= a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$

Let $a = x$ and let $b = -2y$.

$(x - 2y)^6 = x^6 + 6x^5(-2y) + 15x^4(-2y)^2 + 20x^3(-2y)^3 + 15x^2(-2y)^4 + 6x(-2y)^5 + (-2y)^6$

$= x^6 - 12x^5y + 15x^4(4y^2) - 20x^3(8y^3) + 15x^2(16y^4) - 6x(32y^5) + 64y^6$

$= x^6 - 12x^5y + 60x^4y^2 - 160x^3y^3 + 240x^2y^4 - 192xy^5 + 64y^6$

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20. $(x + 3y)^5 =$

21. $(4x - y)^7 =$

22. $(3x + 2y)^4 =$

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23. $(-x + 5y)^8 =$

24. $(2x - 3)^6 =$

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