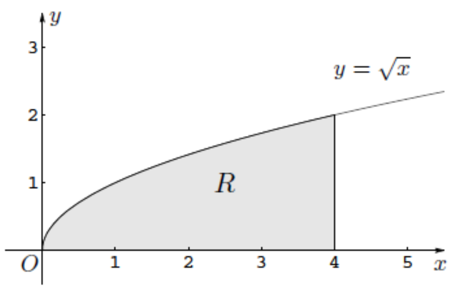
Calculus AB Chapter 7 Review

1. Let R be the region bounded by the x–axis, the graph of $y = \sqrt{x}$, and the line $x = 4$.
   (a) Find the area of the region R.
   (b) Find the value of h such that the vertical line $x = h$ divides the region R into two regions of equal area.
   (c) Find the volume of the solid generated when R is revolved about the x–axis.
   (d) The vertical line $x = k$ divides the region R into two regions such that when these two regions are revolved about the x–axis, they generate solids with equal volumes. Find the value of k.

(a)



$$A = \int_0^4 \sqrt{x}\,dx = \frac{2}{3}x^{3/2}\Big|_0^4 = \frac{16}{3} \text{ or } 5.333$$

$$2 \begin{cases} 1: & A = \int_0^4 \sqrt{x}\,dx \\ 1: & \text{answer} \end{cases}$$

(b) $\int_0^h \sqrt{x}\,dx = \frac{8}{3}$ –or– $\int_0^h \sqrt{x}\,dx = \int_h^4 \sqrt{x}\,dx$

$\frac{2}{3}h^{3/2} = \frac{8}{3}$ –or– $\frac{2}{3}h^{3/2} = \frac{16}{3} - \frac{2}{3}h^{3/2}$

$h = \sqrt[3]{16}$ or 2.520 or 2.519

$$2 \begin{cases} 1: & \text{equation in } h \\ 1: & \text{answer} \end{cases}$$

(c) $V = \pi\int_0^4 (\sqrt{x})^2\,dx = \pi\frac{x^2}{2}\Big|_0^4 = 8\pi$

or 25.133 or 25.132

$$3 \begin{cases} 1: & \text{limits and constant} \\ 1: & \text{integrand} \\ 1: & \text{answer} \end{cases}$$

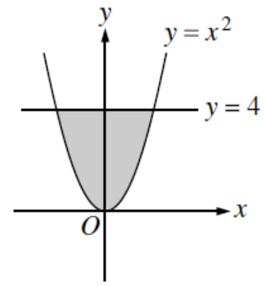
(d) $\pi\int_0^k (\sqrt{x})^2\,dx = 4\pi$ –or– $\pi\int_0^k (\sqrt{x})^2\,dx = \pi\int_k^4 (\sqrt{x})^2\,dx$

$\pi\frac{k^2}{2} = 4\pi$ –or– $\pi\frac{k^2}{2} = 8\pi - \pi\frac{k^2}{2}$

$k = \sqrt{8}$ or 2.828

$$2 \begin{cases} 1: & \text{equation in } k \\ 1: & \text{answer} \end{cases}$$

2. The shaded region, R, is bounded by the graph of $y = x^2$ and the line $y = 4$, as shown in the figure above.



(a) Find the area of R.

(b) Find the volume of the solid generated by revolving R about the x–axis.

(c) There exists a number k, $k > 4$, such that when R is revolved about the line $y = k$, the resulting solid has the same volume as the solid in part (b). Write, but do not solve, an equation involving an integral expression that can be used to find the value of k.

(a) Area $= \int_{-2}^{2} (4 - x^2)\,dx$

$$= 2\int_{0}^{2} (4 - x^2)\,dx$$

$$= 2\left[4x - \frac{x^3}{3}\right]_0^2$$

$$= \frac{32}{3} = 10.666 \text{ or } 10.667$$

2 { 1: integral; 1: answer }

(b) Volume $= \pi \int_{-2}^{2} \left(4^2 - (x^2)^2\right) dx$

$$= 2\pi \int_{0}^{2} (16 - x^4)\,dx$$

$$= 2\pi \left[16x - \frac{x^5}{5}\right]_0^2$$

$$= \frac{256\pi}{5} = 160.849 \text{ or } 160.850$$

3 { 1: limits and constant; 1: integrand; 1: answer }

(c) $\pi \int_{-2}^{2} \left[(k - x^2)^2 - (k - 4)^2\right]\,dx = \frac{256\pi}{5}$

4 { 1: limits and constant; 2: integrand <−1> each error; 1: equation }

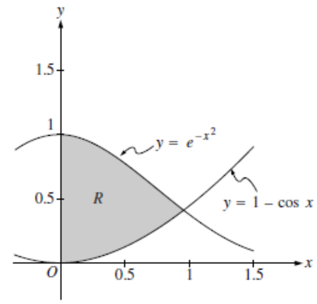
3.

Let R be the shaded region in the first quadrant enclosed by the graphs of $y = e^{-x^2}$, $y = 1 - \cos x$, and the y-axis, as shown in the figure above.

(a) Find the area of the region R.

(b) Find the volume of the solid generated when the region R is revolved about the x-axis.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.



Region R $e^{-x^2} = 1 - \cos x$ at $x = 0.941944 = A$	1 : Correct limits in an integral in (a), (b), or (c).
(a) Area $= \int_0^A (e^{-x^2} - (1 - \cos x))\,dx$ $= 0.590$ or 0.591	2 { 1 : integrand 1 : answer
(b) Volume $= \pi \int_0^A \left(\left(e^{-x^2}\right)^2 - (1 - \cos x)^2 \right) dx$ $= 0.55596\pi = 1.746$ or 1.747	3 { 2 : integrand and constant < −1> each error 1 : answer
(c) Volume $= \int_0^A \left(e^{-x^2} - (1 - \cos x)\right)^2 dx$ $= 0.461$	3 { 2 : integrand < −1> each error Note: 0/2 if not of the form $k \int_c^d (f(x) - g(x))^2\,dx$ 1 : answer

The other best Means of studying is the Classworks we did.

Suggested problems: p. 511 #'s 1-7 odd, 11, 23, 24