Review Exercise Set 20

Exercise 1: Use Gaussian elimination to find the solution for the given system of equations.

$$3x + y - z = 1$$
$$x - y + z = -3$$
$$2x + y + z = 0$$

Exercise 2: Use Gaussian elimination to find the solution for the given system of equations.

$$2x + 5y = 9$$
$$x + 2y - z = 3$$
$$-3x - 4y + 7z = 1$$

Review Exercise Set 20 Answer Key

Exercise 1: Use Gaussian elimination to find the solution for the given system of equations.

$3x + y - z = 1$
$x - y + z = -3$
$2x + y + z = 0$

Setup the augmented matrix

$$\left[\begin{array}{ccc|c} 3 & 1 & -1 & 1 \\ 1 & -1 & 1 & -3 \\ 2 & 1 & 1 & 0 \end{array}\right]$$

Perform row operations to reduce the matrix

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -3 \\ 3 & 1 & -1 & 1 \\ 2 & 1 & 1 & 0 \end{array}\right] R_1 \leftrightarrow R_2$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -3 \\ 0 & 4 & -4 & 10 \\ 2 & 1 & 1 & 0 \end{array}\right] -3R_1 + R_2 \rightarrow R_2$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -3 \\ 0 & 4 & -4 & 10 \\ 0 & 3 & -1 & 6 \end{array}\right] -2R_1 + R_3 \rightarrow R_3$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -3 \\ 0 & 1 & -1 & \frac{5}{2} \\ 0 & 3 & -1 & 6 \end{array}\right] R_2 \div 4 \rightarrow R_2$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -\frac{1}{2} \\ 0 & 1 & -1 & \frac{5}{2} \\ 0 & 3 & -1 & 6 \end{array}\right] R_2 + R_1 \rightarrow R_1$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -\frac{1}{2} \\ 0 & 1 & -1 & \frac{5}{2} \\ 0 & 0 & 2 & -\frac{3}{2} \end{array}\right] -3R_2 + R_3 \rightarrow R_3$$

Exercise 1 (Continued):

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -\frac{1}{2} \\ 0 & 1 & -1 & \frac{5}{2} \\ 0 & 0 & 1 & -\frac{3}{4} \end{array}\right] R_3 \div 2 \rightarrow R_3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -\frac{1}{2} \\ 0 & 1 & 0 & \frac{7}{4} \\ 0 & 0 & 1 & -\frac{3}{4} \end{array}\right] R_3 + R_2 \rightarrow R_2$$

The solution set (x, y, z) is $\left(-\frac{1}{2}, \frac{7}{4}, -\frac{3}{4}\right)$

Exercise 2: Use Gaussian elimination to find the solution for the given system of equations.

2x + 5y = 9
x + 2y - z = 3
-3x - 4y + 7z = 1

Setup the augmented matrix

$$\left[\begin{array}{ccc|c} 2 & 5 & 0 & 9 \\ 1 & 2 & -1 & 3 \\ -3 & -4 & 7 & 1 \end{array}\right]$$

Perform row operations to reduce the matrix

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 2 & 5 & 0 & 9 \\ -3 & -4 & 7 & 1 \end{array}\right] R_1 \leftrightarrow R_2$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & 1 & 2 & 3 \\ -3 & -4 & 7 & 1 \end{array}\right] -2R_1 + R_2 \rightarrow R_2$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & 1 & 2 & 3 \\ 0 & 2 & 4 & 10 \end{array}\right] 3R_1 + R_3 \rightarrow R_3$$

Exercise 2 (Continued):

$$\left[\begin{array}{ccc|c} 1 & 0 & -5 & -3 \\ 0 & 1 & 2 & 3 \\ 0 & 2 & 4 & 10 \end{array}\right] -2R_2 + R_1 \to R_1$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -5 & -3 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 4 \end{array}\right] -2R_2 + R_3 \to R_3$$

$0 \neq 4$ so this system is inconsistent and has no solution.

Exercise 3: The circle given by the equation $x^2 + y^2 + ax + by + c = 0$ passes through the points (-2, 0), (-1, 7), and (5, -1). Find a, b, and c.

Substitute the given points into the equation to find the system of equations

$x^2 + y^2 + ax + by + c = 0$
$(-2)^2 + (0)^2 + a(-2) + b(0) + c = 0$
$4 - 2a + c = 0$
$-2a + c = -4$

$x^2 + y^2 + ax + by + c = 0$
$(-1)^2 + (7)^2 + a(-1) + b(7) + c = 0$
$1 + 49 - a + 7b + c = 0$
$-a + 7b + c = -50$

$x^2 + y^2 + ax + by + c = 0$
$(5)^2 + (-1)^2 + a(5) + b(-1) + c = 0$
$25 + 1 + 5a - b + c = 0$
$5a - b + c = -26$

$-2a + c = -4$
$-a + 7b + c = -50$
$5a - b + c = -26$

Setup the augmented matrix

$$\left[\begin{array}{ccc|c} -2 & 0 & 1 & -4 \\ -1 & 7 & 1 & -50 \\ 5 & -1 & 1 & -26 \end{array}\right]$$