Matrix Algebra Tutor - Worksheet 5 – Gaussian Elimination and Gauss-Jordan Elimination

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1. Use the Gaussian Elimination method to solve this system of equations.

$$\begin{aligned} -5x+8y-4z&=38 \\ 6y+3z&=-9 \\ -4z&=20 \end{aligned}$$

2. Use the Gauss-Jordan Elimination method to solve this system of equations.

$$\begin{aligned} 7x+3y-3z&=-4 \\ -6y+2z&=24 \\ 10z&=30 \end{aligned}$$

3. Use the Gaussian Elimination method to solve this system of equations.

$$\begin{aligned} 4x + 2y + 3z &= 5 \\ -2y + 5z &= -9 \\ 12z &= -12 \end{aligned}$$

4. Use the Gauss-Jordan Elimination method to solve this system of equations.

$$\begin{aligned} 5x + 3y + 2z &= 15 \\ 2y + 4z &= 24 \\ y - 4z &= -42 \end{aligned}$$

5. Use the Gaussian Elimination method to solve this system of equations.

$$\begin{aligned} -2x - y - z &= -11 \\ 3x + 4y + z &= 19 \\ 3x + 6y + 5z &= 43 \end{aligned}$$

6. Use the Gauss-Jordan Elimination method to solve this system of equations.

$$\begin{aligned} 2x + y + z &= 11 \\ 3x + 5y + 5z &= 34 \\ -5x - 6y - 3z &= -42 \end{aligned}$$

7. Use the Gauss-Jordan Elimination method to solve this system of equations.

$$2x+5y-3z=-10$$
$$-2x-2y+2z=2$$
$$-x+-5y+z=17$$

Answers – Matrix Algebra Tutor - Worksheet 5 – Gaussian Elimination and Gauss-Jordan Elimination

As we go through the solutions to these problems, bear in mind that there are multiple ways to solve each problem.

1. Use the Gaussian Elimination method to solve this system of equations.

$$\begin{aligned} -5x+8y-4z&=38 \\ 6y+3z&=-9 \\ -4z&=20 \end{aligned}$$

Write the system of equations as an augmented matrix:

$$\begin{bmatrix} -5 & 8 & -4 & \vdots & 38 \\ 0 & 6 & 3 & \vdots & -9 \\ 0 & 0 & -4 & \vdots & 20 \end{bmatrix}$$

The Gaussian Elimination method requires the matrix to be transformed into a row equivalent matrix of this form in which all elements that are not specified as 1 or 0 can be any number which results from transforming the matrix:

$$\begin{bmatrix} 1 & & & \vdots & \\ 0 & 1 & & \vdots & \\ 0 & 0 & 1 & \vdots & \end{bmatrix}$$

Transform the matrix:

$$\begin{bmatrix} -5 & 8 & -4 & \vdots & 38 \\ 0 & 6 & 3 & \vdots & -9 \\ 0 & 0 & -4 & \vdots & 20 \end{bmatrix} \begin{matrix} \\ \leftarrow \frac{1}{6}R_2 \\ \leftarrow -\frac{1}{4}R_3 \end{matrix}$$

The symbol $\leftarrow \frac{1}{6}R_2$ means multiply Row 2 by $\frac{1}{6}$ and put the result in Row 2. The symbol $\leftarrow -\frac{1}{4}R_3$ means multiply Row 3 by $-\frac{1}{4}$ and put the result in Row 3.

The math for this manipulation and the resulting row equivalent matrix is:

$$\begin{bmatrix} -5 & 8 & -4 & \vdots & 38 \\ \frac{1}{6}\cdot 0 & \frac{1}{6}\cdot 6 & \frac{1}{6}\cdot 3 & \vdots & \frac{1}{6}\cdot -9 \\ -\frac{1}{4}\cdot 0 & -\frac{1}{4}\cdot 0 & -\frac{1}{4}\cdot -4 & \vdots & -\frac{1}{4}\cdot 20 \end{bmatrix} = \begin{bmatrix} -5 & 8 & -4 & \vdots & 38 \\ 0 & 1 & \frac{1}{2} & \vdots & -\frac{3}{2} \\ 0 & 0 & 1 & \vdots & -5 \end{bmatrix}$$

Next:

$$\begin{bmatrix} -5 & 8 & -4 & \vdots & 38 \\ 0 & 1 & \frac{1}{2} & \vdots & -\frac{3}{2} \\ 0 & 0 & 1 & \vdots & -5 \end{bmatrix} \begin{matrix} \leftarrow -\frac{1}{5}R_1 \\ \\ \\ \end{matrix}$$

The symbol $\leftarrow -\frac{1}{5}R_1$ means multiply Row 1 by $-\frac{1}{5}$ and put the result in Row 1. The math for this manipulation and the resulting row equivalent matrix is:

$$\begin{bmatrix} -5\cdot -\frac{1}{5} & 8\cdot -\frac{1}{5} & -4\cdot -\frac{1}{5} & \vdots & 38\cdot -\frac{1}{5} \\ 0 & 1 & \frac{1}{2} & \vdots & -\frac{3}{2} \\ 0 & 0 & 1 & \vdots & -5 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{8}{5} & \frac{4}{5} & \vdots & -\frac{38}{5} \\ 0 & 1 & \frac{1}{2} & \vdots & -\frac{3}{2} \\ 0 & 0 & 1 & \vdots & -5 \end{bmatrix}$$

Use this augmented matrix to rewrite the system of equations.

$$\begin{aligned} x-\frac{8}{5}y+\frac{4}{5}z &= -\frac{38}{5} \\ y+\frac{1}{2}z &= -\frac{3}{2} \\ z &= -5 \end{aligned}$$

We know that $z = -5$. Substitute this value into the second equation.

$$y+\frac{1}{2}(-5) = -\frac{3}{2} \rightarrow y-\frac{5}{2} = -\frac{3}{2} \rightarrow y = 1$$

Now we know that $z = -5$ and $y = 1$. Substitute these values into the first equation.

$$x-\frac{8}{5}(1)+\frac{4}{5}(-5) = -\frac{38}{5} \rightarrow x-\frac{8}{5}-\frac{20}{5} = -\frac{38}{5} \rightarrow x = -\frac{10}{5} = -2$$

Answer: $x = -2, y = 1, z = -5$

2. Use the Gauss-Jordan Elimination method to solve this system of equations.

$$\begin{aligned} 7x+3y-3z&=-4 \\ -6y+2z&=24 \\ 10z&=30 \end{aligned}$$

Write the system of equations as an augmented matrix:

$$\begin{bmatrix} 7 & 3 & -3 & \vdots & -4 \\ 0 & -6 & 2 & \vdots & 24 \\ 0 & 0 & 10 & \vdots & 30 \end{bmatrix}$$

The Gauss-Jordan Elimination method requires the matrix to be transformed into a row equivalent matrix of this form in which all elements that are not specified as 1 or 0 can be any number which results from transforming the matrix:

$$\begin{bmatrix} 1 & 0 & 0 & \vdots & \\ 0 & 1 & 0 & \vdots & \\ 0 & 0 & 1 & \vdots & \end{bmatrix}$$

Transform the matrix:

$$\begin{bmatrix} 7 & 3 & -3 & \vdots & -4 \\ 0 & -6 & 2 & \vdots & 24 \\ 0 & 0 & 10 & \vdots & 30 \end{bmatrix} \begin{matrix} \\ \leftarrow -\frac{1}{6}R_2 \\ \leftarrow \frac{1}{10}R_3 \end{matrix}$$

The symbol $\leftarrow -\frac{1}{6}R_2$ means multiply Row 2 by $-\frac{1}{6}$ and put the result in Row 2. The symbol $\leftarrow \frac{1}{10}R_3$ means multiply Row 3 by $\frac{1}{10}$ and put the result in Row 3. The resulting row equivalent matrix is:

$$\begin{bmatrix} 7 & 3 & -3 & \vdots & -4 \\ 0\cdot -\frac{1}{6} & -6\cdot -\frac{1}{6} & 2\cdot -\frac{1}{6} & \vdots & 24\cdot -\frac{1}{6} \\ 0\cdot \frac{1}{10} & 0\cdot \frac{1}{10} & 10\cdot \frac{1}{10} & \vdots & 30\cdot \frac{1}{10} \end{bmatrix} = \begin{bmatrix} 7 & 3 & -3 & \vdots & -4 \\ 0 & 1 & -\frac{1}{3} & \vdots & -4 \\ 0 & 0 & 1 & \vdots & 3 \end{bmatrix}$$

Next:

$$\begin{bmatrix} 7 & 3 & -3 & \vdots & -4 \\ 0 & 1 & -\frac{1}{3} & \vdots & -4 \\ 0 & 0 & 1 & \vdots & 3 \end{bmatrix} \begin{matrix} \leftarrow -3R_2 + R_1 \\ \leftarrow \frac{1}{3}R_3 + R_2 \\ \\ \end{matrix}$$

The symbol $\leftarrow -3R_2 + R_1$ means multiply Row 2 by -3, add Row 1, and put the result in Row 1. The symbol $\leftarrow \frac{1}{3}R_3 + R_2$ means multiply Row 3 by $\frac{1}{3}$, add Row 2, and put the result in Row 2. The resulting row equivalent matrix is:

$$\begin{bmatrix} -3\cdot 0+7 & -3\cdot 1+3 & -3\cdot -\frac{1}{3}+-3 & \vdots & -3\cdot -4+-4 \\ \frac{1}{3}\cdot 0+0 & \frac{1}{3}\cdot 0+1 & \frac{1}{3}\cdot 1+-\frac{1}{3} & \vdots & \frac{1}{3}\cdot 3+-4 \\ 0 & 0 & 1 & \vdots & 3 \end{bmatrix}$$
$$= \begin{bmatrix} 7 & 0 & -2 & \vdots & 8 \\ 0 & 1 & 0 & \vdots & -3 \\ 0 & 0 & 1 & \vdots & 3 \end{bmatrix}$$

Next:

$$\begin{bmatrix} 7 & 0 & -2 & \vdots & 8 \\ 0 & 1 & 0 & \vdots & -3 \\ 0 & 0 & 1 & \vdots & 3 \end{bmatrix} 2R_3 + R_1$$

The symbol $\leftarrow 2R_3 + R_1$ means multiply Row 3 by 2, add Row 1, and put the result in Row 1. The resulting row equivalent matrix is:

$$\begin{bmatrix} 2\cdot 0+7 & 2\cdot 0+0 & 2\cdot 1+-2 & \vdots & 2\cdot 3+8 \\ 0 & 1 & 0 & \vdots & -3 \\ 0 & 0 & 1 & \vdots & 3 \end{bmatrix} = \begin{bmatrix} 7 & 0 & 0 & \vdots & 14 \\ 0 & 1 & 0 & \vdots & -3 \\ 0 & 0 & 1 & \vdots & 3 \end{bmatrix}$$

Next:

$$\begin{bmatrix} 7 & 0 & 0 & \vdots & 14 \\ 0 & 1 & 0 & \vdots & -3 \\ 0 & 0 & 1 & \vdots & 3 \end{bmatrix} \leftarrow \frac{1}{7}R_1$$

The symbol $\leftarrow \frac{1}{7}R_1$ means multiply Row 1 by $\frac{1}{7}$ and put the result in Row 1. The resulting row equivalent matrix is:

$$\begin{bmatrix} 7\cdot\frac{1}{7} & 0\cdot\frac{1}{7} & 0\cdot\frac{1}{7} & \vdots & 14\cdot\frac{1}{7} \\ 0 & 1 & 0 & \vdots & -3 \\ 0 & 0 & 1 & \vdots & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \vdots & 2 \\ 0 & 1 & 0 & \vdots & -3 \\ 0 & 0 & 1 & \vdots & 3 \end{bmatrix}$$

Answer: $x = 2, y = -3, z = 3$

3. Use the Gaussian Elimination method to solve this system of equations.

$$\begin{aligned} 4x+2y+3z&=5 \\ -2y+5z&=-9 \\ 12z&=-12 \end{aligned}$$

Write the system of equations as an augmented matrix:

$$\begin{bmatrix} 4 & 2 & 3 & \vdots & 5 \\ 0 & -2 & 5 & \vdots & -9 \\ 0 & 0 & 12 & \vdots & -12 \end{bmatrix}$$

The Gaussian Elimination method requires the matrix to be transformed into a row equivalent matrix of this form in which all elements that are not specified as 1 or 0 can be any number which results from transforming the matrix:

$$\begin{bmatrix} 1 & & & \vdots & \\ 0 & 1 & & \vdots & \\ 0 & 0 & 1 & \vdots & \end{bmatrix}$$

Transform the matrix:

$$\begin{bmatrix} 4 & 2 & 3 & \vdots & 5 \\ 0 & -2 & 5 & \vdots & -9 \\ 0 & 0 & 12 & \vdots & -12 \end{bmatrix} \leftarrow \frac{1}{12}R_3$$

The symbol $\leftarrow \frac{1}{12}R_3$ means multiply Row 3 by $\frac{1}{12}$ and put the result in Row 3. The resulting row equivalent matrix is:

$$\begin{bmatrix} 4 & 2 & 3 & \vdots & 5 \\ 0 & -2 & 5 & \vdots & -9 \\ 0 \cdot \frac{1}{12} & 0 \cdot \frac{1}{12} & 12 \cdot \frac{1}{12} & \vdots & -12 \cdot \frac{1}{12} \end{bmatrix} = \begin{bmatrix} 4 & 2 & 3 & \vdots & 5 \\ 0 & -2 & 5 & \vdots & -9 \\ 0 & 0 & 1 & \vdots & -1 \end{bmatrix}$$

Next:

$$\begin{bmatrix} 4 & 2 & 3 & \vdots & 5 \\ 0 & -2 & 5 & \vdots & -9 \\ 0 & 0 & 1 & \vdots & -1 \end{bmatrix} \begin{matrix} \leftarrow R_2 + R_1 \\ \\ \\ \end{matrix}$$

The symbol $\leftarrow R_2 + R_1$ means add Row 2 and Row 1 and put the result in Row 1. The resulting row equivalent matrix is:

$$\begin{bmatrix} 0+4 & -2+2 & 5+3 & \vdots & -9+5 \\ 0 & -2 & 5 & \vdots & -9 \\ 0 & 0 & 1 & \vdots & -1 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 8 & \vdots & -4 \\ 0 & -2 & 5 & \vdots & -9 \\ 0 & 0 & 1 & \vdots & -1 \end{bmatrix}$$

Next,

$$\begin{bmatrix} 4 & 0 & 8 & \vdots & -4 \\ 0 & -2 & 5 & \vdots & -9 \\ 0 & 0 & 1 & \vdots & -1 \end{bmatrix} \begin{matrix} \leftarrow \frac{1}{4}R_1 \\ \leftarrow -\frac{1}{2}R_2 \\ \end{matrix}$$

The symbol $\leftarrow \frac{1}{4}R_1$ means multiply Row 1 by $\frac{1}{4}$ and put the result in Row 1. The symbol $\leftarrow -\frac{1}{2}R_2$ means multiply Row 2 by $-\frac{1}{2}$ and put the result in Row 2. The resulting row equivalent matrix is:

$$\begin{bmatrix} 4\cdot\frac{1}{4} & 0\cdot\frac{1}{4} & 8\cdot\frac{1}{4} & \vdots & -4\cdot\frac{1}{4} \\ 0\cdot-\frac{1}{2} & -2\cdot-\frac{1}{2} & 5\cdot-\frac{1}{2} & \vdots & -9\cdot-\frac{1}{2} \\ 0 & 0 & 1 & \vdots & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 2 & \vdots & -1 \\ 0 & 1 & -\frac{5}{2} & \vdots & \frac{9}{2} \\ 0 & 0 & 1 & \vdots & -1 \end{bmatrix}$$

Use this augmented matrix to rewrite the system of equations.

$$\begin{aligned} x+2z&=-1 \\ y-\frac{5}{2}z&=\frac{9}{2} \\ z&=-1 \end{aligned}$$

We know that $z=-1$. Substitute this value into the second equation.

$$y-\frac{5}{2}(-1)=\frac{9}{2} \rightarrow y+\frac{5}{2}=\frac{9}{2} \rightarrow y=2$$

Now we know that $z=-1$ and $y=2$. Substitute these values into the first equation.

$$x+2(-1)=-1 \rightarrow x-2=-1 \rightarrow x=1$$

Answer: $x=1, y=2, z=-1$

4. Use the Gauss-Jordan Elimination method to solve this system of equations.

$$\begin{aligned} 5x+3y+2z &= 15 \\ 2y+4z &= 24 \\ y-4z &= -42 \end{aligned}$$

Write the system of equations as an augmented matrix:

$$\begin{bmatrix} 5 & 3 & 2 & \vdots & 15 \\ 0 & 2 & 4 & \vdots & 24 \\ 0 & 1 & -4 & \vdots & -42 \end{bmatrix}$$

The Gauss-Jordan Elimination method requires the matrix to be transformed into a row equivalent matrix of this form in which all elements that are not specified as 1 or 0 can be any number which results from transforming the matrix:

$$\begin{bmatrix} 1 & 0 & 0 & \vdots & \\ 0 & 1 & 0 & \vdots & \\ 0 & 0 & 1 & \vdots & \end{bmatrix}$$

Transform the matrix:

$$\begin{bmatrix} 5 & 3 & 2 & \vdots & 15 \\ 0 & 2 & 4 & \vdots & 24 \\ 0 & 1 & -4 & \vdots & -42 \end{bmatrix} \begin{matrix} \leftarrow -3R_3 + R_1 \\ \leftarrow -R_3 + R_2 \\ \end{matrix}$$

The symbol $\leftarrow -3R_3 + R_1$ means multiply Row 3 by -3, add Row 1, and put the result in Row 1. The symbol $\leftarrow -R_3 + R_2$ means multiply Row 3 by -1, add Row 2, and put the result in Row 2. The resulting row equivalent matrix is:

$$\begin{bmatrix} -3\cdot 0+5 & -3\cdot 1+3 & -3\cdot -4+2 & \vdots & -3\cdot -42+15 \\ -0+0 & -1+2 & -(-4)+4 & \vdots & -(-42)+24 \\ 0 & 1 & -4 & \vdots & -42 \end{bmatrix}$$
$$= \begin{bmatrix} 5 & 0 & 14 & \vdots & 141 \\ 0 & 1 & 8 & \vdots & 66 \\ 0 & 1 & -4 & \vdots & -42 \end{bmatrix}$$

Next,

$$\begin{bmatrix} 5 & 0 & 14 & \vdots & 141 \\ 0 & 1 & 8 & \vdots & 66 \\ 0 & 1 & -4 & \vdots & -42 \end{bmatrix} \begin{matrix} \\ \\ \leftarrow -R_3 + R_2 \end{matrix}$$

The symbol $\leftarrow -R_3 + R_2$ means multiply Row 3 by -1, add Row 2, and put the result in Row 3. The resulting row equivalent matrix is:

$$\begin{bmatrix} 5 & 0 & 14 & \vdots & 141 \\ 0 & 1 & 8 & \vdots & 66 \\ -0+0 & -1+1 & -(-4)+8 & \vdots & -(-42)+66 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 14 & \vdots & 141 \\ 0 & 1 & 8 & \vdots & 66 \\ 0 & 0 & 12 & \vdots & 108 \end{bmatrix}$$

Next,

$$\begin{bmatrix} 5 & 0 & 14 & \vdots & 141 \\ 0 & 1 & 8 & \vdots & 66 \\ 0 & 0 & 12 & \vdots & 108 \end{bmatrix} \begin{matrix} \leftarrow -R_3 + R_1 \\ \\ \leftarrow \frac{1}{12} R_3 \end{matrix}$$

The symbol $\leftarrow -R_3 + R_1$ means multiply Row 3 by -1, add Row 1, and put the result in Row 1. The symbol $\leftarrow \frac{1}{12} R_3$ means multiply Row 3 by $\frac{1}{12}$, and put the result in Row 3. The resulting row equivalent matrix is:

$$\begin{bmatrix} -0+5 & -0+0 & -12+14 & \vdots & -108+141 \\ 0 & 1 & 8 & \vdots & 66 \\ \frac{1}{12}\cdot 0 & \frac{1}{12}\cdot 0 & \frac{1}{12}\cdot 12 & \vdots & \frac{1}{12}\cdot 108 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 2 & \vdots & 33 \\ 0 & 1 & 8 & \vdots & 66 \\ 0 & 0 & 1 & \vdots & 9 \end{bmatrix}$$

Next,

$$\begin{bmatrix} 5 & 0 & 2 & \vdots & 33 \\ 0 & 1 & 8 & \vdots & 66 \\ 0 & 0 & 1 & \vdots & 9 \end{bmatrix} \begin{matrix} \leftarrow -2R_3 + R_1 \\ \leftarrow -8R_3 + R_2 \\ \\ \end{matrix}$$

The symbol $\leftarrow -2R_3 + R_1$ means multiply Row 3 by -2, add Row 1, and put the result in Row 1. The symbol $\leftarrow -8R_3 + R_2$ means multiply Row 3 by -8, add Row 2 and put the result in Row 2. The resulting row equivalent matrix is:

$$\begin{bmatrix} -2\cdot 0+5 & -2\cdot 0+0 & -2\cdot 1+2 & \vdots & -2\cdot 9+33 \\ -8\cdot 0+0 & -8\cdot 0+1 & -8\cdot 1+8 & \vdots & -8\cdot 9+66 \\ 0 & 0 & 1 & \vdots & 9 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 0 & \vdots & 15 \\ 0 & 1 & 0 & \vdots & -6 \\ 0 & 0 & 1 & \vdots & 9 \end{bmatrix}$$

Next,

$$\begin{bmatrix} 5 & 0 & 0 & \vdots & 15 \\ 0 & 1 & 0 & \vdots & -6 \\ 0 & 0 & 1 & \vdots & 9 \end{bmatrix} \leftarrow \frac{1}{5}R_1$$

The symbol $\leftarrow \frac{1}{5}R_1$ means multiply Row 1 by $\frac{1}{5}$, and put the result in Row 1. The resulting row equivalent matrix is:

$$\begin{bmatrix} \frac{1}{5}\cdot 5 & \frac{1}{5}\cdot 0 & \frac{1}{5}\cdot 0 & \vdots & \frac{1}{5}\cdot 15 \\ 0 & 1 & 0 & \vdots & -6 \\ 0 & 0 & 1 & \vdots & 9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \vdots & 3 \\ 0 & 1 & 0 & \vdots & -6 \\ 0 & 0 & 1 & \vdots & 9 \end{bmatrix}$$

Answer: $x = 3, y = -6, z = 9$

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5. Use the Gauss-Jordan Elimination method to solve this system of equations.

$$\begin{aligned} -2x - y - z &= -11 \\ 3x + 4y + z &= 19 \\ 3x + 6y + 5z &= 43 \end{aligned}$$

Write the system of equations as an augmented matrix:

$$\begin{bmatrix} -2 & -1 & -1 & \vdots & -11 \\ 3 & 4 & 1 & \vdots & 19 \\ 3 & 6 & 5 & \vdots & 43 \end{bmatrix}$$

The Gaussian Elimination method requires the matrix to be transformed into a row equivalent matrix of this form in which all elements that are not specified as 1 or 0 can be any number which results from transforming the matrix:

$$\begin{bmatrix} 1 & 0 & 0 & \vdots & \\ 0 & 1 & 0 & \vdots & \\ 0 & 0 & 1 & \vdots & \end{bmatrix}$$

Transform the matrix:

$$\begin{bmatrix} -2 & -1 & -1 & \vdots & -11 \\ 3 & 4 & 1 & \vdots & 19 \\ 3 & 6 & 5 & \vdots & 43 \end{bmatrix} \begin{matrix} \leftarrow R_1 + R_3 \\ \leftarrow R_2 + (-R_3\ ) \\ \end{matrix}$$

The symbol $\leftarrow R_1 + R_3$ means add Row 1 and Row 3 and put the result in Row 1. The symbol $\leftarrow R_2 + (-R_3)$ means multiply Row 3 by -1, add Row 2, and put the result in Row 2. The resulting row equivalent matrix is:

$$\begin{bmatrix} -2+3 & -1+6 & -1+5 & \vdots & -11+43 \\ 3+-3 & 4+-6 & 1+-5 & \vdots & 19+-43 \\ 3 & 6 & 5 & \vdots & 43 \end{bmatrix} = \begin{bmatrix} 1 & 5 & 4 & \vdots & 32 \\ 0 & -2 & -4 & \vdots & -24 \\ 3 & 6 & 5 & \vdots & 43 \end{bmatrix}$$

Next,

$$\begin{bmatrix} 1 & 5 & 4 & \vdots & 32 \\ 0 & -2 & -4 & \vdots & -24 \\ 3 & 6 & 5 & \vdots & 43 \end{bmatrix} \begin{matrix} \\ \leftarrow -\frac{1}{2}R_2 \\ \leftarrow -3R_1 + R_3 \end{matrix}$$

The symbol $\leftarrow -\frac{1}{2}R_2$ means multiply Row 1 by $-\frac{1}{2}$, and put the result in Row 2. The symbol $\leftarrow -3R_1 + R_3$ means multiply Row 1 by -3, add Row 3, and put the result in Row 3. The resulting row equivalent matrix is:

$$\begin{bmatrix} 1 & 5 & 4 & \vdots & 32 \\ -\frac{1}{2}\cdot 0 & -\frac{1}{2}\cdot -2 & -\frac{1}{2}\cdot -4 & \vdots & -\frac{1}{2}\cdot -24 \\ -3\cdot 1+3 & -3\cdot 5+6 & -3\cdot 4+5 & \vdots & -3\cdot 32+43 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 5 & 4 & \vdots & 32 \\ 0 & 1 & 2 & \vdots & 12 \\ 0 & -9 & -7 & \vdots & -53 \end{bmatrix}$$

Next,

$$\begin{bmatrix} 1 & 5 & 4 & \vdots & 32 \\ 0 & 1 & 2 & \vdots & 12 \\ 0 & -9 & -7 & \vdots & -53 \end{bmatrix} \begin{matrix} \leftarrow R_1 + (-5R_2) \\ \\ \leftarrow 9R_2 + R_3 \end{matrix}$$

The symbol $\leftarrow R_1 + (-5R_2)$ means multiply Row 2 by -5, add Row 1, and put the result in Row 1. The symbol $\leftarrow 9R_2 + R_3$ means multiply Row 2 by 9, add Row 3, and put the result in Row 3. The resulting row equivalent matrix is:

$$\begin{bmatrix} 1+-5\cdot 0 & 5+-5\cdot 1 & 4+-5\cdot 2 & \vdots & 32+-5\cdot 12 \\ 0 & 1 & 2 & \vdots & 12 \\ 9\cdot 0+0 & 9\cdot 1+-9 & 9\cdot 2+-7 & \vdots & 9\cdot 12+-53 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -6 & \vdots & -28 \\ 0 & 1 & 2 & \vdots & 12 \\ 0 & 0 & 11 & \vdots & 55 \end{bmatrix}$$

Next,

$$\begin{bmatrix} 1 & 0 & -6 & \vdots & -28 \\ 0 & 1 & 2 & \vdots & 12 \\ 0 & 0 & 11 & \vdots & 55 \end{bmatrix} \begin{matrix} \\ \\ \leftarrow \frac{1}{11}R_3 \end{matrix}$$

The symbol $\leftarrow \frac{1}{11}R_3$ means multiply Row 3 by $\frac{1}{11}$, and put the result in Row 3. The resulting row equivalent matrix is:

$$\begin{bmatrix} 1 & 0 & -6 & \vdots & -28 \\ 0 & 1 & 2 & \vdots & 12 \\ \frac{1}{11}\cdot 0 & \frac{1}{11}\cdot 0 & \frac{1}{11}\cdot 11 & \vdots & \frac{1}{11}\cdot 55 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -6 & \vdots & -28 \\ 0 & 1 & 2 & \vdots & 12 \\ 0 & 0 & 1 & \vdots & 5 \end{bmatrix}$$

Next,

$$\begin{bmatrix} 1 & 0 & -6 & \vdots & -28 \\ 0 & 1 & 2 & \vdots & 12 \\ 0 & 0 & 1 & \vdots & 5 \end{bmatrix} \begin{matrix} 6R_3 + R_1 \\ -2R_3 + R_2 \\ \\ \end{matrix}$$

The symbol $\leftarrow 6R_3 + R_1$ means multiply Row 3 by 6, add Row 1, and put the result in Row 1. The symbol $\leftarrow -2R_3 + R_2$ means multiply Row 3 by -2, add Row 2, and put the result in Row 2. The resulting row equivalent matrix is:

$$\begin{bmatrix} 6\cdot 0+1 & 6\cdot 0+0 & 6\cdot 1+-6 & \vdots & 6\cdot 5+-28 \\ -2\cdot 0+0 & -2\cdot 0+1 & -2\cdot 1+2 & \vdots & -2\cdot 5+12 \\ 0 & 0 & 1 & \vdots & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & \vdots & 2 \\ 0 & 1 & 0 & \vdots & 2 \\ 0 & 0 & 1 & \vdots & 5 \end{bmatrix}$$

Answer: $x = 2, y = 2, z = 5$

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6. Use the Gauss-Jordan Elimination method to solve this system of equations.

$$\begin{aligned} 2x + y + z &= 11 \\ 3x + 5y + 5z &= 34 \\ -5x - 6y - 3z &= -42 \end{aligned}$$

Write the system of equations as an augmented matrix:

$$\begin{bmatrix} 2 & 1 & 1 & \vdots & 11 \\ 3 & 5 & 5 & \vdots & 34 \\ -5 & -6 & -3 & \vdots & -42 \end{bmatrix}$$

The Gauss-Jordan Elimination method requires the matrix to be transformed into a row equivalent matrix of this form in which all elements that are not specified as 1 or 0 can be any number which results from transforming the matrix:

$$\begin{bmatrix} 1 & 0 & 0 & \vdots & \\ 0 & 1 & 0 & \vdots & \\ 0 & 0 & 1 & \vdots & \end{bmatrix}$$

Transform the matrix:

$$\begin{bmatrix} 2 & 1 & 1 & \vdots & 11 \\ 3 & 5 & 5 & \vdots & 34 \\ -5 & -6 & -3 & \vdots & -42 \end{bmatrix} \begin{matrix} \leftarrow R_2 + (-R_1) \\ \\ \leftarrow -R_3 \end{matrix}$$

The symbol $\leftarrow R_2 + (-R_1)$ means multiply Row 1 by -1, add Row 2, and put the result in Row 1. The symbol $\leftarrow -R_3$ means multiply Row 3 by -1, and put the result in Row 3. The resulting row equivalent matrix is:

$$\begin{bmatrix} 3+-2 & 5+-1 & 5+-1 & \vdots & 34+-11 \\ 3 & 5 & 5 & \vdots & 34 \\ -(-5) & -(-6) & -(-3) & \vdots & -(-42) \end{bmatrix} = \begin{bmatrix} 1 & 4 & 4 & \vdots & 23 \\ 3 & 5 & 5 & \vdots & 34 \\ 5 & 6 & 3 & \vdots & 42 \end{bmatrix}$$

Next,

$$\begin{bmatrix} 1 & 4 & 4 & \vdots & 23 \\ 3 & 5 & 5 & \vdots & 34 \\ 5 & 6 & 3 & \vdots & 42 \end{bmatrix} \leftarrow -3R_1 + R_2$$

The symbol $\leftarrow -3R_1 + R_2$ means multiply Row 1 by -3, add Row 2, and put the result in Row 2. The resulting row equivalent matrix is:

$$\begin{bmatrix} 1 & 4 & 4 & \vdots & 23 \\ -3\cdot 1+3 & -3\cdot 4+5 & -3\cdot 4+5 & \vdots & -3\cdot 23+34 \\ 5 & 6 & 3 & \vdots & 42 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 4 & 4 & \vdots & 23 \\ 0 & -7 & -7 & \vdots & -35 \\ 5 & 6 & 3 & \vdots & 42 \end{bmatrix}$$

Next,

$$\begin{bmatrix} 1 & 4 & 4 & \vdots & 23 \\ 0 & -7 & -7 & \vdots & -35 \\ 5 & 6 & 3 & \vdots & 42 \end{bmatrix} \leftarrow -\frac{1}{7}R_2$$

The symbol $\leftarrow -\frac{1}{7}R_2$ means multiply Row 2 by $-\frac{1}{7}$, and put the result in Row 2. The resulting row equivalent matrix is:

$$\begin{bmatrix} 1 & 4 & 4 & \vdots & 23 \\ -\frac{1}{7}\cdot 0 & -\frac{1}{7}\cdot -7 & -\frac{1}{7}\cdot -7 & \vdots & -\frac{1}{7}\cdot -35 \\ 5 & 6 & 3 & \vdots & 42 \end{bmatrix} = \begin{bmatrix} 1 & 4 & 4 & \vdots & 23 \\ 0 & 1 & 1 & \vdots & 5 \\ 5 & 6 & 3 & \vdots & 42 \end{bmatrix}$$

Next,

$$\begin{bmatrix} 1 & 4 & 4 & \vdots & 23 \\ 0 & 1 & 1 & \vdots & 5 \\ 5 & 6 & 3 & \vdots & 42 \end{bmatrix} \begin{matrix} \leftarrow R_1 + (-4R_2) \\ \\ \leftarrow R_3 + (-5R_1) \end{matrix}$$

The symbol $\leftarrow R_1 + (-4R_2)$ means multiply Row 2 by -4, add Row 1, and put the result in Row 1. The symbol $\leftarrow R_3 + (-5R_1)$ means multiply Row 1 by -5, add Row 3, and put the result in Row 3. The resulting row equivalent matrix is:

$$\begin{bmatrix} 1 + -4 \cdot 0 & 4 + -4 \cdot 1 & 4 + -4 \cdot 1 & \vdots & 23 + -4 \cdot 5 \\ 0 & 1 & 1 & \vdots & 5 \\ 5 + -5 \cdot 1 & 6 + -5 \cdot 4 & 3 + -5 \cdot 4 & \vdots & 42 + -5 \cdot 23 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 0 & 0 & \vdots & 3 \\ 0 & 1 & 1 & \vdots & 5 \\ 0 & -14 & -17 & \vdots & -73 \end{bmatrix}$$

Next,

$$\begin{bmatrix} 1 & 0 & 0 & \vdots & 3 \\ 0 & 1 & 1 & \vdots & 5 \\ 0 & -14 & -17 & \vdots & -73 \end{bmatrix} \leftarrow 14R_2 + R_3$$

The symbol $\leftarrow 14R_2 + R_3$ means multiply Row 2 by 14, add Row 3, and put the result in Row 3. The resulting row equivalent matrix is:

$$\begin{bmatrix} 1 & 0 & 0 & \vdots & 3 \\ 0 & 1 & 1 & \vdots & 5 \\ 14 \cdot 0 + 0 & 14 \cdot 1 + -14 & 14 \cdot 1 + -17 & \vdots & 14 \cdot 5 + -73 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 0 & 0 & \vdots & 3 \\ 0 & 1 & 1 & \vdots & 5 \\ 0 & 0 & -3 & \vdots & -3 \end{bmatrix}$$

Next,

$$\begin{bmatrix} 1 & 0 & 0 & \vdots & 3 \\ 0 & 1 & 1 & \vdots & 5 \\ 0 & 0 & -3 & \vdots & -3 \end{bmatrix} \leftarrow -\frac{1}{3}R_3$$

The symbol $\leftarrow -\frac{1}{3}R_3$ means multiply Row 3 by $-\frac{1}{3}$, and put the result in Row 3. The resulting row equivalent matrix is:

$$\begin{bmatrix} 1 & 0 & 0 & \vdots & 3 \\ 0 & 1 & 1 & \vdots & 5 \\ -\frac{1}{3}\cdot 0 & -\frac{1}{3}\cdot 0 & -\frac{1}{3}\cdot -3 & \vdots & -\frac{1}{3}\cdot -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \vdots & 3 \\ 0 & 1 & 1 & \vdots & 5 \\ 0 & 0 & 1 & \vdots & 1 \end{bmatrix}$$

Next,

$$\begin{bmatrix} 1 & 0 & 0 & \vdots & 3 \\ 0 & 1 & 1 & \vdots & 5 \\ 0 & 0 & 1 & \vdots & 1 \end{bmatrix} \leftarrow -R_3 + R_2$$

The symbol $\leftarrow -R_3 + R_2$ means multiply Row 3 by -1, add Row 2, and put the result in Row 2. The resulting row equivalent matrix is:

$$\begin{bmatrix} 1 & 0 & 0 & \vdots & 3 \\ -0+0 & -0+1 & -1+1 & \vdots & -1+5 \\ 0 & 0 & 1 & \vdots & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \vdots & 3 \\ 0 & 1 & 0 & \vdots & 4 \\ 0 & 0 & 1 & \vdots & 1 \end{bmatrix}$$

Answer: $x = 3, y = 4, z = 1$

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7. Use the Gauss-Jordan Elimination method to solve this system of equations.

$$\begin{aligned} 2x + 5y - 3z &= -10 \\ -2x - 2y + 2z &= 2 \\ -x + -5y + z &= 17 \end{aligned}$$

Write the system of equations as an augmented matrix:

$$\begin{bmatrix} 2 & 5 & -3 & \vdots & -10 \\ -2 & -2 & 2 & \vdots & 2 \\ -1 & -5 & 1 & \vdots & 17 \end{bmatrix}$$

The Gauss-Jordan Elimination method requires the matrix to be transformed into a row equivalent matrix of this form in which all elements that are not specified as 1 or 0 can be any number which results from transforming the matrix:

$$\begin{bmatrix} 1 & 0 & 0 & \vdots & \\ 0 & 1 & 0 & \vdots & \\ 0 & 0 & 1 & \vdots & \end{bmatrix}$$

Transform the matrix:

$$\begin{bmatrix} 2 & 5 & -3 & \vdots & -10 \\ -2 & -2 & 2 & \vdots & 2 \\ -1 & -5 & 1 & \vdots & 17 \end{bmatrix} \begin{matrix} \leftarrow R_1 + R_3 \\ \leftarrow -\frac{1}{2}R_2 \\ \\ \end{matrix}$$

The symbol $\leftarrow R_1 + R_3$ means add Row 1 to Row 3 and put the result in Row 1. The symbol $\leftarrow \frac{1}{2}R_2$means multiply Row 2 by $-\frac{1}{2}$, and put the result in Row 2. The resulting row equivalent matrix is:

$$\begin{bmatrix} 2+-1 & 5+-5 & -3+1 & \vdots & -10+17 \\ -\frac{1}{2}\cdot -2 & -\frac{1}{2}\cdot -2 & -\frac{1}{2}\cdot 2 & \vdots & -\frac{1}{2}\cdot 2 \\ -1 & -5 & 1 & \vdots & 17 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -2 & \vdots & 7 \\ 1 & 1 & -1 & \vdots & -1 \\ -1 & -5 & 1 & \vdots & 17 \end{bmatrix}$$

Next,

$$\begin{bmatrix} 1 & 0 & -2 & \vdots & 7 \\ 1 & 1 & -1 & \vdots & -1 \\ -1 & -5 & 1 & \vdots & 17 \end{bmatrix} \begin{matrix} \\ \leftarrow -R_1 + R_2 \\ \leftarrow R_1 + R_3 \end{matrix}$$

The symbol $\leftarrow -R_1 + R_2$ means multiply Row 1 by -1, add it to Row 2, and put the result in Row 2. The symbol $\leftarrow R_1 + R_3$means add Row 1 and Row 3 and put the result in Row 3. The resulting row equivalent matrix is:

$$\begin{bmatrix} 1 & 0 & -2 & \vdots & 7 \\ -1+1 & -0+1 & -(-2)+-1 & \vdots & -7+-1 \\ 1+-1 & 0+-5 & -2+1 & \vdots & 7+17 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -2 & \vdots & 7 \\ 0 & 1 & 1 & \vdots & -8 \\ 0 & -5 & -1 & \vdots & 24 \end{bmatrix}$$

Next,

$$\begin{bmatrix} 1 & 0 & -2 & \vdots & 7 \\ 0 & 1 & 1 & \vdots & -8 \\ 0 & -5 & -1 & \vdots & 24 \end{bmatrix} \begin{matrix} \\ \\ \leftarrow 5R_2 + R_3 \end{matrix}$$

The symbol $\leftarrow 5R_2 + R_3$ means multiply Row 2 by 5, add it to Row 3, and put the result in Row 3. The resulting row equivalent matrix is:

$$\begin{bmatrix} 1 & 0 & -2 & \vdots & 7 \\ 0 & 1 & 1 & \vdots & -8 \\ 5\cdot 0+0 & 5\cdot 1+-5 & 5\cdot 1+-1 & \vdots & 5\cdot -8+24 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -2 & \vdots & 7 \\ 0 & 1 & 1 & \vdots & -8 \\ 0 & 0 & 4 & \vdots & -16 \end{bmatrix}$$

Next,

$$\begin{bmatrix} 1 & 0 & -2 & \vdots & 7 \\ 0 & 1 & 1 & \vdots & -8 \\ 0 & 0 & 4 & \vdots & -16 \end{bmatrix} \begin{matrix} \\ \\ \leftarrow \frac{1}{4}R_3 \end{matrix}$$

The symbol $\leftarrow \frac{1}{4}R_3$ means multiply Row 3 by $\frac{1}{4}$ and put the result in Row 3. The resulting row equivalent matrix is:

$$\begin{bmatrix} 1 & 0 & -2 & \vdots & 7 \\ 0 & 1 & 1 & \vdots & -8 \\ \frac{1}{4}\cdot 0 & \frac{1}{4}\cdot 0 & \frac{1}{4}\cdot 4 & \vdots & \frac{1}{4}\cdot -16 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -2 & \vdots & 7 \\ 0 & 1 & 1 & \vdots & -8 \\ 0 & 0 & 1 & \vdots & -4 \end{bmatrix}$$

Next,

$$\begin{bmatrix} 1 & 0 & -2 & \vdots & 7 \\ 0 & 1 & 1 & \vdots & -8 \\ 0 & 0 & 1 & \vdots & -4 \end{bmatrix} \begin{matrix} \leftarrow 2R_3 + R_1 \\ \leftarrow -R_3 + R_2 \\ \\ \end{matrix}$$

The symbol $\leftarrow 2R_3 + R_1$ means multiply Row 3 by 2, add Row 1, and put the result in Row 1. The symbol $\leftarrow -R_3 + R_2$ means multiply Row 3 by -1, add Row 2, and put the result in Row 2. The resulting row equivalent matrix is:

$$\begin{bmatrix} 2\cdot 0+1 & 2\cdot 0+0 & 2\cdot 1+-2 & \vdots & 2\cdot -4+7 \\ -0+0 & -0+1 & -1+1 & \vdots & -(-4)+-8 \\ 0 & 0 & 1 & \vdots & -4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \vdots & -1 \\ 0 & 1 & 0 & \vdots & -4 \\ 0 & 0 & 1 & \vdots & -4 \end{bmatrix}$$

Answer: $x = -1, y = -4, z = -4$