Answer Key

Worksheet #2: Trig Form, Multiplying & Dividing Complex Numbers

$z_1 = 7 - 2i$ $\quad z_2 = -1 + i\sqrt{3}$ $\quad z_3 = 4\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

$z_4 = 3\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$ $\quad z_5 = \left[\cos\left(-\frac{\pi}{2}\right) + i\sin\left(-\frac{\pi}{2}\right)\right]$ $\quad z_6 = -7 + 7i$

Given the complex numbers above, complete the following problems. Be sure to give exact values when possible. Use trig identities when needed.

1. Give the trig form of z_1 $\quad r = \sqrt{49+4} = \sqrt{53}$

$\theta = \tan^{-1}\left(\frac{-2}{7}\right) \approx -15.945^\circ + 360^\circ \approx 344.055^\circ$

$\boxed{\sqrt{53}\left(\cos 344.055^\circ + i\sin 344.055^\circ\right)}$

2. Give the standard form of z_4

$3\left(-\frac{\sqrt{3}}{2}\right) + 3\left(\frac{1}{2}\right)i$

$\boxed{-\frac{3\sqrt{3}}{2} + \frac{3}{2}i}$

3. Find $\frac{z_6}{z_3}$. Give your answer in trig form. $\quad r = \sqrt{49+49} = \sqrt{98} = 7\sqrt{2} \quad \theta = \tan^{-1}(-1) = \frac{3\pi}{4}$

$\frac{7\sqrt{2}}{4}\left[\cos\left(\frac{3\pi}{4} - \frac{\pi}{4}\right) + i\sin\left(\frac{3\pi}{4} - \frac{\pi}{4}\right)\right] = \frac{7\sqrt{2}}{4}(0) + \frac{7\sqrt{2}}{4}(1)i$

$\boxed{SF \rightarrow \frac{7\sqrt{2}}{4}i}$ $\quad \boxed{TF \rightarrow \frac{7\sqrt{2}}{4}\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)}$

4. Find $z_2 z_4$. Give your answer in standard form. $\quad r = \sqrt{1+3} = 2 \quad \theta = \tan^{-1}(-\sqrt{3}) = \frac{2\pi}{3}$

$2(3)\left[\cos\left(\frac{2\pi}{3} + \frac{5\pi}{6}\right) + i\sin\left(\frac{2\pi}{3} + \frac{5\pi}{6}\right)\right] = 6\left(\cos\frac{9\pi}{6} + i\sin\frac{9\pi}{6}\right)$

$6\cos\frac{3\pi}{2} + 6\sin\frac{3\pi}{2}i = 6(0) + 6(-1)i = \boxed{-6i}$

5. Find $\frac{z_3}{z_4}$. Give your answer in standard form.

$\frac{4}{3}\left[\cos\left(\frac{\pi}{4} - \frac{5\pi}{6}\right) + i\sin\left(\frac{\pi}{4} - \frac{5\pi}{6}\right)\right] = \frac{4}{3}\left(\cos\left(-\frac{7\pi}{12}\right) + i\sin\left(-\frac{7\pi}{12}\right)\right)$

$\frac{4}{3}\left[\left(\cos\frac{\pi}{4}\cos\frac{5\pi}{6} + \sin\frac{\pi}{4}\sin\frac{5\pi}{6}\right) + i\left(\sin\frac{\pi}{4}\cos\frac{5\pi}{6} - \cos\frac{\pi}{4}\sin\frac{5\pi}{6}\right)\right]$

$\frac{4}{3}\left[\left(\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{3}}{2}\right) + \frac{\sqrt{2}}{2}\left(\frac{1}{2}\right)\right] + \frac{4}{3}\left[\frac{\sqrt{2}}{2}\left(-\frac{\sqrt{3}}{2}\right) - \frac{\sqrt{2}}{2}\left(\frac{1}{2}\right)\right]$

$\frac{4}{3}\left(\frac{-\sqrt{6}+\sqrt{2}}{4}\right) + \frac{4}{3}\left(\frac{-\sqrt{6}-\sqrt{2}}{4}\right)i = \boxed{\frac{-\sqrt{6}+\sqrt{2}}{3} - \frac{\sqrt{6}+\sqrt{2}}{3}i}$

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6. Find $z_5 z_6$. Give your answer in trig form.

$z_6 = 7\sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

$7\sqrt{2}\left[\cos\left(-\frac{\pi}{2}+\frac{3\pi}{4}\right) + i\sin\left(-\frac{\pi}{2}+\frac{3\pi}{4}\right)\right]$

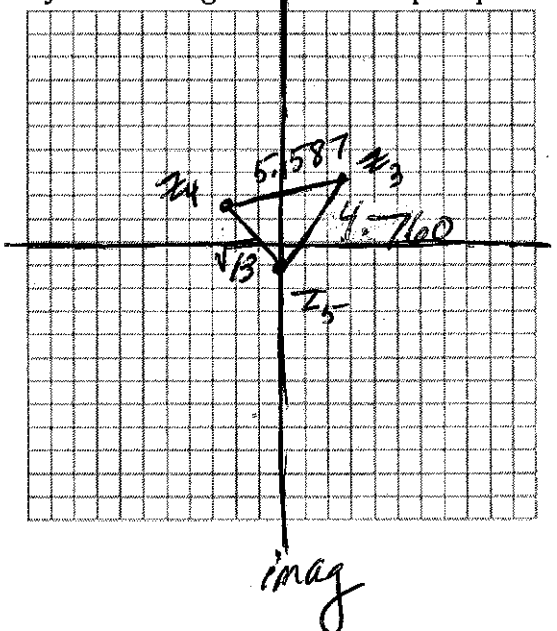
TF $\rightarrow 7\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

SF $\rightarrow 7+7i$

7. Find $|z_2|$

$\sqrt{1+3} = 2$

8. Graph the points z_3, z_4, and z_5 on the grid below. Find the area of the triangle formed by connecting the three complex points.



$z_3 = 2\sqrt{2} + 2\sqrt{2}i$

$z_4 = -\frac{3\sqrt{3}}{2} + \frac{3}{2}i$

$z_5 = -i$

$v_1 = \langle 0,-1\rangle - \langle -\frac{3\sqrt{3}}{2}, \frac{3}{2}\rangle$

$v_1 = \langle \frac{3\sqrt{3}}{2}, \frac{-5}{2}\rangle$

$\|v_1\| = \sqrt{13}$

$v_2 = \langle 2\sqrt{2}, 2\sqrt{2}\rangle - \langle -\frac{3\sqrt{3}}{2}, \frac{3}{2}\rangle$

$v_2 = \langle 5.427, 1.328\rangle$

$\|v_2\| = 5.587$

$v_3 = \langle 0,-1\rangle - \langle 2\sqrt{2}, 2\sqrt{2}\rangle$

$v_3 = \langle -2\sqrt{2}, -1-2\sqrt{2}\rangle$

$\|v_3\| = 4.760$

$s = \frac{\sqrt{13}+5.587+4.760}{2} \approx 6.976$

area $= \sqrt{6.976(6.976-\sqrt{13})(6.976-5.587)(6.976-4.760)}$

$\approx 8.507 \text{ units}^2$