Exercise Set 5.5

Practice Exercises

In Exercises 1–10, use substitution to determine whether the given x-value is a solution of the equation.

1. $\cos x = \frac{\sqrt{2}}{2}, \quad x = \frac{\pi}{4}$

2. $\tan x = \sqrt{3}, \quad x = \frac{\pi}{3}$

3. $\sin x = \frac{\sqrt{3}}{2}, \quad x = \frac{\pi}{6}$

4. $\sin x = \frac{\sqrt{2}}{2}, \quad x = \frac{\pi}{3}$

5. $\cos x = -\frac{1}{2}, \quad x = \frac{2\pi}{3}$

6. $\cos x = -\frac{1}{2}, \quad x = \frac{4\pi}{3}$

7. $\tan 2x = -\frac{\sqrt{3}}{3}, \quad x = \frac{5\pi}{12}$

8. $\cos\frac{2x}{3} = -\frac{1}{2}, \quad x = \pi$

9. $\cos x = \sin 2x, \quad x = \frac{\pi}{3}$

10. $\cos x + 2 = \sqrt{3}\sin x, \quad x = \frac{\pi}{6}$

In Exercises 11–24, find all solutions of each equation.

11. $\sin x = \frac{\sqrt{3}}{2}$

12. $\cos x = \frac{\sqrt{3}}{2}$

13. $\tan x = 1$

14. $\tan x = \sqrt{3}$

15. $\cos x = -\frac{1}{2}$

16. $\sin x = -\frac{\sqrt{2}}{2}$

17. $\tan x = 0$

18. $\sin x = 0$

19. $2\cos x + \sqrt{3} = 0$

20. $2\sin x + \sqrt{3} = 0$

21. $4\sin\theta - 1 = 2\sin\theta$

22. $5\sin\theta + 1 = 3\sin\theta$

23. $3\sin\theta + 5 = -2\sin\theta$

24. $7\cos\theta + 9 = -2\cos\theta$

Exercises 25–38 involve equations with multiple angles. Solve each equation on the interval $[0, 2\pi)$.

25. $\sin 2x = \frac{\sqrt{3}}{2}$

26. $\cos 2x = \frac{\sqrt{2}}{2}$

27. $\cos 4x = -\frac{\sqrt{3}}{2}$

28. $\sin 4x = -\frac{\sqrt{2}}{2}$

29. $\tan 3x = \frac{\sqrt{3}}{3}$

30. $\tan 3x = \sqrt{3}$

31. $\tan\frac{x}{2} = \sqrt{3}$

32. $\tan\frac{x}{2} = \frac{\sqrt{3}}{3}$

33. $\sin\frac{2\theta}{3} = -1$

34. $\cos\frac{2\theta}{3} = -1$

35. $\sec\frac{3\theta}{2} = -2$

36. $\cot\frac{3\theta}{2} = -\sqrt{3}$

37. $\sin\left(2x + \frac{\pi}{6}\right) = \frac{1}{2}$

38. $\sin\left(2x - \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

Exercises 39–52 involve trigonometric equations quadratic in form. Solve each equation on the interval $[0, 2\pi)$.

39. $2\sin^2 x - \sin x - 1 = 0$

40. $2\sin^2 x + \sin x - 1 = 0$

41. $2\cos^2 x + 3\cos x + 1 = 0$

42. $\cos^2 x + 2\cos x - 3 = 0$

43. $2\sin^2 x = \sin x + 3$

44. $2\sin^2 x = 4\sin x + 6$

45. $\sin^2\theta - 1 = 0$

46. $\cos^2\theta - 1 = 0$

47. $4\cos^2 x - 1 = 0$

48. $4\sin^2 x - 3 = 0$

49. $9\tan^2 x - 3 = 0$

50. $3\tan^2 x - 9 = 0$

51. $\sec^2 x - 2 = 0$

52. $4\sec^2 x - 2 = 0$

In Exercises 53–62, solve each equation on the interval $[0, 2\pi)$.

53. $(\tan x - 1)(\cos x + 1) = 0$

54. $(\tan x + 1)(\sin x - 1) = 0$

55. $\left(2\cos x + \sqrt{3}\right)(2\sin x + 1) = 0$

56. $\left(2\cos x - \sqrt{3}\right)(2\sin x - 1) = 0$

57. $\cot x(\tan x - 1) = 0$

58. $\cot x(\tan x + 1) = 0$

59. $\sin x + 2\sin x\cos x = 0$

60. $\cos x - 2\sin x\cos x = 0$

61. $\tan^2 x\cos x = \tan^2 x$

62. $\cot^2 x\sin x = \cot^2 x$

In Exercises 63–84, use an identity to solve each equation on the interval $[0, 2\pi)$.

63. $2\cos^2 x + \sin x - 1 = 0$

64. $2\cos^2 x - \sin x - 1 = 0$

65. $\sin^2 x - 2\cos x - 2 = 0$

66. $4\sin^2 x + 4\cos x - 5 = 0$

67. $4\cos^2 x = 5 - 4\sin x$

68. $3\cos^2 x = \sin^2 x$

69. $\sin 2x = \cos x$

70. $\sin 2x = \sin x$

71. $\cos 2x = \cos x$

72. $\cos 2x = \sin x$

73. $\cos 2x + 5\cos x + 3 = 0$

74. $\cos 2x + \cos x + 1 = 0$

75. $\sin x\cos x = \frac{\sqrt{2}}{4}$

76. $\sin x\cos x = \frac{\sqrt{3}}{4}$

77. $\sin x + \cos x = 1$

78. $\sin x + \cos x = -1$

79. $\sin\left(x + \frac{\pi}{4}\right) + \sin\left(x - \frac{\pi}{4}\right) = 1$

80. $\sin\left(x + \frac{\pi}{3}\right) + \sin\left(x - \frac{\pi}{3}\right) = 1$

81. $\sin 2x\cos x + \cos 2x\sin x = \frac{\sqrt{2}}{2}$

82. $\sin 3x\cos 2x + \cos 3x\sin 2x = 1$

83. $\tan x + \sec x = 1$

84. $\tan x - \sec x = 1$

In Exercises 85–96, use a calculator to solve each equation, correct to four decimal places, on the interval $[0, 2\pi)$.

85. $\sin x = 0.8246$

86. $\sin x = 0.7392$

87. $\cos x = -\frac{2}{5}$

88. $\cos x = -\frac{4}{7}$

89. $\tan x = -3$

90. $\tan x = -5$

91. $\cos^2 x - \cos x - 1 = 0$

92. $3\cos^2 x - 8\cos x - 3 = 0$

93. $4\tan^2 x - 8\tan x + 3 = 0$

94. $\tan^2 x - 3\tan x + 1 = 0$

95. $7\sin^2 x - 1 = 0$

96. $5\sin^2 x - 1 = 0$

In Exercises 97–116, use the most appropriate method to solve each equation on the interval $[0, 2\pi)$. *Use exact values where possible or give approximate solutions correct to four decimal places.*

97. $2\cos 2x + 1 = 0$

98. $2\sin 3x + \sqrt{3} = 0$

99. $\sin 2x + \sin x = 0$

100. $\sin 2x + \cos x = 0$

101. $3\cos x - 6\sqrt{3} = \cos x - 5\sqrt{3}$

102. $\cos x - 5 = 3\cos x + 6$

103. $\tan x = -4.7143$

104. $\tan x = -6.2154$

105. $2\sin^2 x = 3 - \sin x$

106. $2\sin^2 x = 2 - 3\sin x$

107. $\cos x \csc x = 2\cos x$

108. $\tan x \sec x = 2\tan x$

109. $5\cot^2 x - 15 = 0$

110. $5\sec^2 x - 10 = 0$

111. $\cos^2 x + 2\cos x - 2 = 0$

112. $\cos^2 x + 5\cos x - 1 = 0$

113. $5\sin x = 2\cos^2 x - 4$

114. $7\cos x = 4 - 2\sin^2 x$

115. $2\tan^2 x + 5\tan x + 3 = 0$

116. $3\tan^2 x - \tan x - 2 = 0$

Practice Plus

In Exercises 117–120, graph f *and* g *in the same rectangular coordinate system for* $0 \le x \le 2\pi$. *Then solve a trigonometric equation to determine points of intersection and identify these points on your graphs.*

117. $f(x) = 3\cos x, g(x) = \cos x - 1$

118. $f(x) = 3\sin x, g(x) = \sin x - 1$

119. $f(x) = \cos 2x, g(x) = -2\sin x$

120. $f(x) = \cos 2x, g(x) = 1 - \sin x$

In Exercises 121–126, solve each equation on the interval $[0, 2\pi)$.

121. $|\cos x| = \frac{\sqrt{3}}{2}$

122. $|\sin x| = \frac{1}{2}$

123. $10\cos^2 x + 3\sin x - 9 = 0$

124. $3\cos^2 x - \sin x = \cos^2 x$

125. $2\cos^3 x + \cos^2 x - 2\cos x - 1 = 0$ (*Hint:* Use factoring by grouping.)

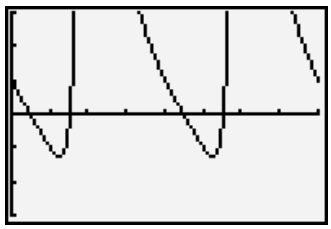
126. $2\sin^3 x - \sin^2 x - 2\sin x + 1 = 0$ (*Hint:* Use factoring by grouping.)

In Exercises 127–128, find the x-intercepts, correct to four decimal places, of the graph of each function. Then use the x-intercepts to match the function with its graph. The graphs are labeled (a) and (b).

127. $f(x) = \tan^2 x - 3\tan x + 1$

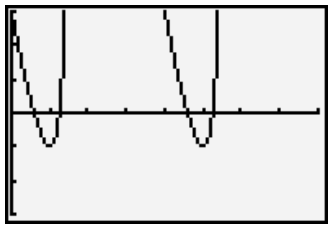
128. $g(x) = 4\tan^2 x - 8\tan x + 3$

a.



$[0, 2\pi, \frac{\pi}{4}]$ by $[-3, 3, 1]$

b.



$[0, 2\pi, \frac{\pi}{4}]$ by $[-3, 3, 1]$

Application Exercises

Use this information to solve Exercises 129–130. Our cycle of normal breathing takes place every 5 seconds. Velocity of air flow, y, *measured in liters per second, after* x *seconds is modeled by*

$$y = 0.6\sin\frac{2\pi}{5}x.$$

Velocity of air flow is positive when we inhale and negative when we exhale.

129. Within each breathing cycle, when are we inhaling at a rate of 0.3 liter per second? Round to the nearest tenth of a second.

130. Within each breathing cycle, when are we exhaling at a rate of 0.3 liter per second? Round to the nearest tenth of a second.

Use this information to solve Exercises 131–132. The number of hours of daylight in Boston is given by

$$y = 3\sin\left[\frac{2\pi}{365}(x - 79)\right] + 12,$$

where x *is the number of days after January 1.*

131. Within a year, when does Boston have 10.5 hours of daylight? Give your answer in days after January 1 and round to the nearest day.

132. Within a year, when does Boston have 13.5 hours of daylight? Give your answer in days after January 1 and round to the nearest day.