5.3 Solving Trig Equations Practice Worksheet #1
Pre-calculus

Name: KEY
Date: _____ Block: ____

Solve for the unknown variable on the interval $0 \le x < 2\pi$.

1. $4\cos^2 x - 3 = 0$

$\cos^2 x = \frac{3}{4}$

$\cos x = \frac{\sqrt{3}}{2}$

$\frac{\pi}{6}, \frac{11\pi}{6}$

2. $\sqrt{2}\sin 2x = 1$

$\sin 2x = \frac{1}{\sqrt{2}}$

$\sin 2x = \frac{\sqrt{2}}{2}$

$2x = \frac{\pi}{4}$ $2x = \frac{3\pi}{4}$

$\frac{\pi}{8}, \frac{3\pi}{8}$

3. $3\cot^2 x - 1 = 0$

$\cot^2 x = \frac{1}{3}$

$\cot x = \frac{1}{\sqrt{3}}$

$\cot x = \frac{\sqrt{3}}{3}$

$\frac{\pi}{3}, \frac{4\pi}{3}$

4. $\cos^3 x = \cos x$

$\cos^3 x - \cos x = 0$

$\cos x(\cos^2 x - 1) = 0$

$\cos x = 0$ $\cos^2 x - 1 = 0$

$\cos^2 x = 1$

$\cos x = \pm 1$

$\frac{\pi}{2}, \frac{3\pi}{2}, 0$

5. $\sin x - 2\sin x \cos x = 0$

$\sin x(1 - 2\cos x) = 0$

$\sin x = 0$ $1 - 2\cos x = 0$

$0, \pi$ $\cos x = \frac{1}{2}$

$\frac{\pi}{3}, \frac{5\pi}{3}$

6. $2\sin^2 x - \sin x - 3 = 0$

$(2\sin x - 3)(\sin x + 1) = 0$

$2\sin x - 3 = 0$ $\sin x + 1 = 0$

$\sin x = \frac{3}{2}$ $\sin x = -1$

$\sin^{-1}\frac{3}{2}, \frac{3\pi}{2}$

7. $\csc^2 x - \csc x - 2 = 0$

$(\csc x - 2)(\csc x + 1) = 0$

$\csc x - 2 = 0$ $\csc x + 1 = 0$

$\csc x = 2$ $\csc x = -1$

$\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$

8. $\cos^2 x = 1 - \sin x$

$\cos^2 x + \sin x - 1 = 0$

$1 - \sin^2 x + \sin x - 1 = 0$

$-\sin^2 x + \sin x = 0$

$\sin x(-\sin x + 1) = 0$

$\sin x = 0$ $-\sin x + 1 = 0$

$\sin x = 1$

$0, \pi, \frac{\pi}{2}$

Solve for the unknown variable on the given interval.

9. $\sqrt{3} + \tan(2x) = 0$ on $[0, 2\pi)$.

$\tan 2x = -\sqrt{3}$

$2x = \frac{2\pi}{3}$ $2x = \frac{5\pi}{3}$

$\frac{\pi}{3}, \frac{5\pi}{6}$

10. $\cos(\pi x) = 0.5$ on $[0, 2)$.

$\pi x = \frac{\pi}{3}$ $\pi x = \frac{5\pi}{3}$

$\frac{1}{3}, \frac{5}{3}$

11. $\sin\left(\frac{x}{2}\right) - 1 = 0$ on $[0, 8\pi)$.

$\sin \frac{x}{2} = 1$

$\frac{x}{2} = \frac{\pi}{2}$

π 5π

5.3 Solving Trig Equations – Worksheet #2
Pre-calculus

Name: _____
Date: _____ Block: ___

Part 1: Solve for the unknown variable. Give all of the exact general solutions.

1. $\sin\theta = \frac{\sqrt{2}}{2}$

$\frac{\pi}{4}, \frac{3\pi}{4}$

2. $\cos\theta = \sin\theta$

$1 = \frac{\sin\theta}{\cos\theta}$

$1 = \tan\theta$

$\frac{\pi}{4}, \frac{5\pi}{4}$

3. $\tan\theta = 1$

$\frac{\pi}{4}, \frac{5\pi}{4}$

4. $1 + \sin\theta = 2\cos^2\theta$

$2\cos^2\theta - \sin\theta - 1 = 0$
$2(1-\sin^2\theta) - \sin\theta - 1 = 0$
$2 - 2\sin^2\theta - \sin\theta - 1 = 0$
$-2\sin^2\theta - \sin\theta + 1 = 0$
$(-2\sin\theta + 1)(\sin\theta + 1) = 0$

$-2\sin\theta + 1 = 0$ $\quad$ $\sin\theta + 1 = 0$
$\sin\theta = \frac{1}{2}$ $\quad$ $\sin\theta = -1$

$\frac{\pi}{6}, \frac{5\pi}{6}$ $\quad$ $\frac{3\pi}{2}$

5. $2\cos^2\theta + \cos\theta = 0$

$\cos\theta(2\cos\theta + 1) = 0$

$\cos\theta = 0$ $\quad$ $2\cos\theta + 1 = 0$
$\cos\theta = -\frac{1}{2}$

$\frac{\pi}{2}, \frac{3\pi}{2}$ $\quad$ $\frac{2\pi}{3}, \frac{4\pi}{3}$

6. $\sin 3\theta = -1$

$3\theta = \frac{3\pi}{2}$

$\frac{\pi}{2}$

7. $\sin^2\theta - 1 = 0$

$\sin^2\theta = 1$

$\sin\theta = \pm 1$

$\frac{\pi}{2}, \frac{3\pi}{2}$

8. $\cos 2\theta = \frac{1}{2}$

$2\theta = \frac{\pi}{3}$ $\quad$ $2\theta = \frac{5\pi}{3}$

$\frac{\pi}{6}, \frac{5\pi}{6}$

9. $2\sin^2\theta - \sin\theta - 1 = 0$

$(2\sin\theta + 1)(\sin\theta - 1) = 0$

$2\sin\theta + 1 = 0$ $\quad$ $\sin\theta - 1 = 0$
$\sin\theta = -\frac{1}{2}$ $\quad$ $\sin\theta = 1$

$\frac{7\pi}{6}, \frac{11\pi}{6}$ $\quad$ $\frac{\pi}{2}$

10. $\tan 4\theta = -1$

$4\theta = \frac{3\pi}{4}$ $\quad$ $4\theta = \frac{7\pi}{4}$

$\frac{3\pi}{16}, \frac{7\pi}{16}$

11. $\tan^2 3x = 3$

$\tan 3x = \sqrt{3}$

$3x = \frac{\pi}{3}$ $\quad$ $3x = \frac{4\pi}{3}$

$\frac{\pi}{9}, \frac{4\pi}{9}$

12. $\cos\frac{x}{2} = \frac{\sqrt{2}}{2}$

$\frac{x}{2} = \frac{\pi}{4}$ $\quad$ $\frac{x}{2} = \frac{7\pi}{4}$

$\frac{\pi}{8}, \frac{7\pi}{8}$

Part 2: Solve by approximating the solutions on the interval [0,2π).

13. $2\sin^2 x + 3\sin x + 1 = 0$

$(2\sin x + 1)(\sin x + 1) = 0$

$2\sin x + 1 = 0 \quad \sin x + 1 = 0$

$\sin x = -\frac{1}{2} \quad \sin x = -1$

$\frac{7\pi}{6}, \frac{11\pi}{6}, \frac{3\pi}{2}$

14. $4\sin^2 x = 2\cos x + 1$

$4\sin^2 x - 2\cos x - 1 = 0$

$4(1-\cos^2 x) - 2\cos x - 1 = 0$

$4 - 4\cos^2 x - 2\cos x - 1 = 0$

$-4\cos^2 x - 2\cos x + 3 = 0$

$\frac{2 \pm \sqrt{4 - -4(4)(3)}}{-8} = \frac{2 \pm \sqrt{52}}{-8}$

$\frac{2 \pm 2\sqrt{13}}{-8} = \frac{1 \pm \sqrt{13}}{-4}$

$\cos^{-1}\left(\frac{-1+\sqrt{13}}{4}\right)$ and $\cos^{-1}\left(\frac{-1-\sqrt{13}}{4}\right)$

15. $\csc x + \cot x = 1$

$\frac{1}{\sin x} + \frac{\cos x}{\sin x} = 1$

$\frac{1+\cos x}{\sin x} = 1$

$1 + \cos x = \sin x$

$1 + \cos x = -\sqrt{1-\cos^2 x}$

$(1+\cos x)^2 = 1 - \cos^2 x$

$(1+\cos x)^2 = (1-\cos x)(1+\cos x)$

$1 + \cos x = 1 - \cos x$

$2\cos x = 0$

$\cos x = 0$

$\frac{\pi}{2}$

16. $\frac{\cos x \cot x}{1 - \sin x} = 3$ $\quad \frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{2}$

$\frac{\cos^2 x}{\sin x} = 3(1 - \sin x)$

$\cos^2 x = 3\sin x(1 - \sin x)$

$1 - \sin^2 x = 3\sin x - 3\sin^2 x$

$2\sin^2 x - 3\sin x + 1 = 0$

$(2\sin x - 1)(\sin x - 1) = 0$

$\sin x = \frac{1}{2} \quad \sin x = 1$

17. $\sec^2 x + 0.5\tan x = 1$

$\tan^2 x + 1 + 0.5\tan x - 1 = 0$

$\tan^2 x + 0.5\tan x = 0$

$\tan x(\tan x + \frac{1}{2}) = 0$

$\tan x = 0 \quad \tan x = -\frac{1}{2}$

$0, \pi \quad \tan^{-1}\left(-\frac{1}{2}\right)$

Part 3: Use the calculator's inverse trig functions to approximate the solutions. Remember that you must also find the other solution by either adding π, subtracting the value from π, or subtracting the value from 2π.

18. $\tan\theta = 4$ $\quad +\pi$

$\tan^{-1} 4 = \theta$

1.3258, 4.4674

19. $\cos\theta = 0.84$ $\quad 2\pi -$

$\cos^{-1} .84 = \theta$

.5735, 5.7097

20. $\sin\theta = 0.63$ $\quad \pi -$

$\sin^{-1} .63 = \theta$

.6816, 2.46