Check Point 9 A region that is 30° north of the Equator averages a minimum of 10 hours of daylight in December. Hours of daylight are at a maximum of 14 hours in June. Let x represent the month of the year, with 1 for January, 2 for February, 3 for March, and 12 for December. If y represents the number of hours of daylight in month x, use a sine function of the form $y = A\sin(Bx - C) + D$ to model the hours of daylight.

Exercise Set 4.5

Practice Exercises

In Exercises 1–6, determine the amplitude of each function. Then graph the function and $y = \sin x$ in the same rectangular coordinate system for $0 \le x \le 2\pi$.

1. $y = 4\sin x$

2. $y = 5\sin x$

3. $y = \frac{1}{3}\sin x$

4. $y = \frac{1}{4}\sin x$

5. $y = -3\sin x$

6. $y = -4\sin x$

In Exercises 7–16, determine the amplitude and period of each function. Then graph one period of the function.

7. $y = \sin 2x$

8. $y = \sin 4x$

9. $y = 3\sin\frac{1}{2}x$

10. $y = 2\sin\frac{1}{4}x$

11. $y = 4\sin \pi x$

12. $y = 3\sin 2\pi x$

13. $y = -3\sin 2\pi x$

14. $y = -2\sin \pi x$

15. $y = -\sin\frac{2}{3}x$

16. $y = -\sin\frac{4}{3}x$

In Exercises 17–30, determine the amplitude, period, and phase shift of each function. Then graph one period of the function.

17. $y = \sin(x - \pi)$

18. $y = \sin\left(x - \frac{\pi}{2}\right)$

19. $y = \sin(2x - \pi)$

20. $y = \sin\left(2x - \frac{\pi}{2}\right)$

21. $y = 3\sin(2x - \pi)$

22. $y = 3\sin\left(2x - \frac{\pi}{2}\right)$

23. $y = \frac{1}{2}\sin\left(x + \frac{\pi}{2}\right)$

24. $y = \frac{1}{2}\sin(x + \pi)$

25. $y = -2\sin\left(2x + \frac{\pi}{2}\right)$

26. $y = -3\sin\left(2x + \frac{\pi}{2}\right)$

27. $y = 3\sin(\pi x + 2)$

28. $y = 3\sin(2\pi x + 4)$

29. $y = -2\sin(2\pi x + 4\pi)$

30. $y = -3\sin(2\pi x + 4\pi)$

In Exercises 31–34, determine the amplitude of each function. Then graph the function and $y = \cos x$ in the same rectangular coordinate system for $0 \le x \le 2\pi$.

31. $y = 2\cos x$

32. $y = 3\cos x$

33. $y = -2\cos x$

34. $y = -3\cos x$

In Exercises 35–42, determine the amplitude and period of each function. Then graph one period of the function.

35. $y = \cos 2x$

36. $y = \cos 4x$

37. $y = 4\cos 2\pi x$

38. $y = 5\cos 2\pi x$

39. $y = -4\cos\frac{1}{2}x$

40. $y = -3\cos\frac{1}{3}x$

41. $y = -\frac{1}{2}\cos\frac{\pi}{3}x$

42. $y = -\frac{1}{2}\cos\frac{\pi}{4}x$

In Exercises 43–52, determine the amplitude, period, and phase shift of each function. Then graph one period of the function.

43. $y = \cos\left(x - \frac{\pi}{2}\right)$

44. $y = \cos\left(x + \frac{\pi}{2}\right)$

45. $y = 3\cos(2x - \pi)$

46. $y = 4\cos(2x - \pi)$

47. $y = \frac{1}{2}\cos\left(3x + \frac{\pi}{2}\right)$

48. $y = \frac{1}{2}\cos(2x + \pi)$

49. $y = -3\cos\left(2x - \frac{\pi}{2}\right)$

50. $y = -4\cos\left(2x - \frac{\pi}{2}\right)$

51. $y = 2\cos(2\pi x + 8\pi)$

52. $y = 3\cos(2\pi x + 4\pi)$

In Exercises 53–60, use a vertical shift to graph one period of the function.

53. $y = \sin x + 2$

54. $y = \sin x - 2$

55. $y = \cos x - 3$

56. $y = \cos x + 3$

57. $y = 2\sin\frac{1}{2}x + 1$

58. $y = 2\cos\frac{1}{2}x + 1$

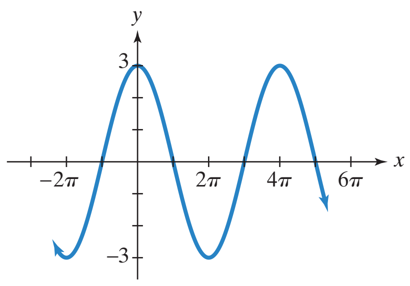
59. $y = -3\cos 2\pi x + 2$

60. $y = -3\sin 2\pi x + 2$

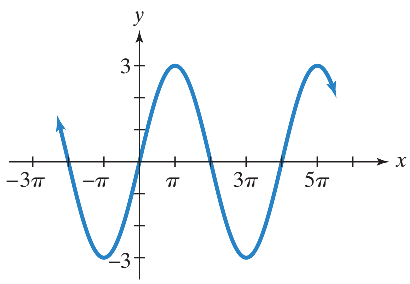
Practice Plus

In Exercises 61–66, find an equation for each graph.

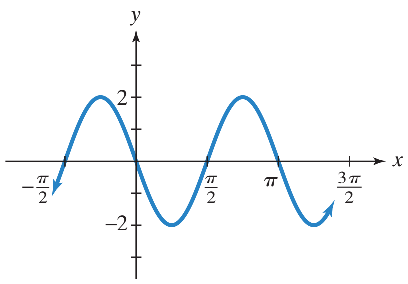
61.



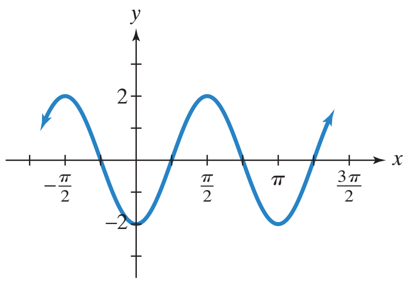
62.



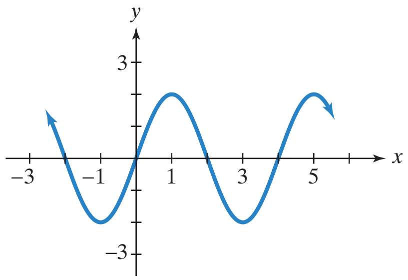
63.



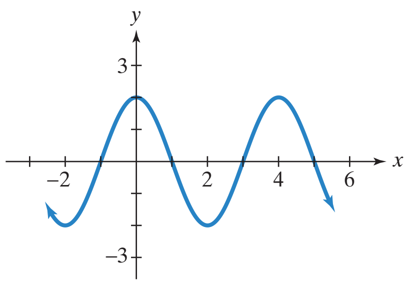
64.



65.



66.



In Exercises 67–70, graph one period of each function.

67. $y = \left|2\cos\frac{x}{2}\right|$

68. $y = \left|3\cos\frac{2x}{3}\right|$

69. $y = -|3\sin \pi x|$

70. $y = -\left|2\sin\frac{\pi x}{2}\right|$

In Exercises 71–74, graph f, g, and h in the same rectangular coordinate system for $0 \le x \le 2\pi$. Obtain the graph of h by adding or subtracting the corresponding y-coordinates on the graphs of f and g.

71. $f(x) = -2\sin x,\ g(x) = \sin 2x,\ h(x) = (f + g)(x)$

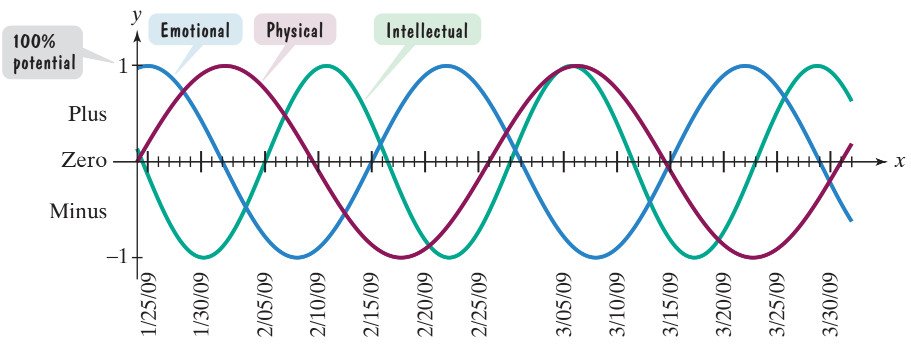
72. $f(x) = 2\cos x,\ g(x) = \cos 2x,\ h(x) = (f + g)(x)$

73. $f(x) = \sin x,\ g(x) = \cos 2x,\ h(x) = (f - g)(x)$

74. $f(x) = \cos x,\ g(x) = \sin 2x,\ h(x) = (f - g)(x)$

Application Exercises

In the theory of biorhythms, sine functions are used to measure a person's potential. You can obtain your biorhythm chart online by simply entering your date of birth, the date you want your biorhythm chart to begin, and the number of months you wish to be included in the plot. Shown below is your author's chart, beginning January 25, 2009, when he was 23,283 days old. We all have cycles with the same amplitudes and periods as those shown here. Each of our three basic cycles begins at birth. Use the biorhythm chart shown to solve Exercises 75–82. The longer tick marks correspond to the dates shown.



75. What is the period of the physical cycle?

76. What is the period of the emotional cycle?

77. What is the period of the intellectual cycle?

78. For the period shown, what is the worst day in February for your author to run in a marathon?

79. For the period shown, what is the best day in March for your author to meet an online friend for the first time?

80. For the period shown, what is the best day in February for your author to begin writing this trigonometry chapter?

81. If you extend these sinusoidal graphs to the end of the year, is there a day when your author should not even bother getting out of bed?

82. If you extend these sinusoidal graphs to the end of the year, are there any days where your author is at near-peak physical, emotional, and intellectual potential?

83. Rounded to the nearest hour, Los Angeles averages 14 hours of daylight in June, 10 hours in December, and 12 hours in March and September. Let x represent the number of months after June and let y represent the number of hours of daylight in month x. Make a graph that displays the information from June of one year to June of the following year.

84. A clock with an hour hand that is 15 inches long is hanging on a wall. At noon, the distance between the tip of the hour hand and the ceiling is 23 inches. At 3 P.M., the distance is 38 inches; at 6 P.M., 53 inches; at 9 P.M., 38 inches; and at midnight the distance is again 23 inches. If y represents the distance between the tip of the hour hand and the ceiling x hours after noon, make a graph that displays the information for $0 \le x \le 24$.

85. The number of hours of daylight in Boston is given by

$$y = 3\sin\frac{2\pi}{365}(x - 79) + 12,$$

where x is the number of days after January 1.

a. What is the amplitude of this function?

b. What is the period of this function?

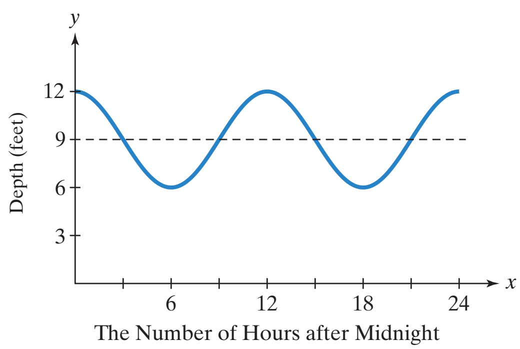
c. How many hours of daylight are there on the longest day of the year?

d. How many hours of daylight are there on the shortest day of the year?

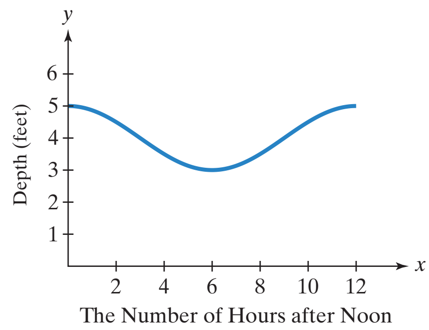
e. Graph the function for one period, starting on January 1.

86. The average monthly temperature, y, in degrees Fahrenheit, for Juneau, Alaska, can be modeled by $y = 16\sin\left(\frac{\pi}{6}x - \frac{2\pi}{3}\right) + 40$, where x is the month of the year (January = 1, February = 2, ... December = 12). Graph the function for $1 \le x \le 12$. What is the highest average monthly temperature? In which month does this occur?

87. The figure shows the depth of water at the end of a boat dock. The depth is 6 feet at low tide and 12 feet at high tide. On a certain day, low tide occurs at 6 A.M. and high tide at noon. If y represents the depth of the water x hours after midnight, use a cosine function of the form $y = A\cos Bx + D$ to model the water's depth.



88. The figure in the next column shows the depth of water at the end of a boat dock. The depth is 5 feet at high tide and 3 feet at low tide. On a certain day, high tide occurs at noon and low tide at 6 P.M. If y represents the depth of the water x hours after noon, use a cosine function of the form $y = A\cos Bx + D$ to model the water's depth.



Writing in Mathematics

89. Without drawing a graph, describe the behavior of the basic sine curve.

90. What is the amplitude of the sine function? What does this tell you about the graph?

91. If you are given the equation of a sine function, how do you determine the period?

92. What does a phase shift indicate about the graph of a sine function? How do you determine the phase shift from the function's equation?

93. Describe a general procedure for obtaining the graph of $y = A\sin(Bx - C)$.

94. Without drawing a graph, describe the behavior of the basic cosine curve.

95. Describe a relationship between the graphs of $y = \sin x$ and $y = \cos x$.

96. Describe the relationship between the graphs of $y = A\cos(Bx - C)$ and $y = A\cos(Bx - C) + D$.

97. Biorhythm cycles provide interesting applications of sinusoidal graphs. But do you believe in the validity of biorhythms? Write a few sentences explaining why or why not.

Technology Exercises

98. Use a graphing utility to verify any five of the sine curves that you drew by hand in Exercises 7–30. The amplitude, period, and phase shift should help you to determine appropriate viewing rectangle settings.

99. Use a graphing utility to verify any five of the cosine curves that you drew by hand in Exercises 35–52.

100. Use a graphing utility to verify any two of the sinusoidal curves with vertical shifts that you drew in Exercises 53–60.

In Exercises 101–104, use a graphing utility to graph two periods of the function.

101. $y = 3\sin(2x + \pi)$

102. $y = -2\cos\left(2\pi x - \frac{\pi}{2}\right)$

103. $y = 0.2\sin\left(\frac{\pi}{10}x + \pi\right)$

104. $y = 3\sin(2x - \pi) + 5$

105. Use a graphing utility to graph $y = \sin x$ and $y = x - \frac{x^3}{6} + \frac{x^5}{120}$ in a $\left[-\pi, \pi, \frac{\pi}{2}\right]$ by $[-2, 2, 1]$ viewing rectangle. How do the graphs compare?

106. Use a graphing utility to graph $y = \cos x$ and $y = 1 - \frac{x^2}{2} + \frac{x^4}{24}$ in a $\left[-\pi, \pi, \frac{\pi}{2}\right]$ by $[-2, 2, 1]$ viewing rectangle. How do the graphs compare?

107. Use a graphing utility to graph

$$y = \sin x + \frac{\sin 2x}{2} + \frac{\sin 3x}{3} + \frac{\sin 4x}{4}$$

in a $\left[-2\pi, 2\pi, \frac{\pi}{2}\right]$ by $[-2, 2, 1]$ viewing rectangle. How do these waves compare to the smooth rolling waves of the basic sine curve?

108. Use a graphing utility to graph

$$y = \sin x - \frac{\sin 3x}{9} + \frac{\sin 5x}{25}$$

in a $\left[-2\pi, 2\pi, \frac{\pi}{2}\right]$ by $[-2, 2, 1]$ viewing rectangle. How do these waves compare to the smooth rolling waves of the basic sine curve?

109. The data show the average monthly temperatures for Washington, D.C.

x (Month)		**Average Monthly Temperature, °F**
1	(January)	**34.6**
2	(February)	**37.5**
3	(March)	**47.2**
4	(April)	**56.5**
5	(May)	**66.4**
6	(June)	**75.6**
7	(July)	**80.0**
8	(August)	**78.5**
9	(September)	**71.3**
10	(October)	**59.7**
11	(November)	**49.8**
12	(December)	**39.4**

Source: U.S. National Oceanic and Atmospheric Administration

a. Use your graphing utility to draw a scatter plot of the data from $x = 1$ through $x = 12$.

b. Use the SINe REGression feature to find the sinusoidal function of the form $y = A\sin(Bx + C) + D$ that best fits the data.

c. Use your graphing utility to draw the sinusoidal function of best fit on the scatter plot.

110. Repeat Exercise 109 for data of your choice. The data can involve the average monthly temperatures for the region where you live or any data whose scatter plot takes the form of a sinusoidal function.

Critical Thinking Exercises

Make Sense? *In Exercises 111–114, determine whether each statement makes sense or does not make sense, and explain your reasoning.*

111. When graphing one complete cycle of $y = A\sin(Bx - C)$, I find it easiest to begin my graph on the x-axis.

112. When graphing one complete cycle of $y = A\cos(Bx - C)$, I find it easiest to begin my graph on the x-axis.

113. Using the equation $y = A\sin Bx$, if I replace either A or B with its opposite, the graph of the resulting equation is a reflection of the graph of the original equation about the x-axis.

114. A ride on a circular Ferris wheel is like riding sinusoidal graphs.

115. Determine the range of each of the following functions. Then give a viewing rectangle, or window, that shows two periods of the function's graph.

a. $f(x) = 3\sin\left(x + \frac{\pi}{6}\right) - 2$

b. $g(x) = \sin 3\left(x + \frac{\pi}{6}\right) - 2$

116. Write the equation for a cosine function with amplitude π, period 1, and phase shift -2.

In Chapter 5, we will prove the following identities:

$$\sin^2 x = \frac{1}{2} - \frac{1}{2}\cos 2x$$

$$\cos^2 x = \frac{1}{2} + \frac{1}{2}\cos 2x.$$

Use these identities to solve Exercises 117–118.

117. Use the identity for $\sin^2 x$ to graph one period of $y = \sin^2 x$.

118. Use the identity for $\cos^2 x$ to graph one period of $y = \cos^2 x$.

Group Exercise

119. This exercise is intended to provide some fun with biorhythms, regardless of whether you believe they have any validity. We will use each member's chart to determine biorhythmic compatibility. Before meeting, each group member should go online and obtain his or her biorhythm chart. The date of the group meeting is the date on which your chart should begin. Include 12 months in the plot. At the meeting, compare differences and similarities among the intellectual sinusoidal curves. Using these comparisons, each person should find the one other person with whom he or she would be most intellectually compatible.

Preview Exercises

Exercises 120–122 will help you prepare for the material covered in the next section.

120. Solve: $-\frac{\pi}{2} < x + \frac{\pi}{4} < \frac{\pi}{2}$.

121. Simplify: $\dfrac{-\frac{3\pi}{4} + \frac{\pi}{4}}{2}$.

122. a. Graph $y = -3\cos\frac{x}{2}$ for $-\pi \le x \le 5\pi$.

b. Consider the reciprocal function of $y = -3\cos\frac{x}{2}$, namely, $y = -3\sec\frac{x}{2}$. What does your graph from part (a) indicate about this reciprocal function for $x = -\pi, \pi, 3\pi$, and 5π?