Verifying Trigonometric Identities

Objective: To verify that two expressions are equivalent. That is, we want to verify that what we have is an identity.

- To do this, we generally pick the expression on one side of the given identity and manipulate that expression until we get the other side.
- In most cases, it is best to start with the more complex looking side and try to simply to match the less complex side.
- You must be very familiar with the fundamental trigonometric identities, especially the Pythagorean Identities. In some cases, a direct substitution using these fundamental identities will verify the identity you are trying to prove (Exercise 8 at the end of this document is one example).
- Some special approaches are useful for certain types of identities, which are provided below.

Identity Type	Verification	Approach
<u>Type 1</u>: Sometimes it is easier if we just rewrite everything in terms of sine and cosine to see if the expression simplifies.	Verify: $$\mathbf{\cot x + 1 = \csc x\,(\cos x + \sin x)}$$ $$\text{RHS} \rightarrow \mathbf{\csc x\,(\cos x + \sin x)} = \frac{1}{\sin x}(\cos x + \sin x) = \frac{\cos x}{\sin x} + \frac{\sin x}{\sin x} = \mathbf{\cot x + 1}$$	• Start with more complex RHS. • Rewrite $\csc x$ in terms of sine or cosine. • Remember, $\csc x = 1/\sin x$ • Also note, $\cos x/\sin x = \cot x$ • The RHS simplifies to original LHS.
<u>Type 2</u>: In some cases, the more complex side involves a fraction that can be split up. Then we rewrite everything in terms of sine and cosine.	Verify: $$\mathbf{\frac{\tan t - \cot t}{\sin t \cos t} = \sec^2 t - \csc^2 t}$$ $$\text{LHS} \rightarrow \mathbf{\frac{\tan t - \cot t}{\sin t \cos t}} = \frac{\tan t}{\sin t \cos t} - \frac{\cot t}{\sin t \cos t} = \tan t \cdot \frac{1}{\sin t \cos t} - \cot t \cdot \frac{1}{\sin t \cos t} = \frac{\sin t}{\cos t} \cdot \frac{1}{\sin t \cos t} - \frac{\cos t}{\sin t} \cdot \frac{1}{\sin t \cos t} = \frac{1}{\cos^2 t} - \frac{1}{\sin^2 t} = \mathbf{\sec^2 t - \csc^2 t}$$	• Start with the more complex LHS. • Rewrite the LHS as difference of two fractions. • Split out $\tan t$ and $\cot t$ to make it easier to simplify. • Notice in the first term, the $\sin t$ cancels out; and in the second term, $\cos t$ cancels out. • The new terms are reciprocal identities • The LHS simplifies to the original RHS.

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Type 3: Using the property of conjugates is sometimes helpful. For an expression like $a+b$, the conjugate would be $a-b$. When you multiply conjugates, you often get a more useful expression, e.g., $(a+b)(a-b)$. Sometimes multiplying by the conjugate will simplify an expression and help in verifying the given identity.	Verify: $$\frac{\mathbf{\cos x}}{\mathbf{1-\sin x}}=\frac{\mathbf{1+\sin x}}{\mathbf{\cos x}}$$ $$\text{RHS} \rightarrow \frac{\mathbf{1+\sin x}}{\mathbf{\cos x}}=\frac{1+\sin x}{\cos x}\left(\frac{1-\sin x}{1-\sin x}\right)$$ $$=\frac{1-\sin^2 x}{\cos x\,(1-\sin x)}$$ $$=\frac{\cos^2 x}{\cos x\,(1-\sin x)}$$ $$=\frac{\cos x\cos x}{\cos x\,(1-\sin x)}$$ $$=\frac{\mathbf{\cos x}}{\mathbf{1-\sin x}}$$	• We could start with either side; but here we will start with the RHS. • The conjugate of the numerator $1+\sin x$ is $1-\sin x$. • Multiply by $\frac{1-\sin x}{1-\sin x}=1$ • Remember, $1-\sin^2 x=\cos^2 x$ • Once we reduce the fraction, we get the LHS of the original identity.
Type 4: Combining fractions before using identities may be an appropriate strategy.	Verify: $$\frac{\mathbf{1}}{\mathbf{1-\sin\alpha}}+\frac{\mathbf{1}}{\mathbf{1+\sin\alpha}}=\mathbf{2\sec^2\alpha}$$ $$\text{LHS} \rightarrow \frac{\mathbf{1}}{\mathbf{1-\sin\alpha}}+\frac{\mathbf{1}}{\mathbf{1+\sin\alpha}}=\frac{1}{1-\sin\alpha}\left(\frac{1+\sin\alpha}{1+\sin\alpha}\right)+\frac{1}{1+\sin\alpha}\left(\frac{1-\sin\alpha}{1-\sin\alpha}\right)$$ $$=\frac{(1+\sin\alpha)+(1-\sin\alpha)}{(1-\sin\alpha)(1+\sin\alpha)}$$ $$=\frac{2}{1-\sin^2\alpha}$$ $$=\frac{2}{\cos^2\alpha}$$ $$=\mathbf{2\sec^2\alpha}$$	• Notice that the denominators of the fractions on the LHS are conjugates. • So we will use the property of conjugates to combine the LHS fractions and simplify.

Verifying Trigonometric Identities

Verify the following trigonometric identities.

1. $\cos x + \sin x \tan x = \sec x$

2. $\dfrac{\csc x - \sin x}{\sin x \csc x} = \csc x - \sin x$

3. $\dfrac{1}{\tan \beta} + \tan \beta = \dfrac{\sec^2 \beta}{\tan \beta}$

4. $\dfrac{1 + \sin \theta}{\cos \theta} + \dfrac{\cos \theta}{1 + \sin \theta} = 2 \sec \theta$

5. $\sec y + \tan y = \dfrac{\cos y}{1 - \sin y}$

6. $\dfrac{\cos^2 x - \sin^2 x}{1 - \tan^2 x} = \cos^2 x$

7. $\dfrac{\sin x}{\cos x + 1} + \dfrac{\cos x - 1}{\sin x} = 0$

8. $\dfrac{\sin^2 \theta + \cos^2 \theta + \cot^2 \theta}{1 + \tan^2 \theta} = \cot^2 \theta$