Chapters 1-4.2 Review

1. $\lim_{z \to -3^-} g(z)$

2. $\lim_{x \to 0^-} h(x)$

$0 - 4 = -4$

3. $\lim_{x \to 2} h(x)$

$\lim_{x \to 2^-} h(x) = -3(2) - 3$
$= -9$

$\lim_{x \to 2^+} h(x) = -(2) - 4$
$= -6$

4. $\lim_{x \to 1} f(x) = -1$

5. $\lim_{x \to -2^-} f(x) = -2$

6. $\lim_{x \to -2} f(x) = DNE$

7. $\lim_{x \to 1^-} f(x) = -1$

$\lim_{x \to 1^+} f(x) = -1$

$f(1) = -1$

$\therefore$ yes it is continuous

8. $\lim_{x \to -2^-} f(x) = -2$

$\lim_{x \to -2^+} f(x) = 2$

$\lim_{x \to -2^-} \neq \lim_{x \to -2^+}$

$\therefore$ it is not continuous

9. $\lim_{x \to 2} \frac{4x(x-2)}{(x-2)}$

$\lim_{x \to 2} 4x = 8$

10. $\lim_{x \to -2} 5 = 5$

11. $\lim_{x \to 1/4} \frac{\sqrt{x+4} - 2}{4x - 1} = \frac{\sqrt{17/4} - 2}{0} = DNE$

12. $\lim_{x \to \frac{-\pi}{3}} \sin x =$

$\sin \frac{-\pi}{3} = \frac{-\sqrt{3}}{2}$

13. $\lim_{x \to -1} g(f(x)) = g(f(-1))$

$= -57$

14. $\lim_{x \to \infty} \frac{6x + 2\sin x}{9x + 7} = \frac{2}{3}$

15. $\lim_{x \to 2^-} g(x) = -2$

$\lim_{x \to 2^+} g(x) = -3$

Not continuous since $\lim_{x \to 2^-} g(x) \neq \lim_{x \to 2^+} g(x)$

16. $f(x) = \frac{(2x-1)(-2x+1)(x+1)}{(x-3)(x-3)(x+4)}$

Vert. Asymp: $x = 3, -4$

17. $f(x) = \frac{-4x^3 + 3x - 1}{x^3 - 2x^2 - 15x + 36}$

Horz. Asymp: $y = -4$

18. $w'(x) = -15x^2 + 4x + 3$

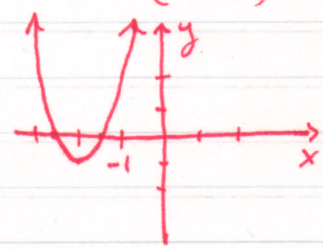
slope: $w'(-1) = -15 - 4 + 3 = -16$

19. $r'(x) = -6x + 3$

20. $t'(x) = \frac{(3x-4)4 - (4x-3)3}{(3x-4)^2} = 12x - 16 - 12x + 9 = \frac{-7}{(3x-4)^2}$

$t'(-2) = \frac{-7}{(-6-4)^2} = \frac{-7}{100}$

21.



22. Not Differentiable since it has a cusp at $x = -2$.

23. @ $x = 10$ the function is continuous.

$\lim_{x \to 10} h(x) = -23$

$\therefore$ yes it is differentiable

24. No it is not continuous

25. No it is not continuous

26. Both

27. Continuous but not differentiable since cusp @ $x = -2$.

28. $a'(x) = -4x - 4$

$a'(1) = -8$

$y = mx + b \quad (1, -4)$

$-4 = -8(1) + b \qquad b = 4$

$y = -8x + 4$

29. $\sqrt{x^2 - 2} = (x^2 - 2)^{1/2}$

$f'(x) = \frac{1}{2}(x^2 - 2)^{-1/2}(2x)$

$= \frac{x}{(x^2 - 2)^{1/2}}$

30. $f'(x) = \dfrac{2x(2)(x^2+1)(2x) - (x^2+1)^2(2)}{(2x)^2}$

$= \dfrac{2(x^2+1)[4x^2 - (x^2+1)]}{4x^2} = \dfrac{(x^2+1)(3x^2-1)}{2x^2}$

31. $f(x) = (\sin(3x^4-2x)^2)^3$

$f'(x) = 3(\sin(3x^4-2x)^2)^2(\cos(3x^4-2x)^2)(2)(3x^4-2x)(12x-2)$

$= 6(3x^4-2x)(12x-2)\sin^2(3x^4-2x)^2 \cos(3x^4-2x)^2$

32. $f(x) = (3x^3-5x)^2 \cdot (\cos 2x)^2$

$= 2(3x^3-5x)(9x^2-5)(\cos 2x)^2 + (3x^3-5x)^2(\cos 2x)(-\sin 2x)$

$= 2(3x^3-5x)\cos 2x\,[(9x^2-5)(\cos 2x) + (3x^3-5x)(-\sin 2x)]$

33. $f'(x) = \dfrac{3x^4+2x^2-1}{2x^2} = \dfrac{3x^4}{2x^2} + \dfrac{2x^2}{2x^2} - \dfrac{1}{2x^2} = \dfrac{3x^2}{2} + 1 - \dfrac{1}{2}x^{-}$

$f''(x) = 3x + x^{-3} = 3x + \dfrac{1}{x^3} = \dfrac{3x^4+1}{x^3}$

34. Ave. Vel $= \dfrac{v(b)-v(a)}{b-a} = \dfrac{s'(b) - s'(a)}{b-a}$

$s'(t) = -6t-2$ $\qquad = \dfrac{-14-(-8)}{2-1} = -6$

35. $s'(2) = -14$

36. $f'(x) = 1$ $\qquad \dfrac{f'(69) - f'(20)}{69-20} = \dfrac{1-1}{39} = 0$

37. $u'(x) = -24x^2 + 54x - 30 = -6(4x^2 - 9x + 5)$
$= -6(4x-5)(x-1)$

Critical points at $x = 1, 5/4$ $\therefore (1,8); (5/4, 129/16)$

38. $f'(x) = \dfrac{(x+2)(2x) - x^2(1)}{(x+2)^2} = \dfrac{2x^2+4x-x^2}{(x+2)^2} = \dfrac{x^2+4x}{(x+2)^2} = \dfrac{x(x+4}{(x+2)}$

critical #'s at $x=0, -4$

$f(3) = 9/5$
$f(8) = 32/5$ Max : $(8, \frac{32}{5})$

39. $q(x) = -5x^2 + 30x - 48$ over $[1,5]$
cont. ✓
Differentiable ✓ $q'(x) = -10x + 30 = -10(x-3)$
$f(1) = -23$
$f(5) = -23$ $q'(x) = 0$ @ $x = 3$

40. $a(x) = 3x^2 + x - 3$ over $[-1, 3]$
continuous ✓
Differentiable ✓ $a'(x) =$

$a'(x) = 6x + 1$

41. $k'(x) = -10x + 4$ $[-\infty, 2/5]$ $[2/5, \infty]$
critical #'s : $2/5$ $f'(0) > 0$ $f'(3) < 0$
Increasing decreasing

42. $n'(x) = 3x^2 + 12x + 9 = 3(x^2 + 4x + 3) = 3(x+3)(x+1)$
$(-\infty, -3)$ $(-3, -1)$ $(-1, \infty)$
$f'(-4)$ $f'(-2)$ $f'(0)$
$+$ $-$ $+$
local max at $x = -3$ local min at $x = -1$

43. $s(t) = -16x^2 + 96x + 2$ $\qquad s'(t) = -32x + 96 = 0$

$x = 3$

max height @ $x = 3$ which is 146 ft

44. $m(x) = x^3 - 4x^2 - x - 2$ $\qquad (-\infty, 4/3) \quad (4/3, \infty)$

$m'(x) = 3x^2 - 8x - 1$ $\qquad f''(0) < 0 \quad f''(2) > 0$

$m''(x) = 6x - 8$ $\qquad$ concave ↓ $\quad$ concave ↑

point of inflection at $x = 4/3$

45. $w(x) = x^3 - 3x^2 - 4x - 4$

$w'(x) = 3x^2 - 6x - 4$ $\qquad$ point of inflection:

$w''(x) = 6x - 6$ $\qquad (1, -10)$

46. $p'(x) = 3x^2 + 30x + 75$ $\qquad p''(x) = 6x + 30$

$p'(x) = 3(x^2 + 10x + 25)$ $\qquad p''(-5) = 0 \Rightarrow$ No conclusion can be made

$p'(x) = 0$ @ $x = -5$ $\qquad \therefore p'(-6) > 0 \quad p'(-4) > 0$

The second Derivative is inconclusive. The First derivative test shows that there is no local maxi

47. $P = 2x + 2y = 30$ $\qquad A = xy = x(15 - x) = 15x - x^2$

$y = 15 - x$ $\qquad A' = 15 - 2x = 0$ @ $x = 7.5$

$x = 7.5$ ft $\quad y = 7.5$ ft

48. $A = 2x^2 + 4xy$ $\qquad Cost = 2x^2 + 4(2)xy = 2x^2 + 8xy$

$V = x^2 y \Rightarrow y = \frac{V}{x^2} = \frac{170}{x^2}$ $\qquad \cancel{x} =$

$C = 2x^2 + 8x\left(\frac{170}{x^2}\right)$ $\qquad C' = 4x - \frac{1360}{x^2} = 0$ $\qquad x^3 = 340$ $\quad y = 3.48$ $\quad x = 6.98$

49. $p(x) = -2x^3 - 3x^2 + 3x + 3$ $\qquad p'(x) = -6x^2 - 6x + 3$

1) $x_2 = -1 - \frac{-1}{3} = \frac{-2}{3}$ $\qquad x_3 = \frac{-2}{3} - \frac{.26}{4.33} = -.7264$

$x_4 = -.7264 - \frac{.00441}{4.1925} = -.7275$ $\qquad x_5 = -.7274$

50. $\int \frac{2}{5} x^6 \, dx = \dfrac{2x^7}{35} + C$

51. $\int \dfrac{2x^4}{2x^{1/2}} + \dfrac{3x}{2x^{1/2}} - \dfrac{6}{2x^{1/2}} \, dx = \int x^{7/2} + \dfrac{3}{2}x^{1/2} - 3x^{-1/2} \, dx$

$= \dfrac{2}{9} x^{9/2} + x^{3/2} - 6x^{1/2} + C$

52. $\int \dfrac{x^{3/4}}{x^{1/2}} \, dx = \int x^{1/4} \, dx = \dfrac{4}{5} x^{5/4} + C$

53. $\int 3x^{-4} \, dx - \int \sin x \, dx$

$-x^{-3} + \cos x + C$

54. $F(x) = \dfrac{2}{3}x^3 + \dfrac{1}{2}x^2 + C \qquad -2 = \dfrac{2}{3} + \dfrac{1}{2} + C \qquad C = -\dfrac{19}{6}$

$F(x) = \dfrac{2x^3}{3} + \dfrac{x^2}{2} - \dfrac{19}{6}$

55. $\dfrac{n^2(n+1)^2}{4} - 3\dfrac{n(n+1)}{2} - 45n$

$\dfrac{20^2(21)^2}{4} - 3\dfrac{(20)(21)}{2} - 45(20) = \dfrac{400(441)}{4} - 3(10)(21) - 900$

$= 42570$

56. $\Delta x = \dfrac{1}{n} \qquad c_i = 2 + \dfrac{i}{n}$

$\lim\limits_{n\to\infty} \sum f(c_i)\Delta x = \lim\limits_{n\to\infty} \sum\limits_{1}^{n} 2\left(2+\dfrac{i}{n}\right)^2 - 1 \left(\dfrac{1}{n}\right)$

$= \sum 2\left(4 + \dfrac{4i}{n} + \dfrac{i^2}{n^2}\right) - 1 = \sum \left(7 + \dfrac{8i}{n} + \dfrac{2i^2}{n^2}\right)\dfrac{1}{n}$

$= \sum \dfrac{7}{n} + \dfrac{8i}{n^2} + \dfrac{2i^2}{n^3} = \dfrac{7n}{n} + \dfrac{8(n(n+1))}{2n^2} + \dfrac{2n(n+1)(2n+1)}{6n^3}$

$= 7 + 4 + \dfrac{2}{3} = 11\frac{2}{3}$