

Even and Odd Functions

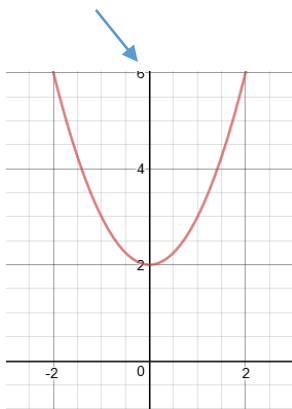
A **Function** can be classified as **Even**, **Odd** or **Neither**. This classification can be determined *graphically* or *algebraically*.

Graphical Interpretation -

Even Functions:

Have a graph that is symmetric with respect to the **Y-Axis**.

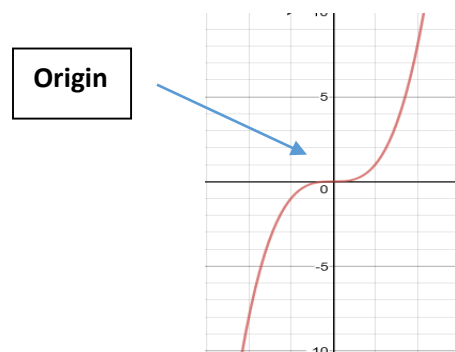
Y-Axis – acts like a mirror



Odd Functions:

Have a graph that is symmetric with respect to the **Origin**.

Origin – If you spin the picture upside down about the Origin, the graph looks the same!



Algebraic Test – Substitute $(-x)$ in for x everywhere in the function and analyze the results of $f(-x)$, by comparing it to the original function $f(x)$.

Even Function: $y = f(x)$ is **Even** when, for each x in the domain of $f(x)$, $f(-x) = f(x)$

Odd Function: $y = f(x)$ is **Odd** when, for each x in the domain of $f(x)$, $f(-x) = -f(x)$

Examples:

a. $f(x) = x^2 + 4$

$$f(-x) = (-x)^2 + 4$$

$$f(-x) = x^2 + 4$$

$$f(-x) = f(x)$$



Even Function!

b. $f(x) = x^3 - 2x$

$$f(-x) = (-x)^3 - 2(-x)$$

$$f(-x) = -x^3 + 2x$$

$$f(-x) = -(x^3 - 2x) = -f(x)$$



Odd Function!

c. $f(x) = x^2 - 3x + 4$

$$f(x) = (-x)^2 - 3(-x) + 4$$

$$f(-x) = x^2 + 3x + 4$$

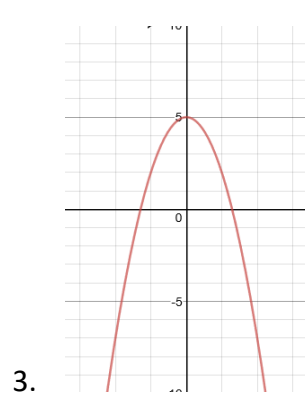
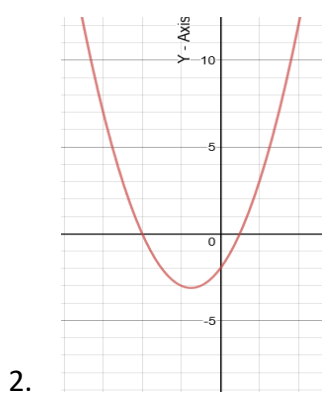
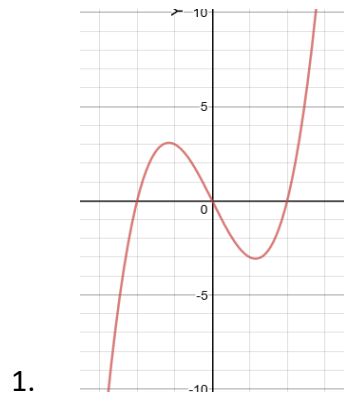
$$f(-x) \neq f(x) \neq -f(x)$$



Neither!

Even and Odd Functions - Practice Problems

A. Graphically determine whether the following functions are Even, Odd, or Neither



B. Algebraically determine whether the following functions are Even, Odd, or Neither

1. $f(x) = x^3 - x^2 + 4x + 2$

2. $f(x) = -x^2 + 10$

3. $f(x) = x^3 + 4x$

4. $f(x) = -x^3 + 5x - 2$

5. $f(x) = \sqrt{x^4 - x^2} + 4$

6. $f(x) = |x + 4|$

7. $f(x) = |x| + 4$

8. $f(x) = x^4 - 2x^2 + 4$

9. $f(x) = \sqrt[3]{x}$

10. $f(x) = x\sqrt{x^2 - 1}$

11. Given the point $(2, -5)$ is on the graph of $f(x)$.

A) If $f(x)$ is an even function, what is another point that is on the graph of $f(x)$?

B) If $f(x)$ is an odd function, what is another point that is on the graph of $f(x)$?

Answers:

Section A (Graphs)

1. Odd
2. Neither
3. Even

Section B (Algebra)

- | | |
|---|--|
| <ol style="list-style-type: none"> 1. Neither 2. Even 3. Odd 4. Neither 5. Even 6. Neither 7. Even 8. Even 9. Odd 10. Odd | <ol style="list-style-type: none"> 11. <ol style="list-style-type: none"> A) $(-2, -5)$ B) $(-2, 5)$ |
|---|--|