Calculus BC HW # 2

1.

Express the number $\frac{1}{2}+\frac{1}{3+\frac{3}{4}}$ in simplest form (as an irreducible quotient of two integers).

2.

The expression

$$\frac{x+1-\frac{1}{x+1}}{\frac{1}{x+1}},$$

such that $x \neq -1$, can be written in the form ax^2+bx. Express a and b as integers.

3.

The value of $\sin\left(\frac{\pi}{4}+\frac{\pi}{6}\right)$ can be written in the form $\frac{\sqrt{2}(\sqrt{3}+1)}{\alpha}$. Express α as an integer.

4.

The value of $\cos\left(\frac{\pi}{4}+\frac{\pi}{6}\right)$ can be written in the form $\frac{\sqrt{2}(\sqrt{3}-1)}{\alpha}$. Express α as an integer.

5.

For some real number θ, $0 \leq \theta \leq 2\pi$, it is known that $\sin(\theta) = \frac{\sqrt{3}}{2}$ and $\tan(\theta) = -\sqrt{3}$. The value of θ can be written in the form $\alpha\pi$. Express α as an irreducible quotient of two integers.

6.

For some real number θ, it is known that $\sin(\theta) = \frac{1}{4}$ and $\cos(\theta) > 0$. The value of $\sin(2\theta)$ can be written in the form $\frac{\sqrt{15}}{\alpha}$. Express α as an integer.

7.

For some real number x, $\cos(x) = \frac{1}{3}$ and $\sin\left(\frac{x}{2}\right) > 0$. The value of $\sin\left(\frac{x}{2}\right)$ can be written as $\frac{1}{\sqrt{\alpha}}$. Express α as an integer.

8.

For some real number x, $\sin(x) = \frac{4}{5}$. Express the value of $\sin(-x)$ as an irreducible quotient of two integers.

9.

The value of $\sin(\theta)$ is $\frac{1}{5}$. The value of $\cos^2(\theta)$ can be written in the form $6(\alpha^2)$. Express the value of α^2 in simplest form (as an irreducible quotient of two integers).

10. Express as an integer the value of $3^{\log_2 4}$.
11. Express as an integer the value of $\frac{1}{2}\log_{\frac{1}{4}} 80 - \frac{1}{2}\log_{\frac{1}{4}} 5$.
12. Express as an integer the value of α that satisfies $(2^{\log_4 2})(4^{\log_2 4}) = 2^{\alpha/2}$.

For each of the following questions enter a T (true) or F (false), as appropriate, on the line at the beginning of the statement.

_____(13) For $-\frac{\pi}{2} < x < \frac{\pi}{2}$, it follows that $1 + \tan^2(x) = \sec^2(x)$.

_____(14) For all real values of x, it follows that $\sin\left(x + \frac{\pi}{2}\right) = \cos(x)$.

_____(15) For all real values of x, it follows that $\sin(x) + \cos(x) = \cos(2x)$.

_____(16) For all real values of x, it follows that $2\sin(x)\cos(x) = \sin(2x)$.

_____(17) For all real values of x, it follows that $1 - 2\sin^2(x) = \cos(2x)$.

_____(18) For all real values of x, it follows that $2\cos^2(x) - 1 = \cos(2x)$.

_____(19) For all real values of θ, it follows that $\cos\left(\frac{\theta}{2}\right) = \frac{\cos(\theta)+1}{2}$.

___ (20) If $\ln a < 0$ for some real number a, then $a < 0$.

___ (21) For positive real numbers a and b,

$$\frac{\log_{10} b}{a} = \log_{10}(b^{1/a}).$$