

# 4.3 Modeling with Arithmetic Sequences



Resource Locker

**Essential Question:** How can you solve real-world problems using arithmetic sequences?

## Explore Interpreting Models of Arithmetic Sequences

You can model real-world situations and solve problems using models of arithmetic sequences. For example, suppose watermelons cost \$6.50 each at the local market. The total cost, in dollars, of  $n$  watermelons can be found using  $c(n) = 6.5n$ .



- (A) Complete the table of values for 1, 2, 3, and 4 watermelons.

|                        |   |   |   |   |
|------------------------|---|---|---|---|
| Watermelons $n$        | 1 | 2 | 3 | 4 |
| Total cost (\$) $c(n)$ |   |   |   |   |

- (B) What is the common difference?

---



---

- (C) What does  $n$  represent in this context?

---



---

- (D) What are the dependent and independent variables in this context?

---



---

- (E) Find  $c(7)$ . What does this value represent?

---



---

### Reflect

1. **Discussion** What domain values make sense for  $c(n) = 6.5n$  in this situation?

---



---

## Explain 1 Modeling Arithmetic Sequences From a Table

Given a table of data values from a real-world situation involving an arithmetic sequence, you can construct a function model and use it to solve problems.

**Example 1** Construct an explicit rule in function notation for the arithmetic sequence represented in the table. Then interpret the meaning of a specific term of the sequence in the given context.

- A** Suppose the table shows the cost, in dollars, of postage per ounce of a letter.

| Number of ounces     | $n$    | 1    | 2    | 3    | 4    |
|----------------------|--------|------|------|------|------|
| Cost (\$) of postage | $f(n)$ | 0.35 | 0.55 | 0.75 | 0.95 |

Determine the value of  $f(9)$ , and tell what it represents in this situation.

Find the common difference,  $d$ .  $d = 0.55 - 0.35 = 0.20$

Substitute 0.35 for  $f(1)$  and 0.20 for  $d$ .

$$f(n) = f(1) + d(n - 1)$$

$$f(n) = 0.35 + 0.20(n - 1)$$

$$f(9) = 0.35 + 0.20(8) = 1.95$$

So, the cost of postage for a 9-ounce letter is \$1.95.

- B** The table shows the cumulative total interest paid, in dollars, on a loan after each month.

| Number of months      | $n$    | 1   | 2   | 3   | 4   |
|-----------------------|--------|-----|-----|-----|-----|
| Cumulative total (\$) | $f(n)$ | 160 | 230 | 300 | 370 |

Determine the value of  $f(20)$  and tell what it represents in this situation.

Find the common difference,  $d$ .  $d = \square - 160 = \square$

Substitute  $\square$  for  $f(1)$  and  $\square$  for  $d$ .

$$f(n) = f(1) + d(n - 1)$$

$$f(n) = \square + \square(n - 1)$$

Find  $f(20)$  and interpret the value in context.

$$f(n) = f(1) + d(n - 1)$$

$$f(\square) = \square + \square(\square) = \square$$

So, the cumulative total \_\_\_\_\_ paid after \_\_\_\_\_ months is \_\_\_\_\_.

### Your Turn

Construct an explicit rule in function notation for the arithmetic sequence represented in the table. Then interpret the meaning of a specific term of the sequence in the given context.

2. The table shows  $f(n)$ , the distance, in miles, from the store after Mila has traveled for  $n$  hours.

| Time (h)      | $n$    | 1  | 2  | 3  | 4  |
|---------------|--------|----|----|----|----|
| Distance (mi) | $f(n)$ | 20 | 32 | 44 | 56 |

Determine the value of  $f(10)$  and tell what it represents in this situation.

3. The table below shows the total cost, in dollars, of purchasing  $n$  battery packs.

|                                |        |      |      |       |       |
|--------------------------------|--------|------|------|-------|-------|
| <b>Number of battery packs</b> | $n$    | 1    | 2    | 3     | 4     |
| <b>Total cost (\$)</b>         | $f(n)$ | 4.90 | 8.90 | 12.90 | 16.90 |

Determine the value of  $f(18)$  and tell what it represents in this situation.

## Explain 2 Modeling Arithmetic Sequences From a Graph

Given a graph of a real-world situation involving an arithmetic sequence, you can construct a function model and use it to solve problems.

**Example 2** Construct an explicit rule in function notation for the arithmetic sequence represented in the graph, and use it to solve the problem.

- A** D'Andre collects feather pens. The graph shows the number of feather pens D'Andre has collected over time, in weeks. According to this pattern, how many feather pens will D'Andre have collected in 12 weeks?

Represent the sequence in a table.

|        |    |    |    |    |
|--------|----|----|----|----|
| $n$    | 1  | 2  | 3  | 4  |
| $f(n)$ | 18 | 37 | 56 | 75 |

Find the common difference.

$$d = 37 - 18 = 19$$

Use the general explicit rule for an arithmetic sequence to write the rule in function notation. Substitute 18 for  $f(1)$  and 19 for  $d$ .

$$f(n) = f(1) + d(n - 1)$$

$$f(n) = 18 + 19(n - 1)$$

To determine the number of feather pens D'Andre will have collected after 12 weeks, find  $f(12)$ .

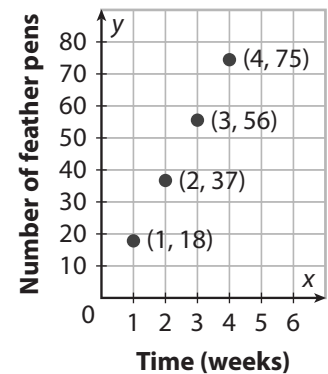
$$f(n) = 18 + 19(n - 1)$$

$$f(12) = 18 + 19(11)$$

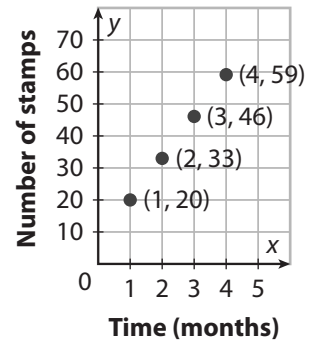
$$f(12) = 18 + 209$$

$$f(12) = 227$$

So, if this pattern continues, D'Andre will have collected 227 feather pens in 12 weeks.



- B** Eric collects stamps. The graph shows the number of stamps that Eric has collected over time, in months. According to this pattern, how many stamps will Eric have collected in 10 months?



Represent the sequence in a table.

|        |   |   |   |   |
|--------|---|---|---|---|
| $n$    | 1 | 2 | 3 | 4 |
| $f(n)$ |   |   |   |   |

Find the common difference.

$$d = \boxed{\phantom{00}} - 20 = \boxed{\phantom{00}}$$

Use the general explicit rule for an arithmetic sequence to write the rule in function notation.

Substitute  $\boxed{\phantom{00}}$  for  $f(1)$  and  $\boxed{\phantom{00}}$  for  $d$ .

$$f(n) = f(1) + d(n - 1)$$

$$f(n) = \boxed{\phantom{00}} + \boxed{\phantom{00}}(n - 1)$$

To determine the number of stamps Eric will have collected in 10 months, find  $f(\boxed{\phantom{00}})$ .

$$f(n) = f(1) + d(n - 1)$$

$$f(\boxed{\phantom{00}}) = \boxed{\phantom{00}} + \boxed{\phantom{00}}(\boxed{\phantom{00}}) = \boxed{\phantom{00}}$$

So, if this pattern continues, Eric will have collected \_\_\_\_\_ in \_\_\_\_\_ months.

### Reflect

4. How do you know which variable is the independent variable and which variable is the dependent variable in a real-world situation involving an arithmetic sequence?

---

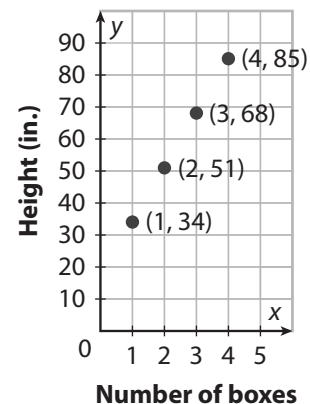


---

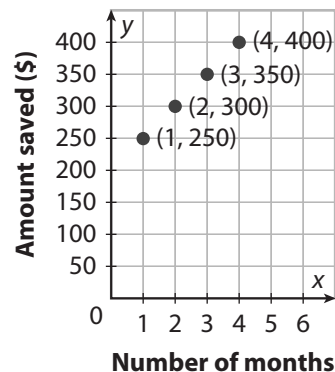
### Your Turn

**Construct an explicit rule in function notation for the arithmetic sequence represented in the graph, and use it to solve the problem.**

5. The graph shows the height, in inches, of a stack of boxes on a table as the number of boxes in the stack increases. Find the height of the stack with 7 boxes.



6. Quynh begins to save the same amount each month to save for a future shopping trip. The graph shows total amount she has saved after each month,  $n$ . What will be the total amount Quynh has saved after 12 months?



### Explain 3 Modeling Arithmetic Sequences From a Description

Given a description of a real-world situation involving an arithmetic sequence, you can construct a function model and use it to solve problems.

**Example 3** Construct an explicit rule in function notation for the arithmetic sequence represented, and use it to solve the problem. Justify and evaluate your answer.

The odometer on a car reads 34,240 on Day 1. Every day the car is driven 57 miles. If this pattern continues, what will the odometer read on Day 15?

#### Analyze Information

- The odometer on the car reads \_\_\_\_\_ miles on Day 1.
- Every day the car is driven \_\_\_\_\_ miles.

$$f(1) = \underline{\hspace{2cm}} \text{ and}$$

$$d = \underline{\hspace{2cm}}$$

#### Formulate a Plan

Write an explicit rule in function notation for the arithmetic sequence, and use it to find \_\_\_\_\_, the odometer reading on Day 15.

#### Solve

$$f(n) = f(1) + d(n - 1)$$

$$f(n) = \boxed{\hspace{1cm}} + \boxed{\hspace{1cm}}(n - 1)$$

$$f(\boxed{\hspace{1cm}}) = \boxed{\hspace{1cm}} + \boxed{\hspace{1cm}}(\boxed{\hspace{1cm}})$$

$$f(\boxed{\hspace{1cm}}) = \boxed{\hspace{1cm}}$$

On the Day 15, the odometer will show \_\_\_\_\_ miles.



## Justify and Evaluate

Using an arithmetic sequence model \_\_\_\_\_ reasonable because the number of miles on the odometer increases by the same amount each day.

By rounding and estimation:

$$34,200 + 60(14) = \boxed{\phantom{000}} + \boxed{\phantom{000}} = \boxed{\phantom{000}} \text{ miles}$$

So \_\_\_\_\_ miles is a reasonable answer.

### Your Turn

**Construct an explicit rule in function notation for the arithmetic sequence represented, and use it to solve the problem. Justify and evaluate your answer.**

- Ruby signed up for a frequent-flier program. She receives 3400 frequent-flier miles for the first round-trip she takes and 1200 frequent-flier miles for all additional round-trips. How many frequent-flier miles will Ruby have after 5 round-trips?
- A gym charges each member \$100 for the first month, which includes a membership fee, and \$50 per month for each month after that. How much money will a person spend on their gym membership for 6 months?



## Elaborate

- What domain values usually make sense for an arithmetic sequence model that represents a real-world situation?

---



---

- When given a graph of an arithmetic sequence that represents a real-world situation, how can you determine the first term and the common difference in order to write a model for the sequence?

---



---



---

- What are some ways to justify your answer when creating an arithmetic sequence model for a real-world situation and using it to solve a problem?

---



---



---

- Essential Question Check-In** How can you construct a model for a real-world situation that involves an arithmetic sequence?

---



---

# ★ Evaluate: Homework and Practice



- Online Homework
- Hints and Help
- Extra Practice

1. A T-shirt at a department store costs \$7.50. The total cost, in dollars, of  $a$  T-shirts is given by the function  $C(a) = 7.5a$ .

- a. Complete the table of values for 4 T-shirts.

| T-shirts  | 1 | 2 | 3 | 4 |
|-----------|---|---|---|---|
| Cost (\$) |   |   |   |   |

- b. Determine the common difference.

- c. What does the variable  $a$  represent? What are the reasonable domain values for  $a$ ?

2. A car dealership sells 5 cars per day. The total number of cars  $C$  sold over time in days is given by the function  $C(t) = 5t$ .

- a. Complete the table of values for the first 4 days of sales.

| Time (days)    | 1 | 2 | 3 | 4 |
|----------------|---|---|---|---|
| Number of Cars |   |   |   |   |



- b. Determine the common difference.

- c. What do the variables represent? What are the reasonable domain and range values for this situation?

3. A telemarketer makes 82 calls per day. The total number of calls made over time, in days, is given by the function  $C(t) = 82t$ .

- a. Complete the table of values for 4 days of calls.

| Time (days)     | 1 | 2 | 3 | 4 |
|-----------------|---|---|---|---|
| Number of Calls |   |   |   |   |

- b. Determine the common difference.

- c. What do the variables represent? What are the reasonable domain and range values for this situation?

Construct an explicit rule in function notation for the arithmetic sequence represented in the table. Then determine the value of the given term, and explain what it means.

4. Darnell starts saving the same amount from each week's paycheck. The table shows the total balance  $f(n)$  of his savings account over time in weeks.

| Time (weeks) $n$                   | 1     | 2     | 3     | 4     |
|------------------------------------|-------|-------|-------|-------|
| Savings Account Balance(\$) $f(n)$ | \$250 | \$380 | \$510 | \$640 |

Determine the value of  $f(9)$ , and explain what it represents in this situation.

5. Juan is traveling to visit universities. He notices mile markers along the road. He records the mile markers every 10 minutes. His father is driving at a constant speed. Complete the table.

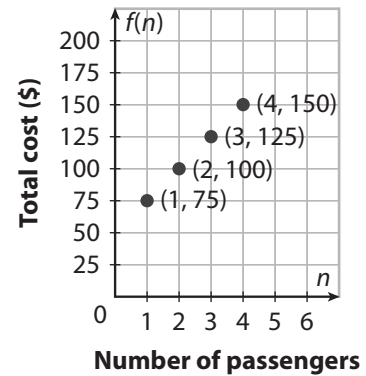
a.

| Time Interval | Mile Marker |
|---------------|-------------|
| 1             | 520         |
| 2             | 509         |
| 3             | 498         |
| 4             |             |
| 5             |             |
| 6             |             |

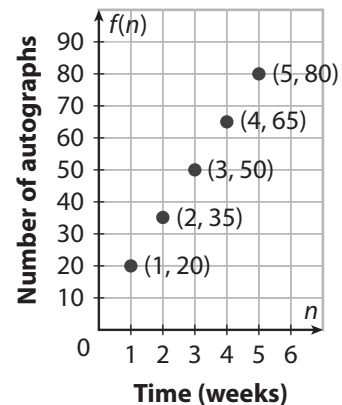
- b. Find  $f(10)$ , and tell what it represents in this situation.

Construct an explicit rule in function notation for the arithmetic sequence represented in the graph. Then determine the value of the given term, and explain what it means.

6. The graph shows total cost of a whitewater rafting trip and the corresponding number of passengers on the trip. Find  $f(8)$ , and explain what it represents.

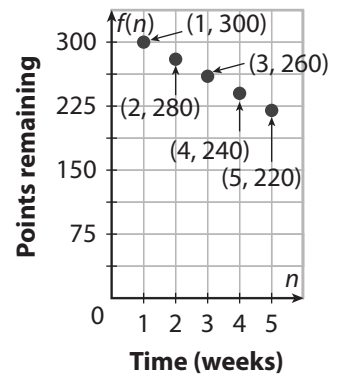


7. Ed collects autographs. The graph shows the total number of autographs that Ed has collected over time, in weeks. Find  $f(12)$ , and explain what it represents.



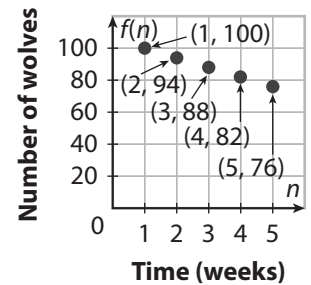


8. **Finance** Bob purchased a bus pass card with 320 points. Each week costs 20 points for unlimited bus rides. The graph shows the points remaining on the card over time in weeks. Determine the value of  $f(10)$ , and explain what it represents.



9. **Biology** The local wolf population is declining. The graph shows the local wolf population over time, in weeks.

Find  $f(9)$ , and explain what it represents.



**Construct an explicit rule in function notation for the arithmetic sequence. Then determine the value of the given term, and explain what it means.**

10. **Economics** To package and ship an item, it costs \$5.75 for the first pound and \$0.75 for each additional pound. Find the 12th term, and explain what it represents.

11. A new bag of cat food weighs 18 pounds. At the end of each day, 0.5 pound of food is removed to feed the cats. Find the 30th term, and explain what it represents.

12. Carrie borrows \$960 interest-free to pay for a car repair. She will repay \$120 monthly until the loan is paid off. How many months will it take Carrie to pay off the loan? Explain.

13. The rates for a go-kart course are shown.

|  |   |   |    |    |
|--|---|---|----|----|
| <b>Number of Laps <math>n</math></b>     | 1 | 2 | 3  | 4  |
| <b>Total cost (\$) <math>f(n)</math></b> | 7 | 9 | 11 | 13 |

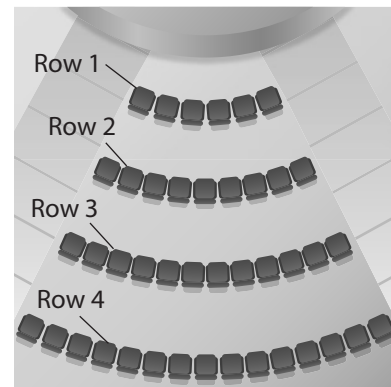
- a. What is the total cost for 15 laps?

- b. Suppose that after paying for 9 laps, the 10th lap is free. Will the sequence still be arithmetic? Explain.

14. **Multi-Part** Seats in a concert hall are arranged in the pattern shown.

- a. The numbers of seats in the rows form an arithmetic sequence. Write a rule for the arithmetic sequence.

- b. How many seats are in Row 15?



- c. Each ticket costs \$40. If every seat in the first 10 rows is filled, what is the total revenue from those seats?

- d. An extra chair is added to each row. Write the new rule for the arithmetic sequence and find the new total revenue from the first 10 rows.

**H.O.T. Focus on Higher Order Thinking**

- 15. Explain the Error** The table shows the number of people who attend an amusement park over time, in days.

|   |    |     |     |     |
|---|----|-----|-----|-----|
| <b>Time (days) <math>n</math></b>         | 1  | 2   | 3   | 4   |
| <b>Number of people <math>f(n)</math></b> | 75 | 100 | 125 | 150 |

Sam writes an explicit rule for this arithmetic sequence:  $f(n) = 25 + 75(n - 1)$

He then claims that according to this pattern, 325 people will attend the amusement park on Day 5. Explain the error that Sam made.

- 16. Communicate Mathematical Ideas** Explain why it may be harder to find the  $n$ th value of an arithmetic sequence from a graph if the points are not labeled.

- 17. Make a prediction** Verona is training for a marathon. The first part of her training schedule is given in the table.

|  |     |   |     |   |     |    |
|--|-----|---|-----|---|-----|----|
| <b>Session <math>n</math></b>          | 1   | 2 | 3   | 4 | 5   | 6  |
| <b>Distance (mi) <math>f(n)</math></b> | 3.5 | 5 | 6.5 | 8 | 9.5 | 11 |

- a.** Is this training schedule an arithmetic sequence? Explain. If it is, write an explicit rule for the sequence.
- b.** If Verona continues this pattern, during which training session will she run 26 miles?

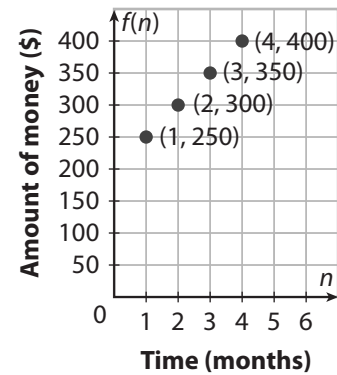


- 18.** If Verona's training schedule starts on a Monday and she runs every third day, on which day will she run 26 miles?

- 19. Multiple Representations** Determine whether the following graph, table, and verbal description all represent the same arithmetic sequence.

| Time (months) $n$           | 1   | 2   | 3   | 4   |
|-----------------------------|-----|-----|-----|-----|
| Amount of money (\$) $f(n)$ | 250 | 300 | 350 | 400 |

A person deposits \$250 dollars into a bank account. Each month, he adds \$25 dollars to the account, and no other transactions occur in the account.



## Lesson Performance Task

The graph shows the population of Ivor's ant colony over the first four weeks. Assume the ant population will continue to grow at the same rate.

- Write an explicit rule in function notation.
- If Ivor's ants have a mass of 1.5 grams each, what will be the total mass of all of his ants in 13 weeks?
- When the colony reaches 1385 ants, Ivor's ant farm will not be big enough for all of them. In how many weeks will the ant population be too large?

