Name _____ Class _____ Date _____

4.2 Constructing Arithmetic Sequences

Essential Question: **What is an arithmetic sequence?**

Resource Locker

Explore Exploring Arithmetic Sequences

You can order tickets for the local theater online. There is a fee of $2 per order. Matinee tickets cost $10 each. The total cost, in dollars, of ordering n matinee tickets online can be found by using $C(n) = 10n + 2$. The table shows the cost of 1, 2, 3, and 4 tickets.

(A) Complete the table of values for $C(n) = 10n + 2$.

Tickets	1	2	3	4
Total Cost ($)				

What is the domain of the sequence? _____

(B) What is the range of the sequence? _____

(C) What is the first term of the sequence? _____

(D) Find the difference between each two consecutive terms in the sequence:

$22 - 12 =$ _____ $32 - 22 =$ _____ $42 - 32 =$ _____

Reflect

1. **Discussion** Suppose you extended the table for up to 15 tickets. Would you expect the difference between consecutive terms to be the same? Explain your reasoning.

_____

_____

_____

2. **Communicate Mathematical Ideas** Explain how the domain is limited in this situation.

_____

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Explain 1 Constructing Rules for Arithmetic Sequences

In an **arithmetic sequence,** the difference between consecutive terms is always equal. This difference, written as d, is called the **common difference.**

An arithmetic sequence can be described in two ways, explicitly and recursively. As you saw earlier, in an **explicit** rule for a sequence, the nth term of the arithmetic sequence is defined as a function of n. In a **recursive** rule for a sequence, the first term of the sequence is given and the nth term is defined by relating it to the previous term. An arithmetic sequence can be defined using either a recursive rule or an explicit rule.

Example 1 **Write a recursive rule and an explicit rule for the sequence described by each table.**

(A) The table shows the monthly balance in a savings account with regular monthly deposits. The savings account begins with $2000, and $500 is deposited each month.

Time (months)	n	1	2	3	4	5
Balance	$f(n)$	2000	2500	3000	3500	4000

Write a recursive rule.

$f(1) = 2000$, and the common difference d is 500.

The recursive rule is $f(1) = 2000$, $f(n) = f(n-1) + 500$ for $n \geq 2$.

Write an explicit rule.

n	$f(n)$	$f(1) + d \cdot x = f(n)$
1	2000	$2000 + 500(0) = 2000$
2	2500	$2000 + 500(1) = 2500$
3	3000	$2000 + 500(2) = 3000$

Since d is always multiplied by a number equal to $(n-1)$, you can generalize the result from the table. The explicit rule is $f(n) = 2000 + 500(n-1)$.

(B) The table shows the monthly balance in a savings account with regular monthly deposits.

Time (months)	n	1	2	3	4	5
Balance	$f(n)$	5000	6000	7000	8000	9000

Write a recursive rule.

$f(1) =$ _____ and the common difference d is _____.

The recursive rule is $f(1) =$ _____, $f(n) = f(n-1) +$ _____ for $n \geq 2$.

Write an explicit rule.

n	$f(n)$	$f(1) + d \cdot x = f(n)$
1	5000	$5000 + 1000(0) = 5000$
2		
3		

Since d is always multiplied by a number equal to _____, you can generalize the result from the table. $f(n) =$ _____ + _____.

Reflect

3. **Critique Reasoning** Jerome says that the sequence 1, 8, 27, 64, 125,... is not an arithmetic sequence. Is that correct? Explain.

4. An arithmetic sequence has a common difference of 3. If you know that the third term of the sequence is 15, how can you find the fourth term? _____

YourTurn

5. The table shows the number of plates left at a buffet after n hours. Write a recursive rule and an explicit rule for the arithmetic sequence represented by the table.

Time (hours)	n	1	2	3	4	5
Number of plates	$f(n)$	155	141	127	113	99

Explain 2 Using a General Form to Construct Rules for Arithmetic Sequences

Arithmetic sequences can be described by a set of general rules. Values can be substituted into these rules to find a recursive and explicit rule for a given sequence.

General Recursive Rule	General Explicit Rule
Given $f(1)$, $f(n) = f(n-1) + d$ for $n \geq 2$	$f(n) = f(1) + d(n-1)$

Example 2 **Write a general recursive and general explicit rule for each arithmetic sequence.**

(A) 100, 88, 76, 64, ...

$f(1) = 100$, common difference $= 88 - 100 = -12$

The recursive rule is $f(1) = 100$, $f(n) = f(n-1) - 12$ for $n \geq 2$.

The explicit rule is $f(n) = 100 - 12(n-1)$.

(B) 0, 8, 16, 24, 32,...

$f(1) =$ _____, common difference $=$ _____ $-$ _____ $=$ _____.

The recursive rule is $f(1) =$ _____, $f(n) = f(n-1) +$ _____ for $n \geq 2$.

The explicit rule is $f(n) =$ _____ $+$ _____ $(n-1)$.

Reflect

6. What is the recursive rule for the sequence $f(n) = 2 + (-3)(n - 1)$? How do you know?

YourTurn

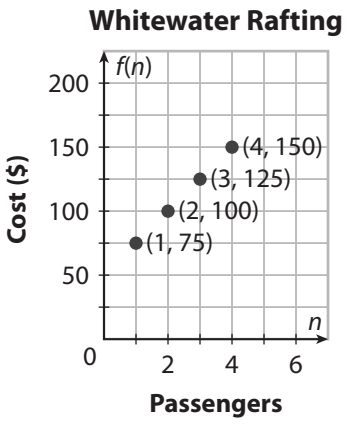
7. Write a recursive rule and an explicit rule for the arithmetic sequence 6, 16, 26, 36,...

Explain 3 Relating Arithmetic Sequences and Functions

The explicit rule for an arithmetic sequence can be expressed as a function. You can use the graph of the function to write an explicit rule.

Example 3 **Write an explicit rule in function notation for each arithmetic sequence.**

(A) The cost of a whitewater rafting trip depends on the number of passengers. The base fee is \$50, and the cost per passenger is \$25. The graph shows the sequence.



Step 1 Represent the sequence in a table.

Number of passengers n	1	2	3	4
Cost (\$) $f(n)$	75	100	125	150

Step 2 Find the common difference.

$f(2) - f(1) = 100 - 75 = 25$

$f(3) - f(2) = 125 - 100 = 25$

$f(4) - f(3) = 150 - 125 = 25$

The common difference d is 25.

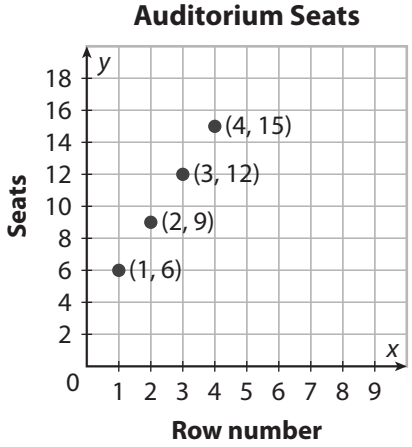
Step 3 Write an explicit rule for the sequence.

Substitute 75 for $f(1)$ and 25 for d.

$f(n) = f(1) + d(n - 1)$

$f(n) = 75 + 25(n - 1)$

Ⓑ The number of seats per row in an auditorium depends on which row it is. The first row has 6 seats, the second row has 9 seats, the third row has 12 seats, and so on. The graph shows the sequence.



Step 1 Represent the sequence in a table.

Row number n	1	2	3	4
Number of seats $f(n)$				

Step 2 Find the common difference.

$f(2) - f(1) = \square - \square = \square$

$f(3) - f(2) = \square - \square = \square$

$f(4) - f(3) = \square - \square = \square$

The common difference is $d =$ _____.

Step 3 Write an explicit rule for the sequence.

$f(n) = f(1) + d(n - 1)$

Substitute _____ for $f(1)$ and _____ for d.

$f(n) = \square + \square(n - 1)$

Reflect

8. **Analyze Relationships** Compare the graph of the function $f(x) = 3 + 5(x - 1)$ and the graph of the sequence $f(n) = 3 + 5(n - 1)$.

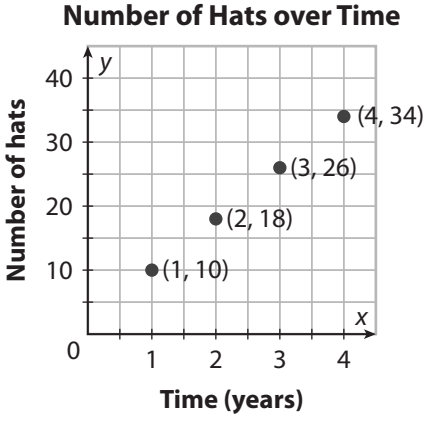
_____

_____

_____

YourTurn

9. Jerry collects hats. The total number of hats in Jerry's collection depends on how many years he has been collecting hats. After the first year, Jerry had 10 hats. Each year he has added the same number of hats to his collection. The graph shows the sequence. Write an explicit rule in function notation for the arithmetic sequence.



Elaborate

10. What information do you need to write a recursive rule for an arithmetic sequence that you do not need to write an explicit rule?

11. Suppose you want to be able to determine the ninetieth term in an arithmetic sequence and you have both an explicit and a recursive rule. Which rule would you use? Explain.

12. **Essential Question Check-In** The explicit equation for an arithmetic sequence and a linear equation have a similar form. How is the value of m in the linear equation $y = mx + b$ similar to the value of d in the explicit equation $f(n) = f(1) + d(n - 1)$?

Evaluate: Homework and Practice

- Online Homework
- Hints and Help
- Extra Practice

1. Farah pays a $25 signup fee to join a car sharing service and a $7 monthly charge. The total cost of using the car sharing service for n months can be found using $C(n) = 25 + 7n$. The table shows the cost of the service for 1, 2, 3, and 4 months.

a. Complete the table for $C(n) = 25 + 7n$

Months	n	1	2	3	4
Cost ($)	$f(n)$				

b. What are the domain and range of the sequence?

c. What is the common difference d?

Tell whether each sequence is an arithmetic sequence.

2.

a. 6, 7, 8, 9, 10,…

b. 5, 10, 20, 35, 55,…

c. 0, −1, 1, −2, 2,…

d. 1, 16, 81, 625, 1296

e. −2, −4, −6, −8, −10, …

3. **Chemistry** A chemist heats up several unknown substances to determine their boiling point. Use the table to determine whether the sequence is arithmetic. If it is arithmetic, write an explicit rule and a recursive rule for the sequence. If not, explain why it is not arithmetic.

Substance	1	2	3	4	5
Boiling Point (°F)	100	135	149	165	188

Write a recursive rule and an explicit rule for the arithmetic sequence described by each table.

4.

Month	n	1	2	3	4	5
Account balance ($)	$f(n)$	35	32	29	26	23

5.

Tickets	n	1	2	3	4	5
Total cost ($)	$f(n)$	58	65	72	79	86

6.

Month	n	1	2	3	4	5
Total deposits ($)	$f(n)$	84	100	116	132	148

7.

Delivery number	n	1	2	3	4	5
Weight of truck (lb)	$f(n)$	4567	3456	2345	1234	123

8.

Week	n	1	2	3	4	5
Account owed ($)	$f(n)$	125	100	75	50	25

9.

Skaters	n	1	2	3	4	5
Charge for lesson ($)	$f(n)$	60	80	100	120	140

Write a recursive rule and an explicit rule for each arithmetic sequence.

10. 95, 90, 85, 80, 75,...

11. 63, 70, 77, 84, 91,...

12. 86, 101, 116, 131, 146,...

13. 112, 110, 108, 106, 104,...

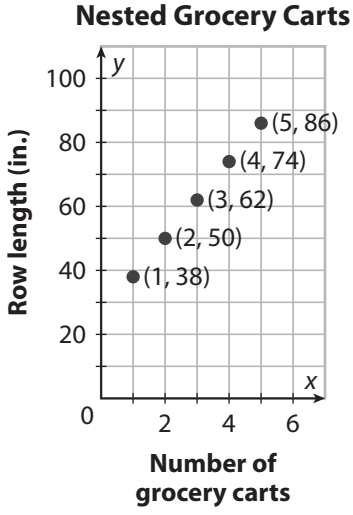
14. 5, 9, 13, 17, 21,...

15. 67, 37, 7, −23, −53,...

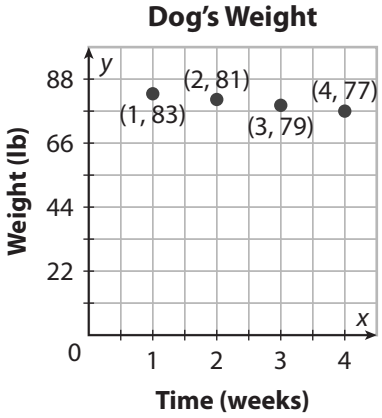
Write an explicit rule in function notation for each arithmetic sequence.

16. A student loan needs to be paid off beginning the first year after graduation. Beginning at Year 1, there is $52,000 remaining to be paid. The graduate makes regular payments of $8,000 each year. The graph shows the sequence.

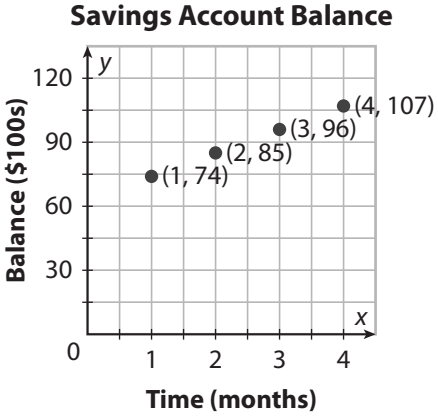
17. A grocery cart is 38 inches long. When the grocery carts are put away in a nested row, the length of the row depends on how many carts are nested together. Each cart added to the row adds 12 inches to the row length. The graph shows the sequence.



18. A dog food for overweight dogs claims that a dog weighing 85 pounds will lose about 2 pounds per week for the first 4 weeks when following the recommended feeding guidelines. The graph shows the sequence.



19. A savings account is opened with $6300. Monthly deposits of $1100 are made. The graph shows the sequence.



20. **Biology** The wolf population in a local wildlife area is currently 12. Due to a new conservation effort, conservationists hope the wolf population will increase by 2 animals each year for the next 50 years. Assume that the plan will be successful. Write an explicit rule for the population sequence. Use the rule to predict the number of animals in the wildlife area in the fiftieth year.

21. How are the terms in the sequence in the table related? Is the sequence an arithmetic sequence? Explain.

n	1	2	3	4	5
$f(n)$	3	9	27	81	243

H.O.T. Focus on Higher Order Thinking

22. Explain the Error The cost of a hamburger is \$2.50. Each additional hamburger costs \$2.00. Sully wrote this explicit rule to explain the sequence of costs: $f(n) = 2 + 2.5(n - 1)$. Using this rule, he found the cost of 12 hamburgers to be \$29.50. Is this number correct? If not, identify Sully's error.

23. Critical Thinking Lucia knows the fourth term in a sequence is 55 and the ninth term in the same sequence is 90. Explain how she can find the common difference for the sequence. Then use the common difference to find the second term of the sequence.

24. Represent Real-World Problems Write and solve a real-world problem involving a situation that can be represented by the sequence $f(n) = 15 + 2(n - 1)$.

Lesson Performance Task

For Carl's birthday, his grandparents gave him a \$50 gift card to a local movie theater. The theater charges \$6 admission for each movie. How can Carl use an arithmetic sequence to determine the value left on his card after each movie he sees?

a. Write an explicit rule for the arithmetic sequence and use it to determine how much value is left on the card after Carl has seen 4 movies.

b. How much is left on the card after Carl has seen the maximum number of movies?