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Limits and Continuity
Unit 1 ↓

- $\lim_{x\to c} f(x)$ is the value f(x) approaches when x -> c from BOTH sides
- $\lim_{x\to c^{\pm}} f(x)$ is the value that f(x) approaches when x -> c from ONLY the right (if +) or left (if -) side
- $\lim_{x\to c}(af(x)) = a\lim_{x\to c} f(x)$
- $\lim_{x\to c}(f(x)\pm g(x)) = \lim_{x\to c} f(x) \pm \lim_{x\to c} g$
- $\lim_{x\to c}(f(x)g(x)) = \lim_{x\to c} f(x)\cdot\lim_{x\to c} g(x)$
- $\lim_{x\to c}\frac{f(x)}{g(x)} = \frac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)}$
- $\lim_{x\to c} f(g(x)) = f(\lim_{x\to c} g(x))$
- Methods to algebraically simplify limits: Completing the square, Rationalization, Factoring
- Order of growth rates from fastest to slowest: $x^x, x!, a^x, x^p, xln(x), ln(x)$
- For f(x)/g(x), if highest power of f > highest power of g: infinite limit DNE, if <, HA at y=0, if =, HA at y = ratio of first terms
- Continuity if $f(c) = \lim_{x\to c} f(x)$
- Removable: hole, Asymptote, and Jump: Piecewise where y-values different
- IVT PROBLEMS: Write "Since f(x) is continuous on [a,b] and f(c) is between f(a) and f(b), by the IVT there is a c in (a,b) such that f(c) = f(c)"

Differentiation: Definition and Fundamental Properties
Unit 2 ↓

- AROC = $\frac{f(x+h)-f(x)}{x+h}$
- $f'(x) = \lim_{x\to h}\frac{f(x+h)-f(x)}{x+h} = \lim_{c\to x}\frac{f(x)-f(c)}{c}$
- When estimating f'(c) from a table, straddle both sides and use AROC, from a graph, slope of tangent line
- All differentiable functions are continuous, but not all continuous functions are differentiable
- Power Rule: $\frac{dy}{dx}x^n = nx^{n-1}$
- $\frac{dy}{dx}c = 0$
- $\frac{dy}{dx}f(x)\pm g(x) = f'(x)\pm g'(x)$
- $\frac{dy}{dx}kf(x) = kf'(x)$
- $\frac{dy}{dx}sin(x) = cos(x)$
- $\frac{dy}{dx}cos(x) = -sin(x)$
- $\frac{dy}{dx}e^x = e^x$
- $\frac{dy}{dx}ln(x) = \frac{1}{x}$
- Product Rule:
  $\frac{dy}{dx}f(x)g(x) = f'(x)g(x) + f(x)g'(x)$
- Quotient Rule:
  $\frac{dy}{dx}\frac{f(x)}{g(x)} = \frac{f'(x)g(x)-f(x)g'(x)}{g^2(x)}$
- $\frac{dy}{dx}tan(x) = sec^2(x)$
- $\frac{dy}{dx}cot(x) = -csc^2(x)$
- $\frac{dy}{dx}sec(x) = sec(x)tan(x)$
- $\frac{dy}{dx}csc(x) = -csc(x)cot(x)$

Differentiation: Composite, Implicit, and Inverse Functions
Unit 3 ↓

- $\frac{dy}{dx}a^x = a^x ln(a)$
- $\frac{dy}{dx}log_a(x) = \frac{1}{ln(a)x}$
- Chain Rule:
  $\frac{dy}{dx}f(g(x)) = g'(x)\cdot f'(g(x))$
- The chain rule is like unpeeling an onion, where you keep going from the outside in, you differentiate the outside function, plug in the inside function, and multiply by the derivative of the inside function.
- Implicit Differentiation: Differentiate each term with respect to the individual variables, and whenever you differentiate a y, multiply by $\frac{dy}{dx}$
  Ex. $\frac{d}{dx}xy = y + x\frac{dy}{dx}$
- Inverses:
- $\frac{dy}{dx}arcsin(x) = \frac{1}{\sqrt{1-x^2}}$
- $\frac{dy}{dx}arctan(x) = \frac{1}{1+x^2}$
- $\frac{dy}{dx}arcsec(x) = \frac{1}{|x|\sqrt{x^2-1}}$
- Derivatives of inverse trig cofunctions are the negative of the derivative of the other 3 inverse trig functions
- Higher order derivatives: Just repeat!
- Second derivatives of implicit functions are functions of $x, y, \frac{dy}{dx}$

Contextual Applications of Differentiation
Unit 4 ↓

- The derivative of a function is the rate of change of that function
- If you are being asked about the rate of change of a rate of change, that's basically the derivative of f'(x), or f"(x)
- Particle motion:
  $\frac{d^2y}{dx^2}x(t) = \frac{dy}{dx}v(t) = a(t)$
- Steps for Related Rates:
  1. Draw picture
  2. List knowns and unknowns
  3. Write an equation to model the situation (DO NOT PLUG IN STUFF THAT CHANGES)
  4. $\frac{d}{dt}$
  5. Substitute for changing values
  6. Solve for desired value
- Linearization:
  $f(c+a) \approx f'(c)\cdot a + f(x)$
- L'Hopital's Rule (LHR):
  $\lim_{x\to c}\frac{f(x)}{g(x)} = \lim_{x\to c}\frac{f'(x)}{g'(x)}$ ONLY IF $\frac{f(x)}{g(x)}$ IS INDETERMINATE, that is $\lim_{x\to c} f(x)$ and $\lim_{x\to c} g(x)$ are both either 0 or ∞
- Sometimes you may need to use LHR multiple times
- Remember to plug in for the limit before doing LHR!!

Analytical Applications of Differentiation
Unit 5 ↓

- MVT PROBLEMS: Write "Since f(x) is continuous on [a,b] and differentiable on (a.b), there exists a c in (a,b) where $f'(c) = \frac{f(b)-f(a)}{b-a}$ by the MVT"
- Rolle's Theorem is MVT where f(b) = f(a)
- EVT PROBLEMS: Write "Since f(x) is continuous on (a,b), by the EVT, there exists at least one local maximum and one local minimum on (a,b).
- Critical Points: Where f'(x) = 0 or undefined
- Local Extrema: Point that is greater/less than surrounding points, always at critical points or endpoints
- Global Extrema: Greatest or Least value of function
- f'(x) > 0: inc, f'(x) < 0: dec
- If asking for whether the rate of change of f(x) is increasing or decreasing, this is asking for the sign of the second derivative!
- Where f'(x) = 0, if f(x): $-\to+$: local min, $+\to-$: local max
- ENDPOINTS CAN BE EXTREMA TOO, REMEMBER THEM WHEN FINDING GLOBAL EXTREMA
- f"(x) > 0: ccu, f"(x) < 0: ccd, f"(x) = 0: inflection point
- Where f'(x) = 0, if f"(x) > 0: min, if f"(x) < 0: max, if f"(x) = 0: indeterminate
- Steps for Optimization:
  1. Draw picture
  2. Write primary equation
  3. Write constraint equation, solve for other variables, and plug into primary equation (if applicable)
  4. Find extrema of primary equation and solve for variables

FRQ Tips↓

- Work on the $\frac{f(x+h)-f(x)}{x+h}$ parts you know you can do first before moving onto other parts!
- Be sure to show all your work still, even though it is a shorter test.
- Shorthand like IVT, MVT, FTOC for Intermediate Value Theorem, Mean Value Theorem, Fundamental Theorem of Calculus is fine!
- Don't simplify your answers if you don't need to! You don't want to unnecessarily lose points on steps you don't need to do!
- Memorize your important theorems and convergence tests! You'll need to know the conditions where the theorems/tests are met!

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Integration and Accumulation of Change
Unit 6 ↓

- Accumulation/Integral is the area between a rate of change graph and the x-axis
- If below x-axis, then accumulating negative area
- Can use geometry to find integral from a graph
- When a function is split into multiple subdivisions, y_1 is the left boundary, y_2 is the right boundary, y_3 is in the middle of an interval, and Δx is the interval width between the values that give y_1 and y_2

  LRS: $\sum y_1 \Delta x$

  RRS: $\sum y_2 \Delta x$

  MRS: $\sum y_3 \Delta x$

  Trap Rule: $\sum \frac{1}{2}(y_1 + y_2)\Delta x$
- Riemann Sum: $\int_a^b f(x)dx \approx \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\Delta x$

  where $\Delta x = \frac{b-a}{n}$ and $x_k = a + k\Delta x$
- FTC Pt 1 (Definite Integrals): If F'(x) = f(x), then $\int_a^b f(x)dx = F(b) - F(a)$
- FTC Pt 2:

  $\frac{d}{dx}\int_{a(x)}^{b(x)} f(x) = b'(x)f(b(x)) - a'(x)f(a(x))$
- Integrals can be used if there are jump or removable discontinuities
- $\int f(x) + g(x)dx = \int f(x)dx + \int g(x)dx$
- $\int cf(x)dx = c\int f(x)dx$
- $\int_a^b f(x)dx = -\int_b^a f(x)dx$
- If b in between a and c:

  $\int_a^c f(x)dx = \int_a^b f(x)dx + \int_b^c f(x)dx$
- U-substitutions are your friend, use them!

Integration and Accumulation of Change
Unit 6 Continued ↓

- (Indefinite integrals) In $\int f(x)dx = F(X) + C$, NEVER FORGET THE +C!!
- $\int sin(x)dx = -cos(x) + C$
- $\int cos(x)dx = sin(x) + C$
- $\int sec^2(x)dx = tan(x) + C$
- $\int csc^2(x)dx = -cot(x) + C$
- $\int sec(x)tan(x)dx = sec(x) + C$
- $\int csc(x)cot(x)dx = -csc(x) + C$
- $\int x^n dx = \frac{1}{n+1}x^{n+1} + C$
- $\int e^x dx = e^x + C$
- $\int a^x dx = \frac{a^x}{ln(a)} + C$

The rest of Unit 6 is BC Only

- IBP: $\int udv = uv - \int vdu$ (Ultraviolet Voodoos)
- Order of precedence for choosing u is LIPET: logs, inverse trig, polynomials, exponential, trig
- PFD: You can break up a complex fraction (after long dividing to make the antiderivative into a sum of logs

  Ex. $\int \frac{D}{(x-a)(x-b)}dx = \int \frac{A}{x-a} + \frac{B}{x-b}dx$

  $= Aln|x-a| + Bln|x-b| + C$
- Improper Integrals: Split your integral when you encounter an asymptote, and write limits for x-values where f(x) is unbounded

  Ex. $\int_0^\infty f(x)dx = \lim_{n\to\infty}\int_0^n f(x)dx$

Differential Equations
Unit 7 ↓

- Slope Fields show tangent line segments to the particular solution through that point
- If you can write the differential equation in the form $g(y)dy = f(x)dx$, it's a separation of variables problem, you can leave your answers implicitly (ex. $xy^2 = 3xln(y)$)
- When all constants from antidifferentiation are replaced with appropriate values, you get a particular solution when there is an initial value condition which the solution must go through
- $\frac{dy}{dx} = ky \to y = ce^{kx}$ where c = y(0) (exponential growth/decay)

The rest of Unit 7 is BC Only

- $\frac{dy}{dx} = ky(L-y) \to y = \frac{L}{1-Ce^{-L}}$ where L is carrying capacity and there is a horizontal asymptote at y = L (Logistic growth/decay)
- Euler's Method: Repeatedly apply linearization at the chunked intervals

Applications of Integration (BC ONLY)
Unit 8 ↓

- Average Value of f(x) = $\frac{1}{b-a}\int_a^b f(x)dx$
- Write "Since f(x) is continuous on (a,b), by the AVT, there must be a c in (a,b) where f(c) = $\frac{1}{b-a}\int_a^b f(x)dx$
- $\int(\int a(t)dt)dt = \int v(t)dt = x(t)$ where x(t) is displacement
- speed = |v(t)|, distance is the integral of speed
- Area: If dx, then integral of top-bottom, if dy, then integral of right - left, same rules apply when finding radii for disk/washer method
- Washer Method Integrand: $\pi(R_{outer} - R_{inner})^2$ dx (or dy)
- Disk method is washer method with inner radius 0
- Sometimes you'll need to split your section (when the curves intersect in the middle)
- Integrand for cross-sections is the area of the cross section
- Arc Length: $\int_a^b \sqrt{1 + (\frac{dy}{dx})^2}dx$

Parametric Equations, Polar Coordinates, and Vector-Valued Functions (BC ONLY)
Unit 9 ↓

- Parametric functions are functions where x and y are independent to each other, connected with the variable *t*
- The derivative dy/dx of a parametric function can be found by dividing dy/dt / dx/dt
- Second derivatives involve using chain rule: $\frac{d}{dx}(\frac{dy/dt}{dx/dt})$
- The arc length of a parametric curve L can be found by the equation

  $L = \int_a^b \sqrt{(x'(t))^2 + (y'(t))^2}dt$
- Motion in x and y directions are defined as vectors, with <x(t), y(t)> representing position
- Polar functions are graphed using (r, θ) rather than (x, y), where x = rcosθ and y = rsinθ
- The derivative of a polar function, dy/dx, can be found using $\frac{rcos\theta + \frac{dr}{d\theta}(sin\theta)}{-rsin\theta + \frac{dr}{d\theta}(cos\theta)}$
- The area under a polar curve A can be found by the equation $A = \int_a^b \frac{1}{2}r^2 d\theta$, where the interval is the period of the trigonometric function
- The area between two curves is simply the area of the top curve - the area of the bottom curve
- The arc length of a polar function can be found using the equation $L = \int_a^b r^2 + (\frac{dr}{d\theta})^2 d\theta$

Infinite Sequences and Series (BC ONLY)
Unit 10 ➡

- GST: For a series with terms ar^n, it converges if |r|<1 and its sum is $\frac{a}{1-r}$, else it diverges
- Harmonic series 1/n diverges, but (-1)ⁿ/n converges
- Power Series with terms $\frac{1}{n^p}$ converges when p>1, else it diverges
- AST: For an alternating series (terms change sign), converges if $\lim_{n\to\infty} a_n = 0$, else it diverges
- Ratio Test: Series converges if $L = \lim_{n\to\infty} |\frac{a_{n+1}}{a_n}| < 1$, diverges if L > 1, inconclusive if L = 1
- If the absolute value of a series converges, it is absolutely convergent, if not, it is conditionally convergent

  $\text{Error} \le \frac{M}{(n+1)!}(x-c)^{n+1}$
- Taylor Polynomials about x=c: $P_n(x) = \sum_{k=0}^{n} \frac{f^{(n)}(x-c)}{n!}(x-c)^n$
- Alternating series error is < the next term in the series
- Lagrange error bound is found using

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