

Accumulation

AP[®] CALCULUS BC 2010 SCORING GUIDELINES

Question 1

There is no snow on Janet's driveway when snow begins to fall at midnight. From midnight to 9 A.M., snow accumulates on the driveway at a rate modeled by $f(t) = 7te^{\cos t}$ cubic feet per hour, where t is measured in hours since midnight. Janet starts removing snow at 6 A.M. ($t = 6$). The rate $g(t)$, in cubic feet per hour, at which Janet removes snow from the driveway at time t hours after midnight is modeled by

$$g(t) = \begin{cases} 0 & \text{for } 0 \leq t < 6 \\ 125 & \text{for } 6 \leq t < 7 \\ 108 & \text{for } 7 \leq t \leq 9. \end{cases}$$

- (a) How many cubic feet of snow have accumulated on the driveway by 6 A.M.?
(b) Find the rate of change of the volume of snow on the driveway at 8 A.M.
(c) Let $h(t)$ represent the total amount of snow, in cubic feet, that Janet has removed from the driveway at time t hours after midnight. Express h as a piecewise-defined function with domain $0 \leq t \leq 9$.
(d) How many cubic feet of snow are on the driveway at 9 A.M.?

(a) $\int_0^6 f(t) dt = 142.274$ or 142.275 cubic feet

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) Rate of change is $f(8) - g(8) = -59.582$ or -59.583 cubic feet per hour.

1 : answer

(c) $h(0) = 0$

For $0 < t \leq 6$, $h(t) = h(0) + \int_0^t g(s) ds = 0 + \int_0^t 0 ds = 0$.

For $6 < t \leq 7$, $h(t) = h(6) + \int_6^t g(s) ds = 0 + \int_6^t 125 ds = 125(t - 6)$.

For $7 < t \leq 9$, $h(t) = h(7) + \int_7^t g(s) ds = 125 + \int_7^t 108 ds = 125 + 108(t - 7)$.

Thus, $h(t) = \begin{cases} 0 & \text{for } 0 \leq t \leq 6 \\ 125(t - 6) & \text{for } 6 < t \leq 7 \\ 125 + 108(t - 7) & \text{for } 7 < t \leq 9 \end{cases}$

3 : $\begin{cases} 1 : h(t) \text{ for } 0 \leq t \leq 6 \\ 1 : h(t) \text{ for } 6 < t \leq 7 \\ 1 : h(t) \text{ for } 7 < t \leq 9 \end{cases}$

(d) Amount of snow is $\int_0^9 f(t) dt - h(9) = 26.334$ or 26.335 cubic feet.

3 : $\begin{cases} 1 : \text{integral} \\ 1 : h(9) \\ 1 : \text{answer} \end{cases}$

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Question 2

The rate at which people enter an auditorium for a rock concert is modeled by the function R given by $R(t) = 1380t^2 - 675t^3$ for $0 \leq t \leq 2$ hours; $R(t)$ is measured in people per hour. No one is in the auditorium at time $t = 0$, when the doors open. The doors close and the concert begins at time $t = 2$.

- (a) How many people are in the auditorium when the concert begins?
 (b) Find the time when the rate at which people enter the auditorium is a maximum. Justify your answer.
 (c) The total wait time for all the people in the auditorium is found by adding the time each person waits, starting at the time the person enters the auditorium and ending when the concert begins. The function w models the total wait time for all the people who enter the auditorium before time t . The derivative of w is given by $w'(t) = (2 - t)R(t)$. Find $w(2) - w(1)$, the total wait time for those who enter the auditorium after time $t = 1$.
 (d) On average, how long does a person wait in the auditorium for the concert to begin? Consider all people who enter the auditorium after the doors open, and use the model for total wait time from part (c).

(a) $\int_0^2 R(t) dt = 980$ people

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) $R'(t) = 0$ when $t = 0$ and $t = 1.36296$
 The maximum rate may occur at 0, $a = 1.36296$, or 2.

$R(0) = 0$
 $R(a) = 854.527$
 $R(2) = 120$

The maximum rate occurs when $t = 1.362$ or 1.363.

3 : $\begin{cases} 1 : \text{considers } R'(t) = 0 \\ 1 : \text{interior critical point} \\ 1 : \text{answer and justification} \end{cases}$

(c) $w(2) - w(1) = \int_1^2 w'(t) dt = \int_1^2 (2 - t)R(t) dt = 387.5$
 The total wait time for those who enter the auditorium after time $t = 1$ is 387.5 hours.

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(d) $\frac{1}{980} w(2) = \frac{1}{980} \int_0^2 (2 - t)R(t) dt = 0.77551$
 On average, a person waits 0.775 or 0.776 hour.

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$