Rational Expressions - Least Common Denominators

Objective: Idenfity the least common denominator and build up denominators to match this common denominator.

As with fractions, the least common denominator or LCD is very important to working with rational expressions. The process we use to find the LCD is based on the process used to find the LCD of integers.

Example 1. Find the LCD of 8 and 6 Consider multiples of the larger number

$8, 16, 24 \ldots$. 24 is the first multiple of 8 that is also divisible by 6

24 Our Solution

Alternate way: 8 breaks down to 2x2x2. 6 breaks down to 2x3. To find the LCD take the highest power of each prime factor: 2^3 x 3^1 = 8 x 3 = 24.

When finding the LCD of several monomials we first find the LCD of the coefficients, then use all variables and attach the highest exponent on each variable.

Example 2.

Find the LCD of $4x^2y^5$ and $6x^4y^3z^6$

First find the LCD of coefficients 4 and 6

12 12 is the LCD of 4 and 6

$x^4y^5z^6$ Use all variables with highest exponents on each variable

$12x^4y^5z^6$ Our Solution

The same pattern can be used on polynomials that have more than one term. However, we must first factor each polynomial so we can identify all the factors to be used (attaching highest exponent if necessary).

Example 3.

Find the LCD of x^2+2x-3 and x^2-x-12 Factor each polynomial

$(x-1)(x+3)$ and $(x-4)(x+3)$ LCD uses all unique factors

$(x-1)(x+3)(x-4)$ Our Solution

Notice we only used $(x+3)$ once in our LCD. This is because it only appears as a factor once in either polynomial. The only time we need to repeat a factor or use an exponent on a factor is if there are exponents when one of the polynomials is factored. Look at the next example.

Example 4.

Find the LCD of $x^2 - 10x + 25$ and $x^2 - 14x + 45$

	Factor each polynomial
$(x-5)^2$ and $(x-5)(x-9)$	LCD uses all unique factors with highest exponent
$(x-5)^2(x-9)$	Our Solution

The previous example could have also been done with factoring the first polyno-mial to $(x-5)(x-5)$. Then we would have used $(x-5)$ twice in the LCD because it showed up twice in one of the polynomials.

Find Least Common Denominators

1) $2a^3, 6a^4b^2, 4a^3b^5$

2) $5x^2y, 25x^3y^5z$

3) $x^2 - 3x,\ x - 3,\ x$

4) $4x - 8,\ x - 2,\ 4$

5) $x + 2,\ x - 4$

6) $x,\ x - 7,\ x + 1$

7) $x^2 - 25,\ x + 5$

8) $x^2 - 9,\ x^2 - 6x + 9$

9) $x^2 + 3x + 2, x^2 + 5x + 6$

10) $x^2 - 7x + 10, x^2 - 2x - 15, x^2 + x - 6$

Answers - Least Common Denominators

1) $12a^4b^5$

2) $25x^3y^5z$

3) $x\,(x-3)$

4) $4(x-2)$

5) $(x+2)(x-4)$

6) $x(x-7)(x+1)$

7) $(x+5)(x-5)$

8) $(x-3)^2(x+3)$

9) $(x+1)(x+2)(x+3)$

10) $(x-2)(x-5)(x+3)$