# UNIT 4 WORKSHEET 12
## Finding Asymptotes of Rational Functions

***Rational functions have various asymptotes. The following will aid in finding all asymptotes of a rational function. The first step to working with rational functions is to completely factor the polynomials. Once in factored form, find all zeros.***

**Vertical Asymptotes**

- **The Vertical Asymptotes of a rational function are found using the zeros of the denominator.**

**For Horizontal Asymptotes use the following guidelines.**

- **If the degree of the numerator is greater than the degree of the denominator by more than one, the graph has no horizontal asymptote.(none)**
- **If the degree of the numerator is equal to the degree of the denominator, the horizontal asymptote is the ratio of the two leading coefficients.(y = #)**
- **If the degree of the numerator is less than the degree of the denominator, the horizontal asymptote is zero. (y = 0)**

**Oblique Asymptotes**

- **If the degree of the numerator is greater than the degree of the denominator by one, there is an oblique asymptote. The asymptote is the quotient numerator divided by the denominator.**

***An asymptote is like an imaginary line that cannot be crossed. All rational functions have vertical asymptotes. A rational function may also have either a horizontal or oblique asymptote. A rational function will never have both a horizontal and oblique asymptote. It is either one or the other. Horizontal asymptotes are the only asymptotes that may be crossed. The vertical asymptotes come from zeroes of the denominator.***

$$f_{(x)} = \frac{x}{(x+2)(x-3)}$$ ***Here is a rational function in completely factored form.***

$x = -2 \quad and \quad x = 3$ ***The zeros of the denominator are -2 and 3. Therefore, these are the vertical asymptotes of the function.***

***Since an x value of -2 or 3 would create a zero in the denominator, the function would be undefined at that location. As a result, these are the vertical asymptotes for this function. In this same function, the degree of the numerator is less than the degree of the denominator, therefore, the horizontal asymptote is*** $y = 0$.

***When finding the oblique asymptote, find the quotient of the numerator and denominator. If there are any remainders, disregard them. You only need the quotient. The graph of the function can have a either a horizontal asymptote, or an oblique asymptote. You can not have one of each. This particular function does not have an oblique asymptote.***

***Here is an example with an oblique asymptote.***

**Find the oblique asymptote of the rational function** $f_{(x)} = \dfrac{x^2 + 8x - 20}{x - 1}$**.**

$$\begin{array}{r|l} & x + 9 \\ \hline x - 1 & x^2 + 8x - 20 \\ & \underline{-x^2 + x} \\ & \quad 9x - 20 \\ & \quad \underline{-9x + 9} \\ & \qquad -11 \end{array}$$

***Dividing the polynomials, the quotient x+9 is found.***

$$y = x + 9$$

***This is the equation for the oblique asymptote of the function. Notice the remainder of the division problem is disregarded. It plays no part in the equation for the oblique asymptote.***

***Finally, let us look at a rational function where the degree of the numerator is equal to the degree of the denominator.***

**Find the horizontal asymptote for the rational function** $f_{(x)} = \dfrac{2x^2 - 4x + 8}{3x^2 - 27}$**.**

$$f_{(x)} = \frac{2x^2 - 4x + 8}{3x^2 - 27}$$

***Notice the degree of the numerator is the same as the degree of the denominator.***

$$y = \frac{2}{3}$$

***Since the degree of the numerator equals that of the denominator, the equation for the horizontal asymptote is the ratio of the two leading coefficients.***

**Find all asymptotes of the following functions. (Do not graph these)**

**A)** $f_{(x)} = \dfrac{x-7}{x+5}$

**B)** $f_{(x)} = \dfrac{3}{x^2-2}$

**C)** $f_{(x)} = \dfrac{x^2}{x-5}$

**D)** $f_{(x)} = \dfrac{2x^2-5x+3}{x-1}$

**E)** $f_{(x)} = \dfrac{7x^2+5x-2}{2x^2-18}$

**F)** $f_{(x)} = \dfrac{2x^2-5x+5}{x-2}$

**G)** $f_{(x)} = \dfrac{1}{3-x}$

**H)** $f_{(x)} = \dfrac{x^2-4}{x^4-81}$

**I)** $f_{(x)} = \dfrac{x^3-2x^2+5}{x^2}$