

Practice 6-5

Theorems About Roots of Polynomial Equations

A polynomial equation with rational coefficients has the given roots.
Find two additional roots.

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| 1. $2 + 3i$ and $\sqrt{7}$ | 2. $3 - \sqrt{2}$ and $1 + \sqrt{3}$ |
| 3. $-4i$ and $6 - i$ | 4. $5 - \sqrt{6}$ and $-2 + \sqrt{10}$ |

Find a fourth-degree polynomial equation with integer coefficients that has the given numbers as roots.

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| 5. $2i$ and $4 - i$ | 6. $\sqrt{2}$ and $2 - \sqrt{3}$ |
| 7. $3i$ and $\sqrt{6}$ | 8. $2 + i$ and $1 - \sqrt{5}$ |

Find the roots of each polynomial equation.

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| 9. $x^3 - 5x^2 + 2x + 8 = 0$ | 10. $x^3 + x^2 - 17x + 15 = 0$ |
| 11. $2x^3 + 13x^2 + 17x - 12 = 0$ | 12. $x^3 - x^2 - 34x - 56 = 0$ |
| 13. $x^3 - 18x + 27 = 0$ | 14. $x^4 - 5x^2 + 4 = 0$ |
| 15. $x^3 - 6x^2 + 13x - 10 = 0$ | 16. $x^3 - 5x^2 + 4x + 10 = 0$ |
| 17. $x^3 - 5x^2 + 17x - 13 = 0$ | 18. $x^3 + x + 10 = 0$ |
| 19. $x^3 - 5x^2 - x + 5 = 0$ | 20. $x^3 - 12x + 16 = 0$ |
| 21. $x^3 - 2x^2 - 5x + 6 = 0$ | 22. $x^3 - 8x^2 - 200 = 0$ |
| 23. $x^3 + x^2 - 5x + 3 = 0$ | 24. $4x^3 - 12x^2 - x + 3 = 0$ |
| 25. $x^3 + x^2 - 7x + 2 = 0$ | 26. $12x^3 + 31x^2 - 17x - 6 = 0$ |

Use the Rational Root Theorem to list all possible rational roots for each polynomial equation. Then find any actual rational roots.

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| 27. $x^3 + 5x^2 - 2x - 15 = 0$ | 28. $36x^3 + 144x^2 - x - 4 = 0$ |
| 29. $2x^3 + 5x^2 + 4x + 1 = 0$ | 30. $12x^4 + 14x^3 - 5x^2 - 14x - 4 = 0$ |
| 31. $5x^3 - 11x^2 + 7x - 1 = 0$ | 32. $x^3 + 81x^2 - 49x - 49 = 0$ |

Find a third-degree polynomial equation with rational coefficients that has the given numbers as roots.

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| 33. $3, 2 - i$ | 34. $5, 2i$ |
| 35. $-1, 3 + i$ | 36. $-7, i$ |
| 37. $-4, 4i$ | 38. $6, 3 - 2i$ |

- Answers
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|--|---|--|
| 1. $2 - 3i, -\sqrt{7}$ | 2. $3 + \sqrt{2}, 1 - \sqrt{3}$ | 3. $4i, 6 + i$ |
| 4. $5 + \sqrt{6}, -2 - \sqrt{10}$ | 5. $x^4 - 8x^3 + 21x^2 - 32x + 68$ | |
| 6. $x^4 - 4x^3 - x^2 + 8x - 2$ | 7. $x^4 + 3x^2 - 54$ | |
| 8. $x^4 - 6x^3 + 9x^2 + 6x - 20$ | 9. $4, 2, -1$ | 10. $3, 1, -5$ |
| 11. $-4, -3, \frac{1}{2}$ | 12. $7, -2, -4$ | 13. $3, \frac{-3 \pm 3\sqrt{5}}{2}$ |
| 14. $-2, -1, 1, 2$ | 15. $2, 2 \pm i$ | 16. $-1, 3 \pm i$ |
| 17. $1, 2 \pm 3i$ | 18. $-2, 1 \pm 2i$ | 19. $1, -1, 5$ |
| 20. $-4, 2$ | 21. $-2, 1, 3$ | |
| 22. $10, -1 \pm i\sqrt{19}$ | 23. $1, -3$ | 24. $3, \frac{1}{2}, -\frac{1}{2}$ |
| 25. $2, \frac{-3 \pm \sqrt{13}}{2}$ | 26. $-3, \frac{2}{3}, \frac{1}{4}$ | |
| 27. $\pm 1, \pm 3, \pm 5, \pm 15$; none | 28. $\pm 1, \pm 2, \pm 4, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}, \pm \frac{1}{4}, \pm \frac{1}{6}, \pm \frac{1}{9}, \pm \frac{2}{9}, \pm \frac{4}{9}, \pm \frac{1}{12}, \pm \frac{1}{18}, \pm \frac{1}{36}, -4, -\frac{1}{6}, \frac{1}{6}$ | |
| 29. $\pm 1, \pm \frac{1}{2}, -1, -\frac{1}{2}$ | 30. $\pm 1, \pm 2, \pm 4, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}, \pm \frac{1}{6}, \pm \frac{1}{12}$; none | 31. $\pm 1, \pm \frac{1}{5}, \frac{1}{5}, 1$ |
| 32. $\pm 1, \pm 7, \pm 49$; none | | |
| 33. $x^3 - 7x^2 + 17x - 15 = 0$ | 34. $x^3 - 5x^2 + 4x - 20 = 0$ | |
| 35. $x^3 - 5x^2 + 4x + 10 = 0$ | 36. $x^3 + 7x^2 + x + 7 = 0$ | |
| 37. $x^3 + 4x^2 + 16x + 64 = 0$ | 38. $x^3 - 12x^2 + 49x - 78 = 0$ | |

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