2017

Module 6 Conceptual WS Key

1a. ① $f(2) = 8 - 16 + 2 + 6 = 0$

Since $f(2) = 0$, 2 is a zero of $f(x)$

②
$$\begin{array}{r|rrrr} 2 & 1 & -4 & 1 & 6 \\ & & 2 & -4 & -6 \\ \hline & 1 & -2 & -3 & 0 \end{array}$$

Since the remainder is 0, 2 is a zero of $f(x)$

③
$$\begin{array}{r} x^2 - 2x - 3 \\ x-2 \overline{\big)\, x^3 - 4x^2 + x + 6} \\ x^3 - 2x^2 \qquad\quad \\ \hline -2x^2 + x \quad \\ -2x^2 + 4x \quad \\ \hline -3x + 6 \\ -3x + 6 \\ \hline 0 \end{array}$$

since the remainder is 0
$x-2$ is a factor ∴
2 is a zero.

b.
$$\begin{array}{r|rrrr} 2 & 1 & -4 & 1 & 6 \\ & & 2 & -4 & -6 \\ \hline & 1 & -2 & -3 & \end{array}$$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$\boxed{x = 3, -1}$

c) $\boxed{(x-2)(x-3)(x+1) = f(x)}$

2a) ① Remainder Theorem

$f(-2) = 16 + 8 + 2 + 3 \quad = 29$

② Synthetic Division

$$\begin{array}{r|rrrrr} \underline{-2} & 1 & 0 & 2 & -1 & 3 \\ & & -2 & 4 & -12 & 26 \\ \hline & 1 & -2 & 6 & -13 & 29 \end{array}$$

③ Long Division

$$\begin{array}{r l} & x^3 - 2x^2 + 6x - 13 \\ x+2 \, \big) & x^4 + 0x^3 + 2x^2 - x + 3 \\ & x^4 + 2x^3 \\ \hline & -2x^3 + 2x^2 \\ & -2x^3 - 4x^2 \\ \hline & 6x^2 - x \\ & 6x^2 + 12x \\ \hline & -13x + 3 \\ & -(-13x - 26) \\ \hline & 29 \end{array}$$

b) Since $f(-2) = 29$ and not zero, $x+2$ is not a factor

c) since $f(-2) \neq 0$, -2 is not a zero

d) $P(x) = (x+2)(x^3 - 2x^2 + 6x - 13) + 29$

3a. Remainder Theorem

$f(3) = 81 - 81 + 24 - 24 = 0$

Long Division

$$\begin{array}{r l}
 & x^3 + 8 \\
x - 3 \;\big) & x^4 - 3x^3 + 0x^2 + 8x - 24 \\
- & x^4 - 3x^2 \\
\hline
 & 0 + 8x - 24 \\
- & 8x - 24 \\
\hline
 & 0
\end{array}$$

synthetic

3	1	-3	0	8	-24
		3	0	0	24
	1	0	0	8	0

b) Since the Remainder is 0, using all 3 techniques, $x - 3$ if a factor.

$(x-3)(x^3+8)$

$(x-3)(x+2)(x^2-2x+4)$

c) Since $f(3) = 0$; 3 is a zero