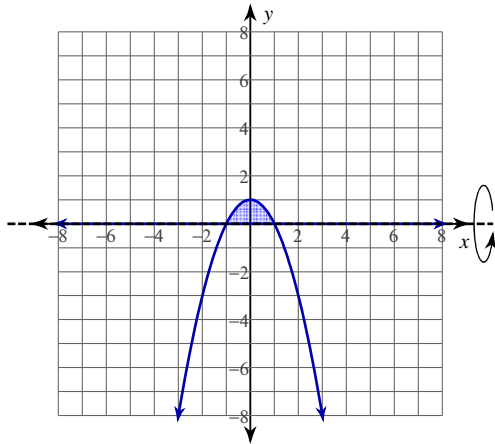


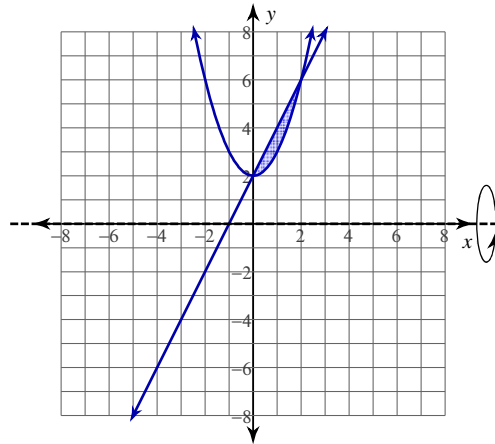
Volumes of Revolution - Washers and Disks

For each problem, find the volume of the solid that results when the region enclosed by the curves is revolved about the the x-axis.

1) $y = -x^2 + 1$
 $y = 0$

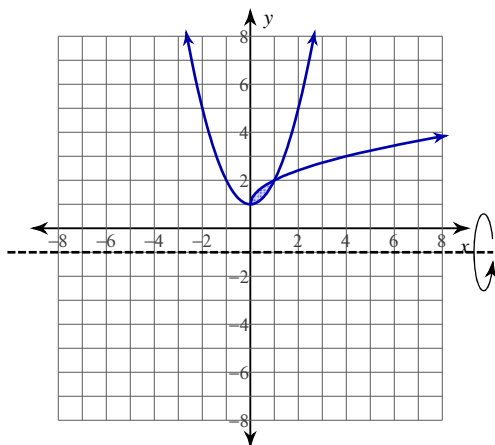


2) $y = 2x + 2$
 $y = x^2 + 2$

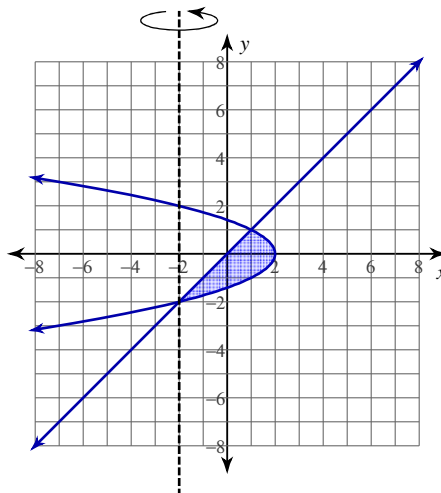


For each problem, find the volume of the solid that results when the region enclosed by the curves is revolved about the the given axis.

3) $y = \sqrt{x} + 1$
 $y = x^2 + 1$
 Axis: $y = -1$



4) $x = -y^2 + 2$
 $x = y$
 Axis: $x = -2$



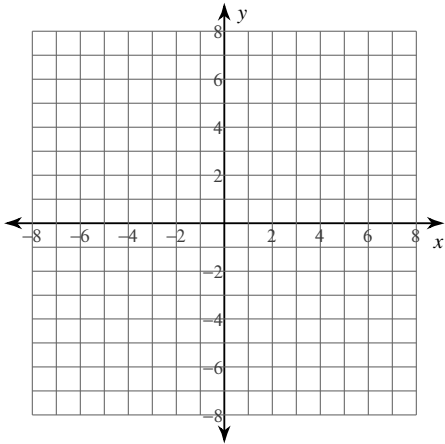
For each problem, find the volume of the solid that results when the region enclosed by the curves is revolved about the the given axis. You may use the provided graph to sketch the curves and shade the enclosed region.

5) $y = x^2 - 2$

$y = -2$

$x = 2$

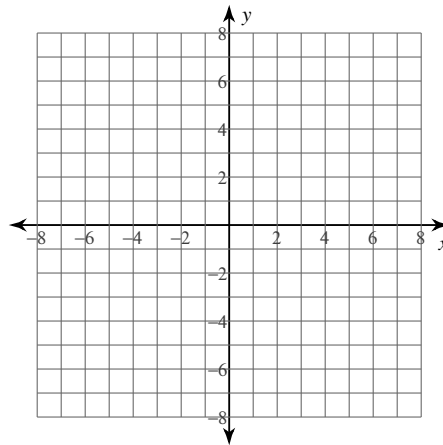
Axis: $y = -2$



6) $x = \sqrt{y} + 3$

$x = \frac{y}{2} + 3$

Axis: $x = 1$



Critical thinking questions:

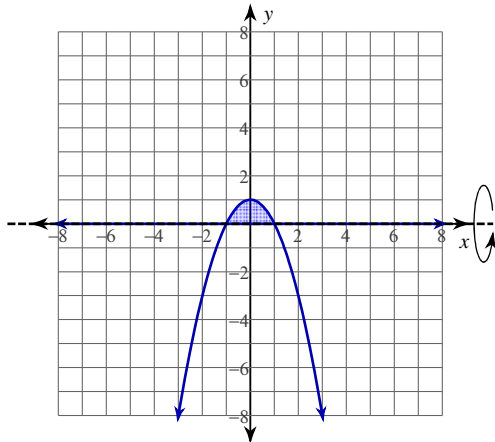
7) Use the method of disks to derive the formula for the volume of a sphere of radius r .

8) A 2 cm diameter drill bit is used to drill a cylindrical hole through a sphere of radius 5 cm. What is the volume of the resulting object?

Volumes of Revolution - Washers and Disks

For each problem, find the volume of the solid that results when the region enclosed by the curves is revolved about the the x-axis.

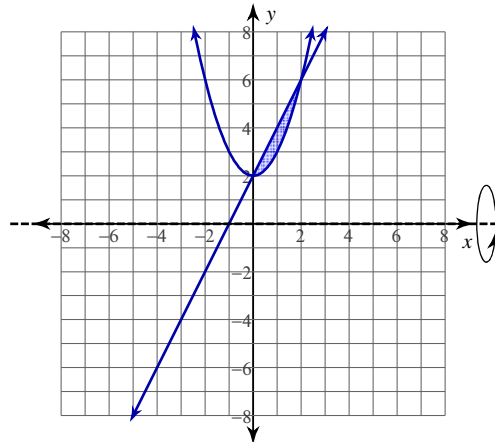
1) $y = -x^2 + 1$
 $y = 0$



$$\pi \int_{-1}^1 (-x^2 + 1)^2 dx$$

$$= \frac{16}{15}\pi \approx 3.351$$

2) $y = 2x + 2$
 $y = x^2 + 2$

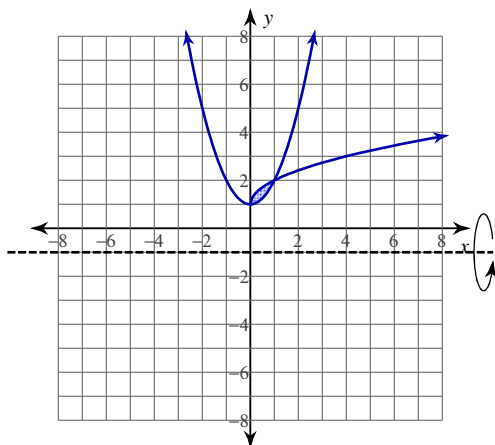


$$\pi \int_0^2 ((2x + 2)^2 - (x^2 + 2)^2) dx$$

$$= \frac{48}{5}\pi \approx 30.159$$

For each problem, find the volume of the solid that results when the region enclosed by the curves is revolved about the the given axis.

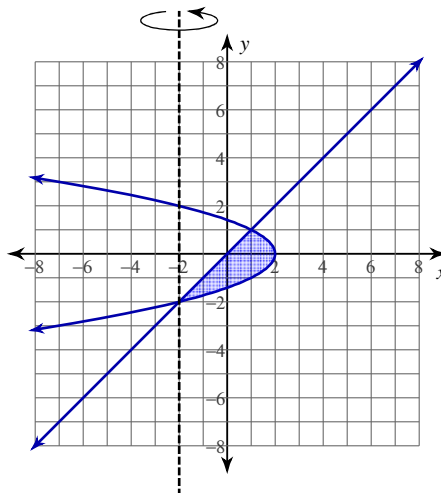
3) $y = \sqrt{x} + 1$
 $y = x^2 + 1$
 Axis: $y = -1$



$$\pi \int_0^1 ((\sqrt{x} + 2)^2 - (x^2 + 2)^2) dx$$

$$= \frac{49}{30}\pi \approx 5.131$$

4) $x = -y^2 + 2$
 $x = y$
 Axis: $x = -2$

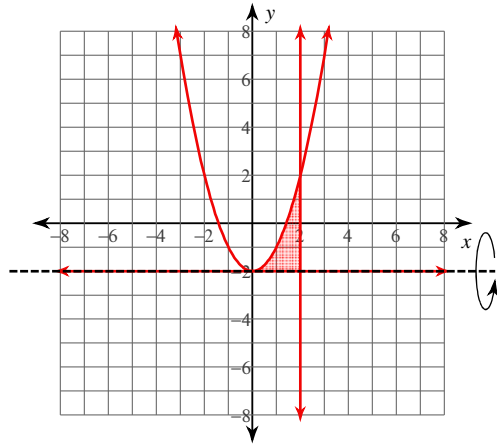


$$\pi \int_{-2}^1 ((-y^2 + 4)^2 - (y + 2)^2) dy$$

$$= \frac{108}{5}\pi \approx 67.858$$

For each problem, find the volume of the solid that results when the region enclosed by the curves is revolved about the the given axis. You may use the provided graph to sketch the curves and shade the enclosed region.

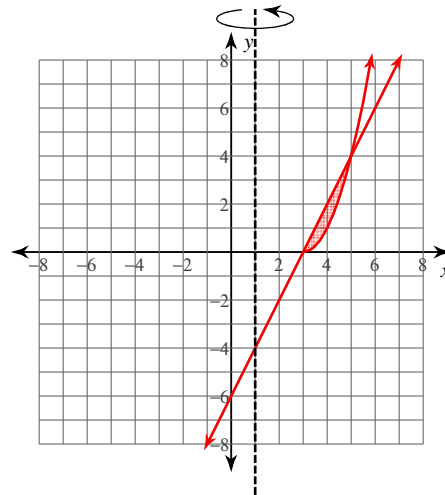
5) $y = x^2 - 2$
 $y = -2$
 $x = 2$
 Axis: $y = -2$



$$\pi \int_0^2 (x^2)^2 dx$$

$$= \frac{32}{5} \pi \approx 20.106$$

6) $x = \sqrt{y} + 3$
 $x = \frac{y}{2} + 3$
 Axis: $x = 1$



$$\pi \int_0^4 \left((\sqrt{y} + 2)^2 - \left(\frac{y}{2} + 2 \right)^2 \right) dy$$

$$= 8\pi \approx 25.133$$

Critical thinking questions:

7) Use the method of disks to derive the formula for the volume of a sphere of radius r .

$$x^2 + y^2 = r^2, y = \sqrt{r^2 - x^2}, V = \pi \int_{-r}^r (\sqrt{r^2 - x^2})^2 dx, V = \left(r^2 x - \frac{x^3}{3} \right) \Big|_{-r}^r, V = \frac{4}{3} \pi r^3$$

8) A 2 cm diameter drill bit is used to drill a cylindrical hole through a sphere of radius 5 cm. What is the volume of the resulting object?

$$V = \pi \int_{-5}^5 \left((\sqrt{25 - x^2})^2 - 1 \right) dx = \frac{470}{3}$$