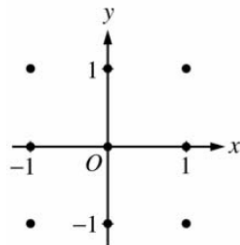


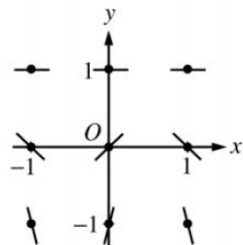
Consider the differential equation $\frac{dy}{dx} = (y-1)^2 \cos(\pi x)$.

- (a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.
 (Note: Use the axes provided in the exam booklet.)



- (b) There is a horizontal line with equation $y = c$ that satisfies this differential equation. Find the value of c .
 (c) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(1) = 0$.

(a)



- (b) The line $y = 1$ satisfies the differential equation, so $c = 1$.

(c)
$$\frac{1}{(y-1)^2} dy = \cos(\pi x) dx$$

$$-(y-1)^{-1} = \frac{1}{\pi} \sin(\pi x) + C$$

$$\frac{1}{1-y} = \frac{1}{\pi} \sin(\pi x) + C$$

$$1 = \frac{1}{\pi} \sin(\pi) + C = C$$

$$\frac{1}{1-y} = \frac{1}{\pi} \sin(\pi x) + 1$$

$$\frac{\pi}{1-y} = \sin(\pi x) + \pi$$

$$y = 1 - \frac{\pi}{\sin(\pi x) + \pi} \text{ for } -\infty < x < \infty$$

$$2 : \begin{cases} 1 : \text{zero slopes} \\ 1 : \text{all other slopes} \end{cases}$$

$$1 : c = 1$$

$$6 : \begin{cases} 1 : \text{separates variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{answer} \end{cases}$$

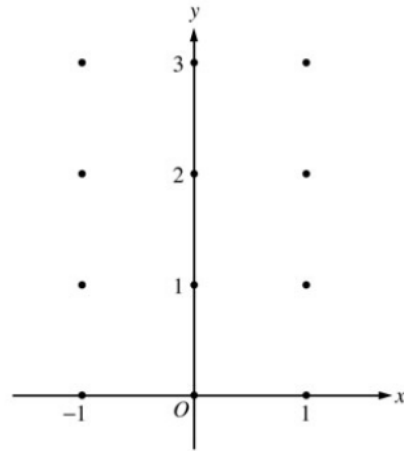
Note: max 3/6 [1-2-0-0-0] if no constant of integration

Note: 0/6 if no separation of variables

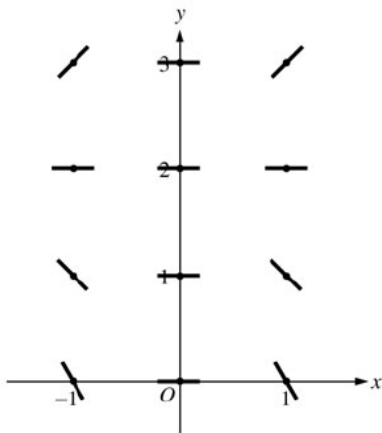
Question 5

Consider the differential equation $\frac{dy}{dx} = x^4(y - 2)$.

- (a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.
(Note: Use the axes provided in the test booklet.)
- (b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy -plane. Describe all points in the xy -plane for which the slopes are negative.
- (c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 0$.



(a)



- (b) Slopes are negative at points (x, y) where $x \neq 0$ and $y < 2$.

(c) $\frac{1}{y-2} dy = x^4 dx$

$$\ln|y-2| = \frac{1}{5}x^5 + C$$

$$|y-2| = e^C e^{\frac{1}{5}x^5}$$

$$y-2 = Ke^{\frac{1}{5}x^5}, K = \pm e^C$$

$$-2 = Ke^0 = K$$

$$y = 2 - 2e^{\frac{1}{5}x^5}$$

- 1 : zero slope at each point (x, y) where $x = 0$ or $y = 2$
- 2 : $\left\{ \begin{array}{l} \text{positive slope at each point } (x, y) \\ \text{where } x \neq 0 \text{ and } y > 2 \end{array} \right.$
- 1 : $\left\{ \begin{array}{l} \text{negative slope at each point } (x, y) \\ \text{where } x \neq 0 \text{ and } y < 2 \end{array} \right.$

1 : description

- 6 : $\left\{ \begin{array}{l} 1 : \text{separates variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } y \\ 0/1 \text{ if } y \text{ is not exponential} \end{array} \right.$

Note: max 3/6 [1-2-0-0-0] if no constant of integration

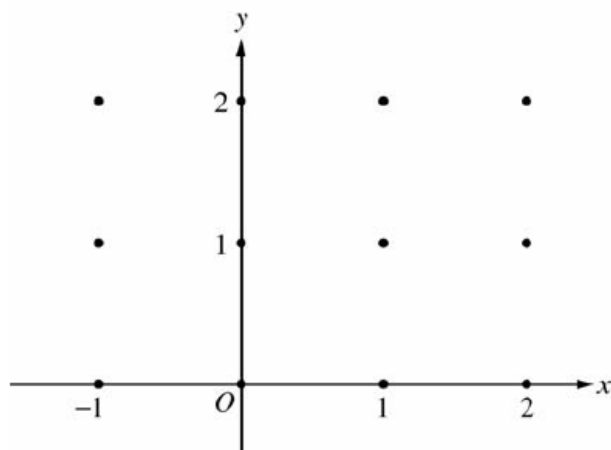
Note: 0/6 if no separation of variables

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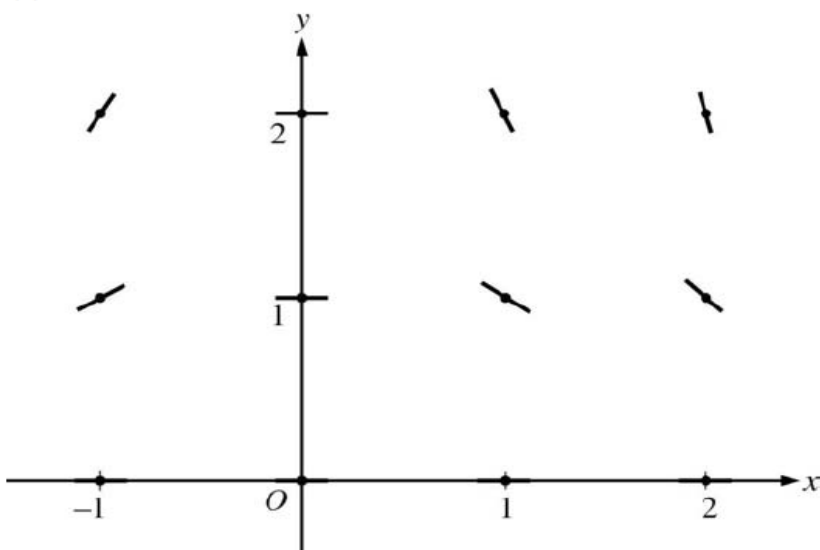
Question 6

Consider the differential equation $\frac{dy}{dx} = \frac{-xy^2}{2}$. Let $y = f(x)$ be the particular solution to this differential equation with the initial condition $f(-1) = 2$.

- (a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.
 (Note: Use the axes provided in the test booklet.)
- (b) Write an equation for the line tangent to the graph of f at $x = -1$.
- (c) Find the solution $y = f(x)$ to the given differential equation with the initial condition $f(-1) = 2$.



(a)



2 : $\begin{cases} 1 : \text{zero slopes} \\ 1 : \text{nonzero slopes} \end{cases}$

(b) Slope = $\frac{-(-1)4}{2} = 2$
 $y - 2 = 2(x + 1)$

1 : equation

(c) $\frac{1}{y^2} dy = -\frac{x}{2} dx$
 $-\frac{1}{y} = -\frac{x^2}{4} + C$
 $-\frac{1}{2} = -\frac{1}{4} + C; C = -\frac{1}{4}$
 $y = \frac{1}{\frac{x^2}{4} + \frac{1}{4}} = \frac{4}{x^2 + 1}$

6 : $\begin{cases} 1 : \text{separates variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } y \end{cases}$

Note: max 3/6 [1-2-0-0-0] if no constant of integration

Note: 0/6 if no separation of variables