



 Answers

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[E Click here for exercises.](#)[S Click here for solutions.](#)

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|--------------------------|--------------------|
| 1. $\frac{1}{4}$         | 2. 5               |
| 3. $\frac{3}{2}$         | 4. $\frac{1}{2}$   |
| 5. 1                     | 6. 2               |
| 7. $\infty$              | 8. 0               |
| 9. 1                     | 10. $\frac{1}{32}$ |
| 11. $\frac{1}{3a^{2/3}}$ | 12. $\ln 3$        |
| 13. 0                    | 14. 0              |
| 15. $\alpha$             | 16. 1              |
| 17. 0                    | 18. $\frac{1}{5}$  |
| 19. $\frac{2}{3}$        | 20. $\frac{1}{3}$  |
| 21. $\frac{m}{n}$        | 22. 1              |
| 23. -2                   | 24. 0              |
| 25. $\frac{1}{2}$        | 26. -3             |
| 27. $\frac{2}{3}$        | 28. 0              |
| 29. $\frac{1}{3}$        | 30. 0              |
| 31. 0                    | 32. $\frac{3}{7}$  |
| 33. 0                    | 34. 1              |
| 35. $-\frac{2}{\pi}$     | 36. $\infty$       |
| 37. 0                    | 38. 1              |
| 39. 0                    | 40. 0              |
| 41. 1                    | 42. 1              |
| 43. 1                    | 44. $\infty$       |
| 45. 1                    |                    |

## Solutions

**E** Click here for exercises.

The use of L'Hospital's Rule is indicated by an H above the equal sign:  $\frac{H}{=}$ .

$$1. \lim_{x \rightarrow 2} \frac{x-2}{x^2-4} = \lim_{x \rightarrow 2} \frac{x-2}{(x-2)(x+2)} = \lim_{x \rightarrow 2} \frac{1}{x+2} = \frac{1}{4}$$

$$2. \lim_{x \rightarrow 1} \frac{x^2+3x-4}{x-1} = \lim_{x \rightarrow 1} \frac{(x-1)(x+4)}{x-1} \\ = \lim_{x \rightarrow 1} (x+4) = 5$$

$$3. \lim_{x \rightarrow -1} \frac{x^6-1}{x^4-1} \stackrel{H}{=} \lim_{x \rightarrow -1} \frac{6x^5}{4x^3} = \frac{-6}{-4} = \frac{3}{2}$$

$$4. \lim_{x \rightarrow 0} \frac{\tan x}{x + \sin x} \stackrel{H}{=} \lim_{x \rightarrow 0} \frac{\sec^2 x}{1 + \cos x} = \frac{1}{1+1} = \frac{1}{2}$$

$$5. \lim_{x \rightarrow 0} \frac{e^x-1}{\sin x} \stackrel{H}{=} \lim_{x \rightarrow 0} \frac{e^x}{\cos x} = \frac{1}{1} = 1$$

$$6. \lim_{x \rightarrow 0} \frac{x + \tan x}{\sin x} \stackrel{H}{=} \lim_{x \rightarrow 0} \frac{1 + \sec^2 x}{\cos x} = \frac{1+1^2}{1} = 2$$

$$7. \lim_{x \rightarrow 0} \frac{\sin x}{x^3} \stackrel{H}{=} \lim_{x \rightarrow 0} \frac{\cos x}{3x^2} = \infty$$

$$8. \lim_{x \rightarrow \pi} \frac{\tan x}{x} = \frac{\tan \pi}{\pi} = \frac{0}{\pi} = 0. \text{ L'Hospital's Rule does not} \\ \text{apply because the denominator doesn't approach 0.}$$

$$9. \lim_{x \rightarrow 3\pi/2} \frac{\cos x}{x - 3\pi/2} \stackrel{H}{=} \lim_{x \rightarrow 3\pi/2} \frac{-\sin x}{1} = -\sin \frac{3\pi}{2} = 1$$

$$10. \lim_{t \rightarrow 16} \frac{\sqrt[4]{t}-2}{t-16} = \lim_{t \rightarrow 16} \frac{\sqrt[4]{t}-2}{(\sqrt[4]{t}+4)(\sqrt[4]{t}-4)} \\ = \lim_{t \rightarrow 16} \frac{\sqrt[4]{t}-2}{(\sqrt[4]{t}+4)(\sqrt[4]{t}+2)(\sqrt[4]{t}-2)} \\ = \lim_{t \rightarrow 16} \frac{1}{(\sqrt[4]{t}+4)(\sqrt[4]{t}+2)} \\ = \frac{1}{(4+4)(2+2)} = \frac{1}{32}$$

$$11. \lim_{x \rightarrow a} \frac{x^{1/3} - a^{1/3}}{x - a} \stackrel{H}{=} \lim_{x \rightarrow a} \frac{(1/3)x^{-2/3}}{1} = \frac{1}{3a^{2/3}}$$

$$12. \lim_{x \rightarrow 0} \frac{6^x - 2^x}{x} \stackrel{H}{=} \lim_{x \rightarrow 0} \frac{6^x(\ln 6) - 2^x(\ln 2)}{1} \\ = \ln 6 - \ln 2 = \ln \frac{6}{2} = \ln 3$$

$$13. \lim_{x \rightarrow \infty} \frac{(\ln x)^3}{x^2} \stackrel{H}{=} \lim_{x \rightarrow \infty} \frac{3(\ln x)^2(1/x)}{2x} = \lim_{x \rightarrow \infty} \frac{3(\ln x)^2}{2x^2} \\ \stackrel{H}{=} \lim_{x \rightarrow \infty} \frac{6(\ln x)(1/x)}{4x} = \lim_{x \rightarrow \infty} \frac{3 \ln x}{2x^2} \\ \stackrel{H}{=} \lim_{x \rightarrow \infty} \frac{3/x}{4x} = \lim_{x \rightarrow \infty} \frac{3}{4x^2} = 0$$

$$14. \lim_{x \rightarrow 0} \frac{\sin x}{e^x} = \frac{0}{1} = 0. \text{ L'Hospital's Rule does not apply.}$$

$$15. \lim_{x \rightarrow 0} \frac{\tan \alpha x}{x} \stackrel{H}{=} \lim_{x \rightarrow 0} \frac{\alpha \sec^2 \alpha x}{1} = \alpha$$

**A** Click here for answers.

$$16. \lim_{x \rightarrow 0} \frac{\sin^2 x}{\tan(x^2)} \stackrel{H}{=} \lim_{x \rightarrow 0} \frac{2 \sin x \cos x}{2x \sec^2(x^2)} \\ = \lim_{x \rightarrow 0} \frac{\sin x}{x} \lim_{x \rightarrow 0} \frac{\cos x}{\sec^2(x^2)} = 1 \cdot 1 = 1$$

$$17. \lim_{x \rightarrow \infty} \frac{\ln \ln x}{\sqrt{x}} \stackrel{H}{=} \lim_{x \rightarrow \infty} \frac{1/(x \ln x)}{1/(2\sqrt{x})} = \lim_{x \rightarrow \infty} \frac{2}{\sqrt{x} \ln x} = 0$$

$$18. \lim_{x \rightarrow \infty} \frac{\ln(1+e^x)}{5x} \stackrel{H}{=} \lim_{x \rightarrow \infty} \frac{e^x/(1+e^x)}{5} \\ = \lim_{x \rightarrow \infty} \frac{e^x}{5(1+e^x)} \stackrel{H}{=} \lim_{x \rightarrow \infty} \frac{e^x}{5e^x} = \frac{1}{5}$$

$$19. \lim_{x \rightarrow 0} \frac{\tan^{-1}(2x)}{3x} \stackrel{H}{=} \lim_{x \rightarrow 0} \frac{2/(1+4x^2)}{3} = \frac{2}{3}$$

$$20. \lim_{x \rightarrow 0} \frac{x}{\sin^{-1}(3x)} \stackrel{H}{=} \lim_{x \rightarrow 0} \frac{1}{3/\sqrt{1-(3x)^2}} \\ = \lim_{x \rightarrow 0} \frac{1}{3} \sqrt{1-9x^2} = \frac{1}{3}$$

$$21. \lim_{x \rightarrow 0} \frac{\sin mx}{\sin nx} \stackrel{H}{=} \lim_{x \rightarrow 0} \frac{m \cos mx}{n \cos nx} = \frac{m}{n}$$

$$22. \lim_{x \rightarrow 0} \frac{\sin^{10} x}{\sin(x^{10})} \stackrel{H}{=} \lim_{x \rightarrow 0} \frac{10 \sin^9 x \cos x}{10x^9 \cos(x^{10})} \\ = \left[ \lim_{x \rightarrow 0} \frac{\sin x}{x} \right]^9 \lim_{x \rightarrow 0} \frac{\cos x}{\cos(x^{10})} \\ = 1^9 \cdot 1 = 1$$

$$23. \lim_{x \rightarrow 0} \frac{x + \sin 3x}{x - \sin 3x} \stackrel{H}{=} \lim_{x \rightarrow 0} \frac{1 + 3 \cos 3x}{1 - 3 \cos 3x} = \frac{1+3}{1-3} = -2$$

$$24. \lim_{x \rightarrow 0} \frac{e^{4x} - 1}{\cos x} = \frac{0}{1} = 0$$

$$25. \lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^3} \stackrel{H}{=} \lim_{x \rightarrow 0} \frac{\sec^2 x - \cos x}{3x^2} \\ \stackrel{H}{=} \lim_{x \rightarrow 0} \frac{2 \sec^2 x \tan x + \sin x}{6x} \\ \stackrel{H}{=} \lim_{x \rightarrow 0} \frac{4 \sec^2 x \tan^2 x + 2 \sec^4 x + \cos x}{6} \\ = \frac{0+2+1}{6} = \frac{1}{2}$$

$$26. \lim_{x \rightarrow 0} \frac{x + \tan 2x}{x - \tan 2x} \stackrel{H}{=} \lim_{x \rightarrow 0} \frac{1 + 2 \sec^2 2x}{1 - 2 \sec^2 2x} \\ = \frac{1+2(1)^2}{1-2(1)^2} = -3$$

$$27. \lim_{x \rightarrow 0} \frac{\tan 2x}{\tanh 3x} \stackrel{H}{=} \lim_{x \rightarrow 0} \frac{2 \sec^2 2x}{3 \operatorname{sech}^2 3x} = \frac{2}{3}$$

$$28. \lim_{x \rightarrow 0} \frac{2x - \sin^{-1} x}{2x + \cos^{-1} x} = \frac{2(0) - 0}{2(0) + \pi/2} = 0. \text{ L'Hospital's Rule} \\ \text{does not apply.}$$

$$\begin{aligned} 29. \lim_{x \rightarrow 0} \frac{2x - \sin^{-1} x}{2x + \tan^{-1} x} &\stackrel{H}{=} \lim_{x \rightarrow 0} \frac{2 - 1/\sqrt{1-x^2}}{2 + 1/(1+x^2)} \\ &= \frac{2-1}{2+1} = \frac{1}{3} \end{aligned}$$

$$\begin{aligned} 30. \lim_{x \rightarrow -\infty} x e^x &= \lim_{x \rightarrow -\infty} \frac{x}{e^{-x}} \stackrel{H}{=} \lim_{x \rightarrow -\infty} \frac{1}{-e^{-x}} \\ &= \lim_{x \rightarrow -\infty} -e^x = 0 \end{aligned}$$

$$\begin{aligned} 31. \lim_{x \rightarrow \infty} e^{-x} \ln x &= \lim_{x \rightarrow \infty} \frac{\ln x}{e^x} \stackrel{H}{=} \lim_{x \rightarrow \infty} \frac{1/x}{e^x} \\ &= \lim_{x \rightarrow \infty} \frac{1}{x e^x} = 0 \end{aligned}$$

$$\begin{aligned} 32. \lim_{x \rightarrow (\pi/2)^-} \sec 7x \cos 3x &= \lim_{x \rightarrow (\pi/2)^-} \frac{\cos 3x}{\cos 7x} \\ &\stackrel{H}{=} \lim_{x \rightarrow (\pi/2)^-} \frac{-3 \sin 3x}{-7 \sin 7x} = \frac{3(-1)}{7(-1)} = \frac{3}{7} \end{aligned}$$

$$33. \lim_{x \rightarrow 0^+} \sqrt{x} \sec x = 0 \cdot 1 = 0$$

$$\begin{aligned} 34. \lim_{x \rightarrow \pi} (x - \pi) \cot x &= \lim_{x \rightarrow \pi} \frac{x - \pi}{\tan x} \stackrel{H}{=} \lim_{x \rightarrow \pi} \frac{1}{\sec^2 x} \\ &= \frac{1}{(-1)^2} = 1 \end{aligned}$$

$$\begin{aligned} 35. \lim_{x \rightarrow 1^+} (x-1) \tan(\pi x/2) &= \lim_{x \rightarrow 1^+} \frac{x-1}{\cot(\pi x/2)} \\ &\stackrel{H}{=} \lim_{x \rightarrow 1^+} \frac{1}{-\csc^2(\pi x/2)} = -\frac{2}{\pi} \end{aligned}$$

$$36. \lim_{x \rightarrow 0} \left( \frac{1}{x^4} - \frac{1}{x^2} \right) = \lim_{x \rightarrow 0} \frac{1-x^2}{x^4} = \infty$$

$$\begin{aligned} 37. \lim_{x \rightarrow \infty} (x - \sqrt{x^2 - 1}) &= \lim_{x \rightarrow \infty} (x - \sqrt{x^2 - 1}) \frac{x + \sqrt{x^2 - 1}}{x + \sqrt{x^2 - 1}} \\ &= \lim_{x \rightarrow \infty} \frac{x^2 - (x^2 - 1)}{x + \sqrt{x^2 - 1}} \\ &= \lim_{x \rightarrow \infty} \frac{1}{x + \sqrt{x^2 - 1}} = 0 \end{aligned}$$

$$\begin{aligned} 38. \lim_{x \rightarrow \infty} (\sqrt{x^2 + x + 1} - \sqrt{x^2 - x}) &= \lim_{x \rightarrow \infty} (\sqrt{x^2 + x + 1} - \sqrt{x^2 - x}) \frac{\sqrt{x^2 + x + 1} + \sqrt{x^2 - x}}{\sqrt{x^2 + x + 1} + \sqrt{x^2 - x}} \\ &= \lim_{x \rightarrow \infty} \frac{(x^2 + x + 1) - (x^2 - x)}{\sqrt{x^2 + x + 1} + \sqrt{x^2 - x}} \\ &= \lim_{x \rightarrow \infty} \frac{2x + 1}{\sqrt{x^2 + x + 1} + \sqrt{x^2 - x}} \\ &= \lim_{x \rightarrow \infty} \frac{2 + 1/x}{\sqrt{1 + 1/x + 1/x^2} + \sqrt{1 - 1/x}} \\ &= \frac{2}{1+1} = 1 \end{aligned}$$

$$\begin{aligned} 39. \lim_{x \rightarrow \infty} \left( \frac{1}{x} - \frac{1}{e^x - 1} \right) &= \lim_{x \rightarrow \infty} \frac{1}{x} - \lim_{x \rightarrow \infty} \frac{1}{e^x - 1} \quad (\text{since} \\ &\text{both limits exist}) = 0 - 0 = 0 \end{aligned}$$

$$\begin{aligned} 40. \lim_{x \rightarrow \infty} \left( \frac{x^3}{x^2 - 1} - \frac{x^3}{x^2 + 1} \right) &= \lim_{x \rightarrow \infty} \frac{x^3(x^2 + 1) - x^3(x^2 - 1)}{(x^2 - 1)(x^2 + 1)} \\ &= \lim_{x \rightarrow \infty} \frac{2x^3}{x^4 - 1} = \lim_{x \rightarrow \infty} \frac{2/x}{1 - 1/x^4} \\ &= 0 \end{aligned}$$

$$\begin{aligned} 41. y = (\sin x)^{\tan x} &\Rightarrow \ln y = \tan x \ln(\sin x), \text{ so} \\ \lim_{x \rightarrow 0^+} \ln y &= \lim_{x \rightarrow 0^+} \tan x \ln(\sin x) = \lim_{x \rightarrow 0^+} \frac{\ln(\sin x)}{\cot x} \\ &\stackrel{H}{=} \lim_{x \rightarrow 0^+} \frac{(\cos x)/\sin x}{-\csc^2 x} \\ &= \lim_{x \rightarrow 0^+} (-\sin x \cos x) = 0 \Rightarrow \\ \lim_{x \rightarrow 0^+} (\sin x)^{\tan x} &= \lim_{x \rightarrow 0^+} e^{\ln y} = e^0 = 1. \end{aligned}$$

$$\begin{aligned} 42. \text{ Let } y = \left(1 + \frac{1}{x^2}\right)^x. &\text{ Then } \ln y = x \ln \left(1 + \frac{1}{x^2}\right) \Rightarrow \\ \lim_{x \rightarrow \infty} \ln y &= \lim_{x \rightarrow \infty} x \ln \left(1 + \frac{1}{x^2}\right) = \lim_{x \rightarrow \infty} \frac{\ln \left(1 + \frac{1}{x^2}\right)}{1/x} \\ &\stackrel{H}{=} \lim_{x \rightarrow \infty} \frac{\left(-\frac{2}{x^3}\right) / \left(1 + \frac{1}{x^2}\right)}{-1/x^2} \\ &= \lim_{x \rightarrow \infty} \frac{2/x}{1 + 1/x^2} = 0, \end{aligned}$$

$$\text{so } \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x^2}\right)^x = \lim_{x \rightarrow \infty} e^{\ln y} = e^0 = 1.$$

$$\begin{aligned} 43. y = (\cot x)^{\sin x} &\Rightarrow \ln y = \sin x \ln(\cot x) \Rightarrow \\ \lim_{x \rightarrow 0^+} \ln y &= \lim_{x \rightarrow 0^+} \frac{\ln(\cot x)}{\csc x} \stackrel{H}{=} \lim_{x \rightarrow 0^+} \frac{(-\csc^2 x)/\cot x}{-\csc x \cot x} \\ &= \lim_{x \rightarrow 0^+} \frac{\csc x}{\cot^2 x} = \lim_{x \rightarrow 0^+} \frac{\sin x}{\cos^2 x} = 0 \\ \text{so } \lim_{x \rightarrow 0^+} (\cot x)^{\sin x} &= \lim_{x \rightarrow 0^+} e^{\ln y} = e^0 = 1. \end{aligned}$$

$$\begin{aligned} 44. \text{ Let } y = (1 + 1/x)^{x^2}. &\text{ Then } \ln y = x^2 \ln(1 + 1/x) \Rightarrow \\ \lim_{x \rightarrow \infty} \ln y &= \lim_{x \rightarrow \infty} x^2 \ln(1 + 1/x) = \lim_{x \rightarrow \infty} \frac{\ln(1 + 1/x)}{1/x^2} \\ &\stackrel{H}{=} \lim_{x \rightarrow \infty} \frac{(-1/x^2) / (1 + 1/x)}{-2/x^3} \\ &= \lim_{x \rightarrow \infty} \frac{x}{2(1 + 1/x)} = \infty \Rightarrow \\ \lim_{x \rightarrow \infty} (1 + 1/x)^{x^2} &= \lim_{x \rightarrow \infty} e^{\ln y} = \infty. \end{aligned}$$

$$\begin{aligned} 45. y = (-\ln x)^x &\Rightarrow \ln y = x \ln(-\ln x), \text{ so} \\ \lim_{x \rightarrow 0^+} \ln y &= \lim_{x \rightarrow 0^+} x \ln(-\ln x) = \lim_{x \rightarrow 0^+} \frac{\ln(-\ln x)}{1/x} \\ &\stackrel{H}{=} \lim_{x \rightarrow 0^+} \frac{(1/(-\ln x))(-1/x)}{-1/x^2} \\ &= \lim_{x \rightarrow 0^+} \frac{-x}{\ln x} = 0 \Rightarrow \\ \lim_{x \rightarrow 0^+} (-\ln x)^x &= e^0 = 1. \end{aligned}$$