Calculus 1 Tutor

Worksheet 14

Derivatives and Integrals of Logarithms

1. Suppose $f(x) = \ln(5x + 3)$. Find $f'(x)$.

2. If $k(x) = \ln(3x^2 + 6x)$, what is $k'(x)$?

3. Find $g'(x)$ if $g(x) = \ln(\sin x)$.

4. If $h(x) = \ln(3\cos x)$, what is $h'(x)$?

5. Suppose $f(x) = x\ln(9x)$. Find $f'(x)$.

6. What is $g'(x)$ if $g(x) = x^2\ln(x^4)$?

7. Suppose $h(x) = \frac{\ln(2x+1)}{\ln(3x-2)}$. What is $h'(x)$?

8. Find $f'(x)$ if $f(x) = (3x^2 + x)\ln(6x + 1)$?

9. What is $k'(x)$ if $k(x) = \sin x \cdot \ln(\cos x)$?

10. Suppose $f(x) = \tan x \cdot \ln(\tan x)$, what is $f'(x)$?

11. Evaluate $\int \frac{2}{2x+5} dx$.

12.Evaluate $\int \frac{4x}{2x^2+9} dx$.

13. Evaluate $\int \frac{5}{3-2x} dx$.

14. Evaluate $\int \frac{\sec^2 x}{\tan x} dx$.

15. Evaluate the definite integral.

$$\int_{2}^{3} \frac{6x}{3x^2+1} dx$$

16. Evaluate $\int(-\cot x)dx$.

17.Evaluate $\int(\tan x)dx$.

18. Evaluate the definite integral.

$$\int_{3}^{4} \frac{4x-2}{2x^2-2x+3}dx$$

19. Evaluate the definite Integral.

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} 5\tan x \, dx$$

20. Evaluate the definite integral.

$$\int_{0}^{2} \frac{2x+8}{x^2+8x+5} \, dx$$

Answers - Calculus 1 Tutor - Worksheet 14 – Derivatives of Logarithms

1. Suppose $f(x) = \ln(5x + 3)$. Find $f'(x)$.

The formula for the derivative of the natural logarithm function is:

$\frac{d}{dx}[\ln x] = \frac{1}{x}$ and $\frac{d}{dx}[\ln u] = \frac{1}{u}du$

Therefore,

$$f'(x) = \frac{1}{5x+3} \cdot (5x+3)' = \frac{1}{5x+3} \cdot 5 = \frac{5}{5x+3}$$

Answer: $f'(x) = \frac{5}{5x+3}$

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2. If $k(x) = \ln(3x^2 + 6x)$, what is $k'(x)$?

The formula for the derivative of the natural logarithm function is:

$\frac{d}{dx}[\ln x] = \frac{1}{x}$ and $\frac{d}{dx}[\ln u] = \frac{1}{u}du$

Therefore,

$$k'(x) = \frac{1}{3x^2+6x} \cdot (3x^2+6x)' = \frac{1}{3x^2+6x} \cdot (6x+6) = \frac{6x+6}{3x^2+6x}$$

Simplify:

$$k'(x) = \frac{6x+6}{3x^2+6x} = \frac{2x+2}{x^2+2x}$$

Answer: $k'(x) = \frac{2x+2}{x^2+2x}$

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3. Find $g'(x)$ if $g(x) = \ln(\sin x)$. The formula for the derivative of the natural logarithm function is:

$\frac{d}{dx}[\ln x] = \frac{1}{x}$ and $\frac{d}{dx}[\ln u] = \frac{1}{u}du$

Therefore,

$$g'(x) = \frac{1}{\sin x} \cdot \cos x = \frac{\cos x}{\sin x} = \cot x$$

Answer: $g'(x) = \cot x$

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4. If $h(x) = \ln(3\cos x)$, what is $h'(x)$? The formula for the derivative of the natural logarithm function is:

$\frac{d}{dx}[\ln x] = \frac{1}{x}$ and $\frac{d}{dx}[\ln u] = \frac{1}{u}du$

Therefore,

$$h'(x) = \frac{1}{3\cos x} \cdot -3\sin x = \frac{-3\sin x}{3\cos x} = -\frac{\sin x}{\cos x} = -\tan x$$

Answer: $h'(x) = -\tan x$

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5. Suppose $f(x) = x\ln(9x)$. Find $f'(x)$. The formula for the derivative of the natural logarithm function is:

$\frac{d}{dx}[\ln x] = \frac{1}{x}$ and $\frac{d}{dx}[\ln u] = \frac{1}{u}du$

Therefore, using this rule and the product rule,

$$f'(x) = x \cdot (\ln(9x))' + x' \cdot \ln(9x)$$

$$f'(x) = x \cdot \frac{1}{9x} \cdot 9 + 1 \cdot \ln(9x)$$

Answer: $f'(x) = 1 + \ln(9x)$

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6. What is $g'(x)$ if $g(x) = x^2\ln(x^4)$? The formula for the derivative of the natural logarithm function is:

$\frac{d}{dx}[\ln x] = \frac{1}{x}$ and $\frac{d}{dx}[\ln u] = \frac{1}{u}du$

Therefore, using this rule and the product rule,

$$g'(x) = x^2 \cdot (\ln(x^4))' + (x^2)' \cdot \ln(x^4)$$

$$g'(x) = x^2 \cdot \frac{1}{x^4} \cdot 4x^3 + 2x \cdot \ln(x^4)$$

Simplify:

$$g'(x) = 4x + 2x\ln(x^4)$$

Answer: $g'(x) = 4x + 2x\ln(x^4)$

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7. Suppose $h(x) = \frac{\ln(2x+1)}{\ln(3x-2)}$. What is $h'(x)$? The formula for the derivative of the natural logarithm function is:

$\frac{d}{dx}[\ln x] = \frac{1}{x}$ and $\frac{d}{dx}[\ln u] = \frac{1}{u}du$

Therefore, using this rule and the quotient rule,

$$h'(x) = \frac{\ln(3x-2) \cdot [\ln(2x+1)]' - \ln(2x+1) \cdot [\ln(3x-2)]'}{[\ln(3x-2)]^2}$$

$$h'(x) = \frac{\ln(3x-2) \cdot \frac{2}{2x+1} - \ln(2x+1) \cdot \frac{3}{3x-2}}{[\ln(3x-2)]^2}$$

$$h'(x) = \frac{\frac{2\ln(3x-2)}{2x+1} - \frac{3\ln(2x+1)}{3x-2}}{[\ln(3x-2)]^2}$$

This expression does not simplify.

Answer:

$$h'(x) = \frac{\frac{2\ln(3x-2)}{2x+1} - \frac{3\ln(2x+1)}{3x-2}}{[\ln(3x-2)]^2}$$

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8. Find $f'(x)$ if $f(x) = (3x^2 + x)\ln(6x + 1)$? The formula for the derivative of the natural logarithm function is:

$\frac{d}{dx}[\ln x] = \frac{1}{x}$ and $\frac{d}{dx}[\ln u] = \frac{1}{u}du$

Therefore, using this rule and the product rule,

$$f'(x) = (3x^2 + x) \cdot [\ln(6x + 1)]' + (3x^2 + x)' \cdot \ln(6x + 1)$$

$$f'(x) = (3x^2 + x) \cdot \frac{6}{6x + 1} + (6x + 1) \cdot \ln(6x + 1)$$

Simplify:

$$f'(x) = \frac{6(3x^2 + x)}{6x + 1} + (6x + 1)\ln(6x + 1)$$

$$f'(x) = \frac{6(3x^2 + x)}{6x + 1} + \frac{(6x + 1)^2 \ln(6x + 1)}{6x + 1}$$

Answer:

$$f'(x) = \frac{6(3x^2 + x) + (6x + 1)^2 \ln(6x + 1)}{6x + 1}$$

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9. What is $k'(x)$ if $k(x) = \sin x \cdot \ln(\cos x)$? The formula for the derivative of the natural logarithm function is:

$\frac{d}{dx}[\ln x] = \frac{1}{x}$ and $\frac{d}{dx}[\ln u] = \frac{1}{u}du$

Therefore, using this rule and the product rule,

$$k'(x) = \sin x \cdot [\ln(\cos x)]' + (\sin x)' \cdot \ln(\cos x)$$

$$k'(x) = \sin x \cdot \frac{1}{\cos x} \cdot -\sin x + (\cos x) \cdot \ln(\cos x)$$

Answer:

$$k'(x) = -\frac{\sin^2 x}{\cos x} + \cos(x)\ln(\cos x)$$

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10. Suppose $f(x) = \tan x \cdot \ln(\tan x)$, what is $f'(x)$? The formula for the derivative of the natural logarithm function is:

$\frac{d}{dx}[\ln x] = \frac{1}{x}$ and $\frac{d}{dx}[\ln u] = \frac{1}{u}du$

Therefore, using this rule and the product rule,

$$f'(x) = \tan x \cdot [\ln(\tan x)]' + (\tan x)' \cdot \ln(\tan x)$$

$$f'(x) = \tan x \cdot \frac{1}{\tan x} \cdot \sec^2 x + \sec^2 x \cdot \ln(\tan x)$$

$$f'(x) = \sec^2 x + \sec^2 x \ln(\tan x)$$

Answer: $f'(x) = \sec^2 x(1 + \ln(\tan x))$

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11. Evaluate $\int \frac{2}{2x+5}dx$.

Integration involving the natural logarithmic function follows the pattern:

$\int \frac{dx}{x} = \ln|x| + C$ and $\int \frac{du}{u} = \ln|u| + C$

Therefore, let $u = 2x + 5$ and $du = 2dx$. Thus,

$$\int \frac{2}{2x+5}dx = \int \frac{(2x+5)'}{2x+5}dx = \ln|2x+5| + C$$

Answer:

$$\int \frac{2}{2x+5}dx = \ln|2x+5| + C$$

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12. Evaluate $\int \frac{4x}{2x^2+9} dx$.

$\int \frac{dx}{x} = \ln|x| + C$ and $\int \frac{du}{u} = \ln|u| + C$

Therefore, let $u = 2x^2 + 9$ and $du = 4xdx$. Thus,

$$\int \frac{4x}{2x^2+9} dx = \int \frac{(2x^2+9)'}{2x^2+9} dx = \ln(2x^2+9) + C$$

Answer:

$$\int \frac{4x}{2x^2+9} dx = \ln(2x^2+9) + C$$

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13. Evaluate $\int \frac{5}{3-2x} dx$.

$\int \frac{dx}{x} = \ln|x| + C$ and $\int \frac{du}{u} = \ln|u| + C$

Therefore, let $u = 3 - 2x$ and $du = -2dx$. Thus, Move the constant 5 out of the integral and multiply by $\frac{-2}{-2}$.

$$\int \frac{5}{3-2x} dx = \frac{5}{-2}\int \frac{-2}{3-2x} dx = -\frac{5}{2}\int \frac{(3-2x)'}{3-2x} dx = -\frac{5}{2}\ln|3-2x| + C$$

Answer: $\int \frac{5}{3-2x} dx = -\frac{5}{2}\ln|3-2x| + C$

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14. Evaluate $\int \frac{\sec^2 x}{\tan x} dx$.

$\int \frac{dx}{x} = \ln|x| + C$ and $\int \frac{du}{u} = \ln|u| + C$

Therefore, let $u = \tan x$ and $du = \sec^2 x dx$.

$$\int \frac{\sec^2 x}{\tan x} dx = \int \frac{(\tan x)'}{\tan x} dx = \ln|\tan x| + C$$

Answer: $\int \frac{\sec^2 x}{\tan x} dx = \ln|\tan x| + C$

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15. Evaluate the definite integral.

$$\int_2^3 \frac{6x}{3x^2 + 1} dx$$

$\int \frac{dx}{x} = \ln|x| + C$ and $\int \frac{du}{u} = \ln|u| + C$

Let $u = 3x^2 + 1$ and $du = 6x dx$. Then, since the integral is a definite integral, change the integration limits so they are in terms of u: $u(2) = 3(2)^2 + 1 = 13$ and $u(3) = 3(3)^2 + 1 = 28$. Therefore,

$$\int_2^3 \frac{6x}{3x^2 + 1} dx = \int_{13}^{28} \frac{du}{u} = \ln(u)\,\Big|_{13}^{28} = \ln 28 - \ln 13$$

Answer: $\int_2^3 \frac{6x}{3x^2+1} dx = \ln 28 - \ln 13$

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16. Evaluate $\int(-\cot x)dx$.

$\int \frac{dx}{x} = \ln|x| + C$ and $\int \frac{du}{u} = \ln|u| + C$

Since $\cot x = \frac{\cos x}{\sin x}$, let $u = \sin x$ and $du = \cos x$. Therefore,

$$\int(-\cot x)dx = \int -\left(\frac{\cos x}{\sin x}\right)dx = -\int\left(\frac{\cos x}{\sin x}\right)dx = -\int\frac{du}{u} = -\ln|u| + C$$

$$-\ln|u| + C = -\ln|\sin x| + C$$

Answer: $\int(-\cot x)dx = -\ln|\sin x| + C$

17. Evaluate $\int(\tan x)dx$.

Change the integrand from $\tan x$ to $\frac{\sin x}{\cos x}$. Then, let $u = \cos x$ and $du = -\sin x dx$.

Now, integrate using the rule: $\int \frac{dx}{x} = \ln|x| + C$ and $\int \frac{du}{u} = \ln|u| + C$

$$\int(\tan x)dx = \int\frac{\sin x}{\cos x}dx$$

The integral needs a negative inside so multiply the integral by a double negative:

$$\int\frac{\sin x}{\cos x}dx = -\int\frac{-\sin x}{\cos x}dx = -\int\frac{(\cos x)'}{\cos x}dx = -\ln|\cos x| + C$$

Answer: $\int(\tan x)dx = -\ln|\cos x| + C$

18. Evaluate the definite integral.

$$\int_{3}^{4} \frac{4x-2}{2x^2-2x+3}\,dx$$

$\int \frac{dx}{x} = \ln|x| + C$ and $\int \frac{du}{u} = \ln|u| + C$

Therefore, let $u = 2x^2 - 2x + 3$ and $du = 4x - 2$. Then, since the integral is a definite integral, change the integration limits so they are in terms of u.

$u(3) = 2(3)^2 - 2(3) + 3 = 15$ and $u(4) = 2(4)^2 - 2(4) + 3 = 27$

Therefore,

$$\int_{3}^{4} \frac{4x-2}{2x^2-2x+3}\,dx = \int_{15}^{27} \frac{du}{u} = \ln u\Big|_{15}^{27} = \ln 27 - \ln 15$$

Answer:

$$\int_{3}^{4} \frac{4x-2}{2x^2-2x+3}\,dx = \ln 27 - \ln 15$$

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19. Evaluate the definite Integral.

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} 5\tan x \, dx$$

$\int \frac{dx}{x} = \ln|x| + C$ and $\int \frac{du}{u} = \ln|u| + C$

Since $\tan x = \frac{\sin x}{\cos x}$, let $u = \cos x$ and $du = -\sin x dx$. Then since the integral is a definite integral, change the integration limits so they are in terms of u.

$u\left(\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$ and $u\left(\frac{\pi}{3}\right) = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$ Therefore,

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} 5\tan x \, dx = 5\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\sin x}{\cos x} dx = -5\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{(\cos x)'}{\cos x} dx = -5\int_{\frac{\sqrt{2}}{2}}^{\frac{1}{2}} \frac{du}{u}$$

$$-5\int_{\frac{\sqrt{2}}{2}}^{\frac{1}{2}} \frac{du}{u} = -5\ln u\Big|_{\frac{\sqrt{2}}{2}}^{\frac{1}{2}} = -5\ln\left(\frac{1}{2}\right) + 5\ln\left(\frac{\sqrt{2}}{2}\right)$$

Answer: $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} 5\tan x \, dx = -5\ln\left(\frac{1}{2}\right) + 5\ln\left(\frac{\sqrt{2}}{2}\right)$

20. Evaluate the definite integral.

$$\int_{0}^{2} \frac{2x+8}{x^2+8x+5}\,dx$$

$\int \frac{dx}{x} = \ln|x| + C$ and $\int \frac{du}{u} = \ln|u| + C$

Let $u = x^2 + 8x + 5$ and $du = (2x+8)dx$. Then, because the integral is a definite integral, change the integration limits so they are in terms of u.

$u(0) = 0^2 + 8(0) + 5 = 5$ and $u(2) = 2^2 + 8(2) + 5 = 25$. Therefore,

$$\int_{0}^{2} \frac{2x+8}{x^2+8x+5}\,dx = \int_{5}^{25} \frac{du}{u} = \ln 25 - \ln 5$$

$$= \ln\left(\frac{25}{5}\right) = \ln(5)$$

Answer: $\ln(5)$