

## 5.2 Integration w/s with ln and u-sub

1.  $\int \frac{2x}{\sqrt{x^2-1}} dx$       $u = x^2 - 1$   
 $du = 2x dx$

$$\int \frac{1}{\sqrt{u}} du$$

$$\int u^{-1/2} du$$

$$2u^{1/2} + C$$

$$\boxed{2(x^2-1)^{1/2} + C}$$

3.  $\int \frac{\sqrt{x} - \sqrt[5]{x}}{x} dx$

$$\int \left( \frac{x^{1/2}}{x} - \frac{x^{1/5}}{x} \right) dx$$

$$\int (x^{-1/2} - x^{-4/5}) dx$$

$$\boxed{2x^{1/2} - 5x^{1/5} + C}$$

5.  $\int \frac{x}{x+1} dx$       $u = x+1$   
 $du = dx$

$$\int \frac{u-1}{u} du \quad x = u-1$$

$$\int 1 du - \int \frac{1}{u} du$$

$$u - \ln|u| + C$$

$$\boxed{(x+1) - \ln|x+1| + C} \quad (+1)$$

or  
 $x - \ln|x+1| + C$  ←  $C$   
is a new C

2.  $\int \sec x \tan x dx$

$$\boxed{\sec x + C}$$

4.  $\int \frac{(x^2-2)}{\sqrt[3]{x}} dx$

$$\int \left( \frac{x^2}{x^{1/3}} - \frac{2}{x^{1/3}} \right) dx$$

$$\int (x^{5/3} - 2x^{-1/3}) dx$$

$$\boxed{\frac{5}{14} x^{14/5} - \frac{5}{2} x^{4/5} + C}$$

6.  $\int \frac{2x}{\sqrt{1+x}} dx$       $u = 1+x$   
 $du = dx$

$$\int \frac{2u-2}{u^{1/2}} du \quad x = u-1$$

$$\int \left( \frac{2u}{u^{1/2}} - \frac{2}{u^{1/2}} \right) du$$

$$\int (2u^{1/2} - 2u^{-1/2}) du$$

$$\frac{4}{3} u^{3/2} - 4u^{1/2} + C$$

$$\boxed{\frac{4}{3} (1+x)^{3/2} - 4(1+x)^{1/2} + C}$$

$$7. \int \sin 3x \, dx \quad \begin{array}{l} u = 3x \\ du = 3 \, dx \end{array}$$

$$\frac{1}{3} \int \sin u \, du$$

$$\frac{1}{3} (-\cos u) + C$$

$$\boxed{-\frac{1}{3} \cos 3x + C}$$

$$9. \int \sec x \, dx$$

$$\boxed{\ln |\sec x + \tan x| + C}$$

$$11. \int \frac{8}{3+4x} \, dx$$

(2)

$$u = 3+4x$$

$$du = 4 \, dx$$

$$8 \int \frac{1}{u} \, dx$$

$$\frac{1}{4} \cdot 8 \int \frac{4}{u} \, dx$$

$$2 \int \frac{1}{u} \, du \quad (+1)$$

$$2 \ln |u| + C \quad (+1)$$

$$\boxed{\ln (3+4x)^2 + C}$$

$$2 \ln |3+4x| + C$$

$$2x - \frac{3}{2} \ln |4x+3| + C$$

$$8. \int x^2 \sqrt{1-2x^3} \, dx \quad \begin{array}{l} u = 1-2x^3 \\ du = -6x^2 \, dx \end{array}$$

$$-\frac{1}{6} \int -6x^2 \sqrt{u} \, dx$$

$$-\frac{1}{6} \int u^{1/2} \, du$$

$$-\frac{1}{6} \cdot \frac{2}{3} u^{3/2} + C$$

$$\boxed{-\frac{1}{9} (1-2x^3)^{3/2} + C}$$

$$10. \int \frac{x+1}{x} \, dx$$

$$\int 1 \, dx + \int \frac{1}{x} \, dx$$

$$\boxed{x + \ln |x| + C}$$

$$12. \int \frac{8x}{3+4x} \, dx$$

(3)

$$u = 3+4x$$

$$du = 4 \, dx$$

$$\int \frac{2x \cdot 4 \, dx}{u} \quad \frac{u-3}{4} = x$$

$$\int \frac{2 \left( \frac{u-3}{4} \right)}{u} \, du$$

$$\frac{1}{2} \int \frac{u-3}{u} \, du \quad (+1)$$

$$\frac{1}{2} \int 1 \, du - 3 \int \frac{1}{u} \, du \quad (+1)$$

$$\frac{1}{2} u - \frac{3}{2} \ln |u| + C$$

$$\boxed{\frac{1}{2} (3+4x) - \frac{3}{2} \ln |3+4x| + C}$$

$$13. \int \frac{8}{\sqrt{3+4x}} dx$$

$$u = 3+4x \\ du = 4 dx$$

$$\int \frac{4 dx}{\sqrt{u}}$$

$$2 \int u^{-1/2} du$$

$$2 \cdot 2 u^{1/2} + C$$

$$\boxed{4(3+4x)^{1/2} + C}$$

$$15. \int \frac{2}{(x+1)^2} dx$$

$$u = x+1 \\ du = dx$$

$$2 \int \frac{1}{u^2} du$$

$$2 \int u^{-2} du$$

$$-2 u^{-1} + C$$

$$\boxed{\frac{-2}{x+1} + C}$$

$$17. \int 9x^2 (3x^3-3)^{-2} dx$$

$$u = 3x^3-3 \\ du = 9x^2 dx$$

$$\int u^{-2} du$$

$$-u^{-1} + C$$

$$\boxed{\frac{-1}{(3x^3-3)} + C}$$

$$14. \int \frac{3x^3-6x+5}{x+2} dx$$

$$\int (3x^2-6x+6) dx - 7 \int \frac{1}{x+2} dx$$

$$\boxed{x^3 - 3x^2 + 6x - 7 \ln|x+2| + C}$$

$$16. \int \frac{1}{-2x-3} dx$$

$$u = -2x-3 \\ du = -2 dx$$

$$-\frac{1}{2} \int \frac{-2}{u} dx$$

$$-\frac{1}{2} \int \frac{1}{u} du$$

$$-\frac{1}{2} \int \frac{1}{u} du$$

$$-\frac{1}{2} \ln|u| + C$$

$$\boxed{-\frac{1}{2} \ln|-2x-3| + C}$$

$$18. \int \frac{\ln(x)^3}{x} dx$$

$$u = \ln x \\ du = \frac{1}{x} dx$$

$$\int 3 \ln x \left( \frac{1}{x} dx \right)$$

$$\int 3u du$$

$$\frac{3}{2} u^2 + C$$

$$\boxed{\frac{3}{2} (\ln x)^2 + C}$$

$$\text{or } \frac{(\ln(x)^3)^2}{6} + C$$

$$\begin{aligned} & (3 \ln x) (3 \ln x) \\ & (3) 2 \cdot 3 \\ & \rightarrow \frac{3}{2} (\ln x)^2 \end{aligned}$$

$$19. \int \frac{(1 + \ln x)^2}{x} dx \quad u = 1 + \ln x \\ du = \frac{1}{x} dx$$

$$\int u^2 du \quad (+1)$$

$$\frac{1}{3} u^3 + C \quad (+1)$$

$$\frac{1}{3} (1 + \ln x)^3 + C$$

$$21. \int e^{-x} \sqrt{7 + e^{-x}} dx \quad u = 7 + e^{-x} \\ du = -e^{-x} dx$$

(2)

$$-\int u^{1/2} du$$

$$-\frac{2}{3} u^{3/2} + C \quad (+2)$$

$$-\frac{2}{3} (7 + e^{-x})^{3/2} + C$$

$$23. \int \frac{e^x + e^{-x}}{e^x - e^{-x}} dx \quad u = e^x - e^{-x} \\ du = e^x + e^{-x} dx$$

(2)

$$\int \frac{1}{u} du \quad (+1)$$

$$\ln |u| + C \quad (+1)$$

$$\ln |e^x - e^{-x}| + C$$

$$20. \int -\tan x dx$$

$$\ln |\cos x| + C$$

$$22. \int \frac{6e^{2x} - e^x}{e^x} dx$$

$$\int (6e^x - 1) dx$$

$$= 6e^x - x + C$$

$$24. \int 5x^2 e^{6x^3} dx$$

$$u = 6x^3 \\ du = 18x^2 dx$$

$$\frac{5}{18} \int e^u du$$

$$\frac{5}{18} e^u + C$$

$$\frac{5}{18} e^{6x^3} + C$$